Binary Frequency Shift Keying (BFSK): Signal Representation

In binary FSK, the frequency of the carrier signal is varied to represent the binary digits and 0 by two distinct frequencies. The amplitude and frequency remain constant during each bit interval.

Signal Representation (coherent FSK)

Send: $s_1(t) = Acos(2\pi(f_c + \Delta f))t)$ if the information bit is "1";

Send: $s_2(t) = Acos(2\pi(f_c - \Delta f))t)$ if the information bit is "0";

Δf is an offset frequency (from the unmodulated carrier f_c) chosen so that $s_1(t)$ and $s_2(t)$ are orthogonal, i.e.,

$$\int_0^{\tau} s_1(t)s_2(t)dt = 0$$

$$\frac{sin(2\pi(2f_c)\tau}{2f_c} + \frac{sin(2\pi(2\Delta f)\tau}{2\Delta f} = 0$$

Binary Frequency Shift Keying (BFSK): Signal Representation

Orthogonality condition

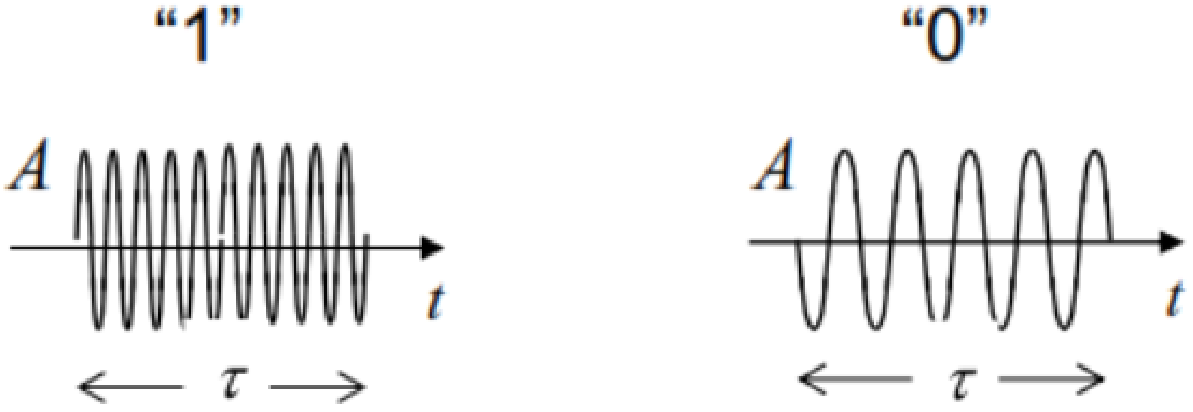
$$\frac{sin(2\pi(2f_c)\tau}{2f_c} + \frac{sin(2\pi(2\Delta f)\tau}{2\Delta f} = 0$$

$$2f_c = \frac{n}{2\tau} = \frac{nR_b}{2}, n = 1, 2, \ldots \quad \boldsymbol{f_c = \frac{nR_b}{4} = kR_b}$$

$$2\Delta f = \frac{mR_b}{2}, m = 1, 2, \ldots \quad \boldsymbol{\Delta f = \frac{mR_b}{4}}$$

Note that $sin(x) = 0$ when $x = n\pi$

The minimum frequency separation $\mathbf{2\Delta f = R_b/2.}$



$s_1(t) = A\cos(2\pi(f_c + \Delta f)t)$ $\quad$ $s_2(t) = A\cos(2\pi(f_c - \Delta f)t)$

$0 \le t \le \tau$ $\quad$ $0 \le t \le \tau$

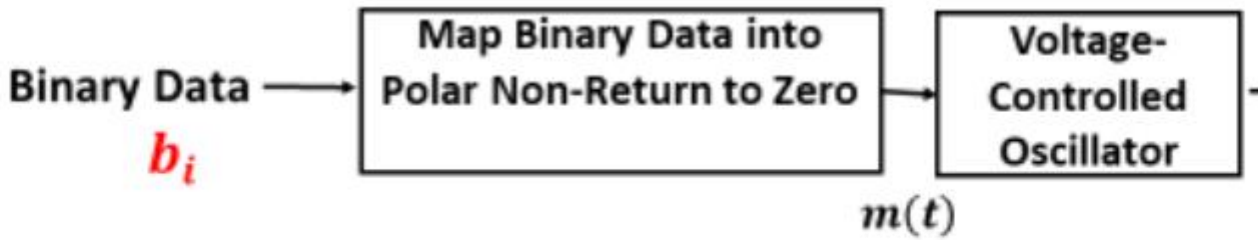
(τ is an integer number of $1/(f_c \pm \Delta f)$)

τ: is the time allocated to transmit the binary digit.

T_c=$1/f_c$ is the carrier period

$R_b = \frac{1}{\tau}$: $Data\ rate$ bits/sec

Binary FSK : Generation using the Single Oscillator Method

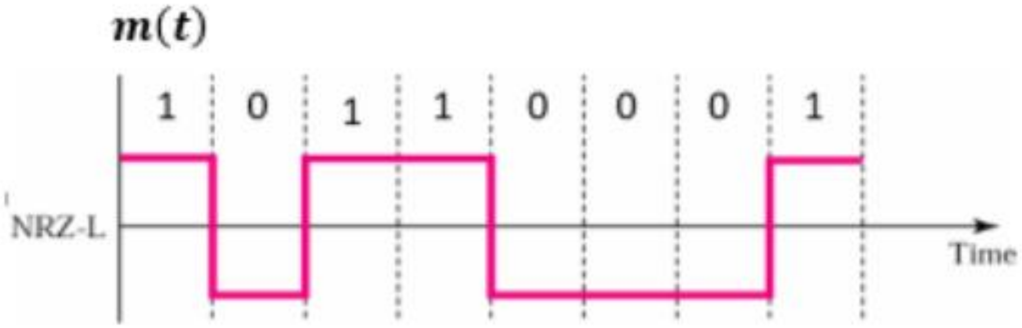


$f_1(t) = f_c + \Delta f; \text{for digit } 1$

$f_2(t) = f_c - \Delta f; \text{for digit } 0$

m(t): Polar non-return to zero

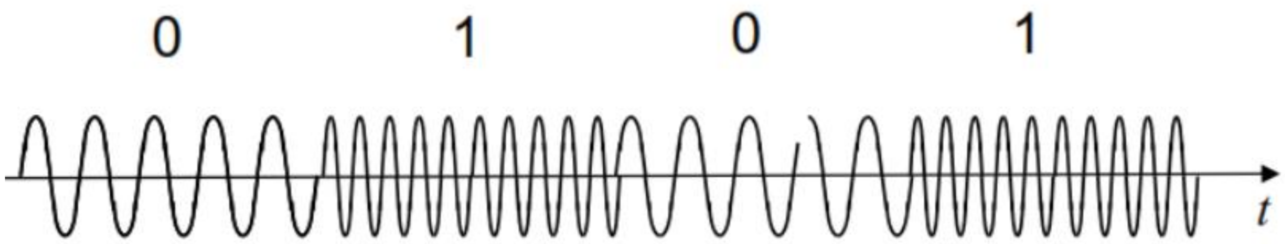
$$f_i(t) = f_c + k_f m(t); \text{for } VCO$$



Polar Non-return to Zero

$s_1(t) = A\cos(2\pi(f_c + \Delta f))t)$

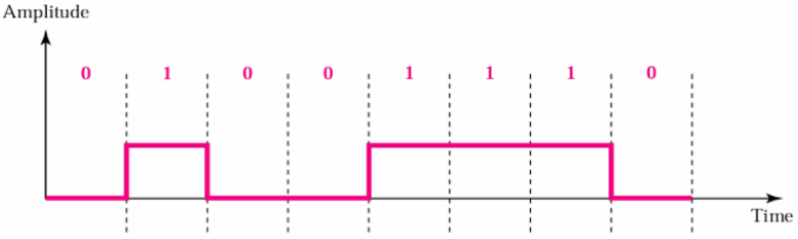
$s_2(t) = A\cos(2\pi(f_c - \Delta f))t)$



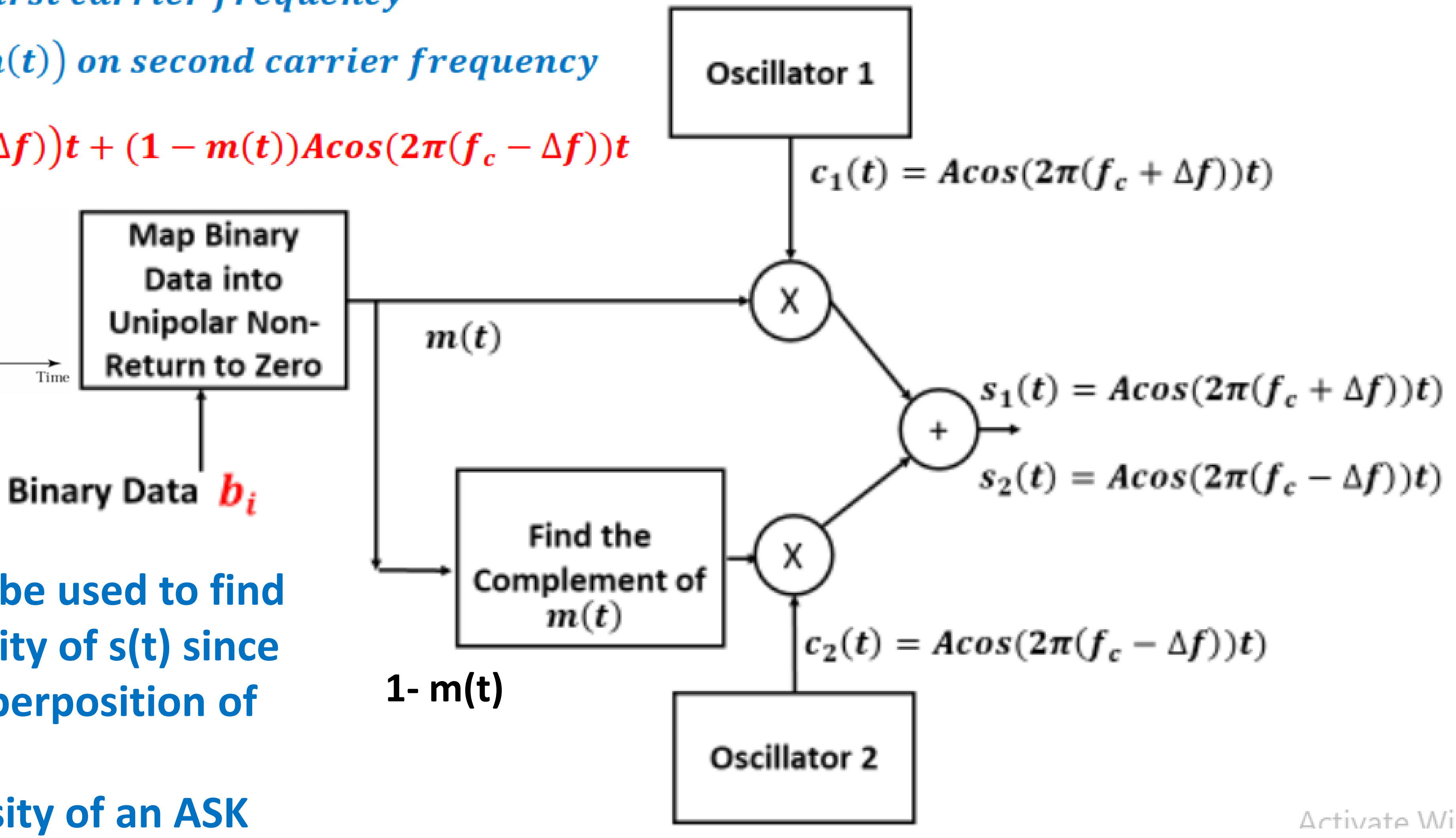
Binary FSK : Generation using the Two-oscillator Method

$$s_{BFSK}(t) = ASK\ of\ m(t)\ on\ first\ carrier\ frequency$$
$$+ ASK\ of\ (1 - m(t))\ on\ second\ carrier\ frequency$$

$$s_{BFSK}(t) = m(t)A\cos\big(2\pi(f_c + \Delta f)\big)t + (1 - m(t))A\cos(2\pi(f_c - \Delta f))t$$



Unipolar Non-Return to Zero Signal

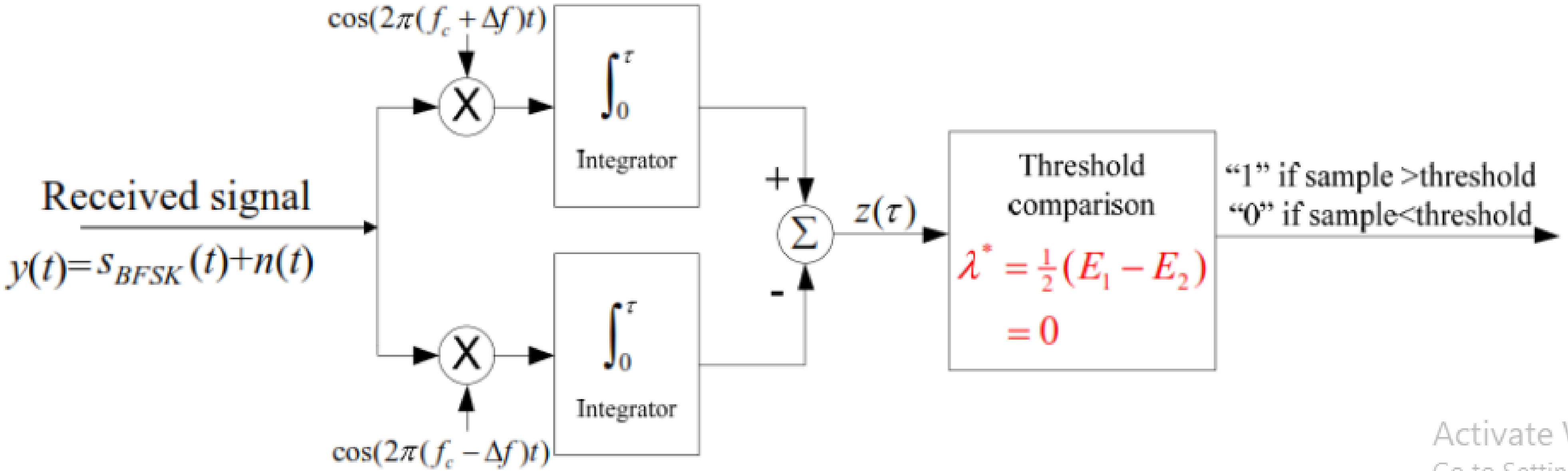


- **This representation will be used to find the power spectral density of s(t) since it is envisaged as the superposition of two ASK signals.**
- **The power spectral density of an ASK signal was derived in a previous video titled: Binary ASK**

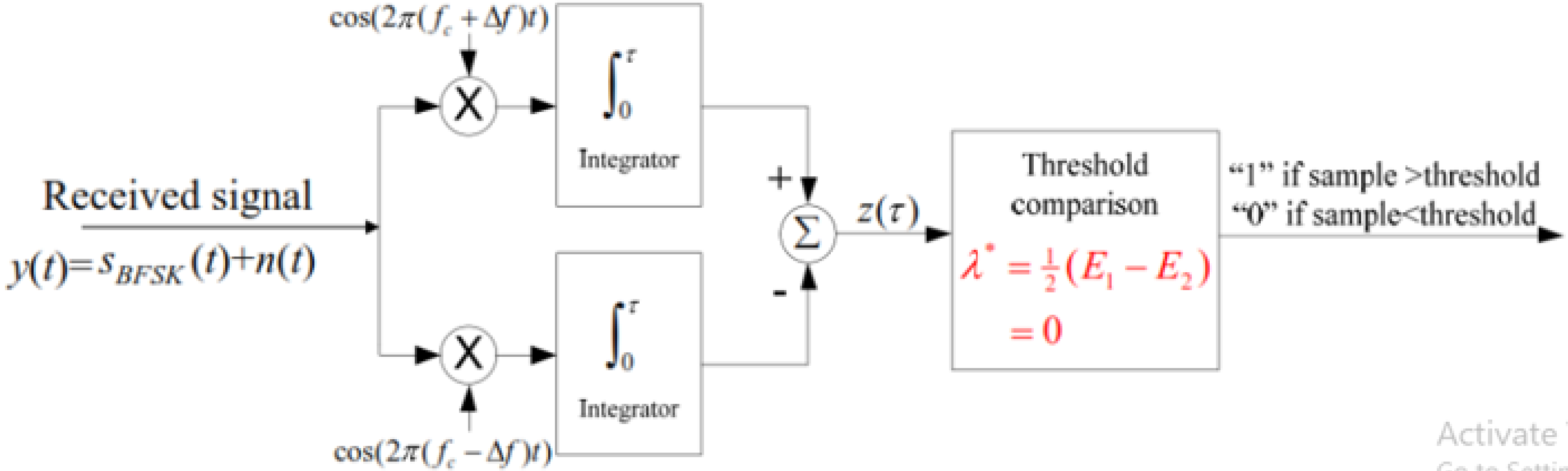
FSK: modeled as a sum of two ASK signals

Binary FSK : Coherent Demodulation

The optimum coherent receiver consists of two correlators. The operation of the receiver makes use of the orthogonality condition imposed on the signals $s_1(t)$ and $s_2(t)$. In the absence of noise, if $s_1(t)$ is received, then the ouput of the upper correlator will have a value greater than zero, while the output of the lower correlator is zero. The converse is true when $s_2(t)$ is received. In the presence of noise, the system decides 1 when $z(\tau) > 0$. That is, when the output of the upper correlator is greater than the output of the lower one. Otherwise, it decides 0.



Binary FSK : Probability of Error



Probability of Error

Energy of $s_i(t)$: $E_1 = E_2 = \frac{1}{2}A^2\tau$

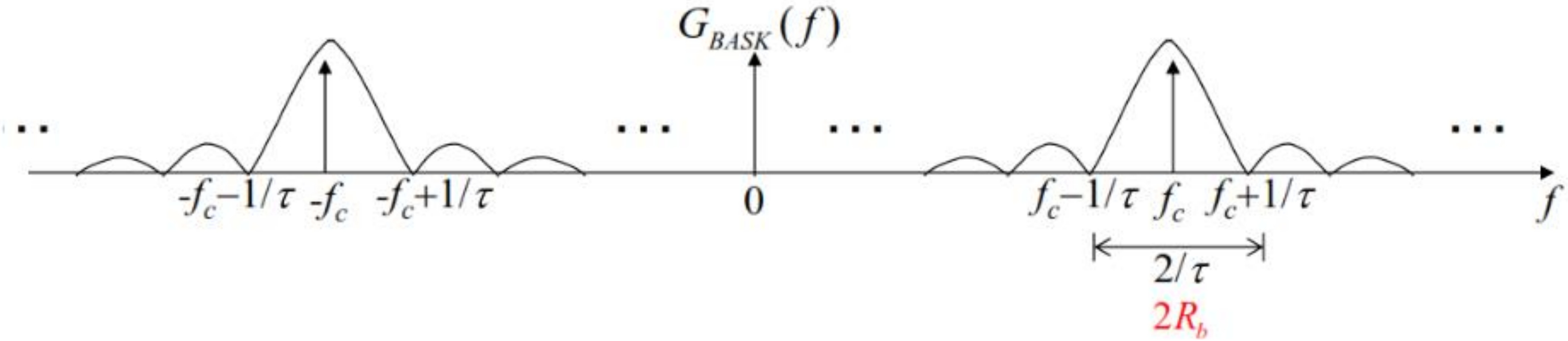
Average Energy per bit: $E_b = \frac{1}{2}(E_1 + E_2) = \frac{1}{2}A^2\tau$

When the signals are orthogonal, i.e., when $\int_0^\tau s_1(t)s_2(t)dt = 0$, the probability of error is given by

$$P_b^* = Q\left(\sqrt{\frac{A^2\tau}{2N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Binary FSK : Power Spectral Density

Since the FSK signal is the superposition of two ASK signals on two orthogonal frequencies, the spectrum is also the superposition of that of the ASK signals. We recall that the spectrum of the ASK signal is as shown below

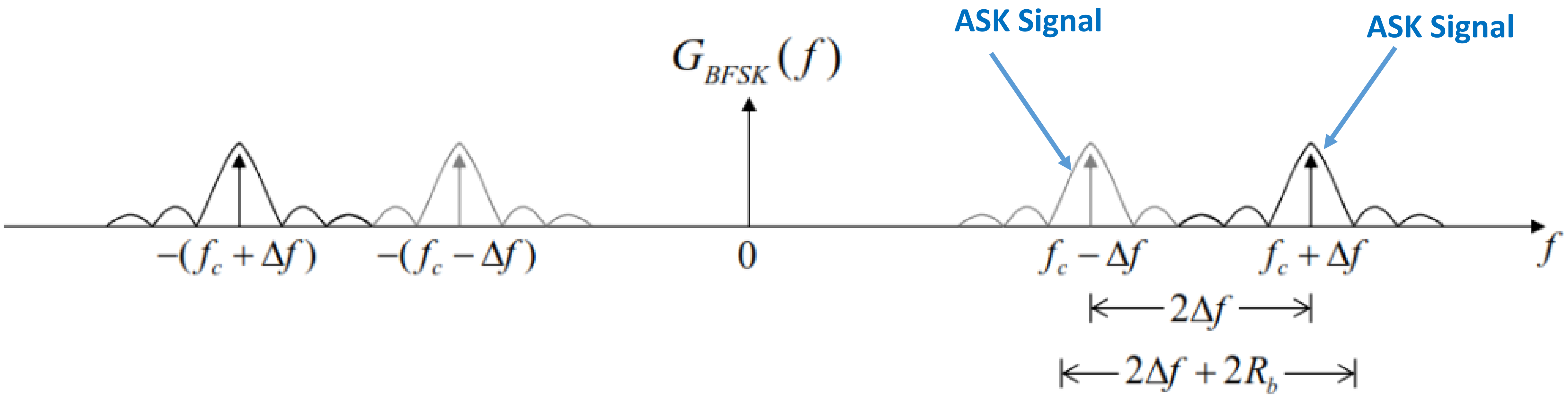


$$s_{BFSK}(t) = ASK\ of\ m(t)\ on\ first\ carrier\ frequency$$

$$+\ ASK\ of\ \left(1 - m(t)\right)\ on\ second\ carrier\ frequency$$

$$s_{BFSK}(t) = m(t)A\cos\left(2\pi(f_c + \Delta f)\right)t + (1 - m(t))A\cos(2\pi(f_c - \Delta f))t$$

Binary FSK : Power Spectral Density and Bandwidth

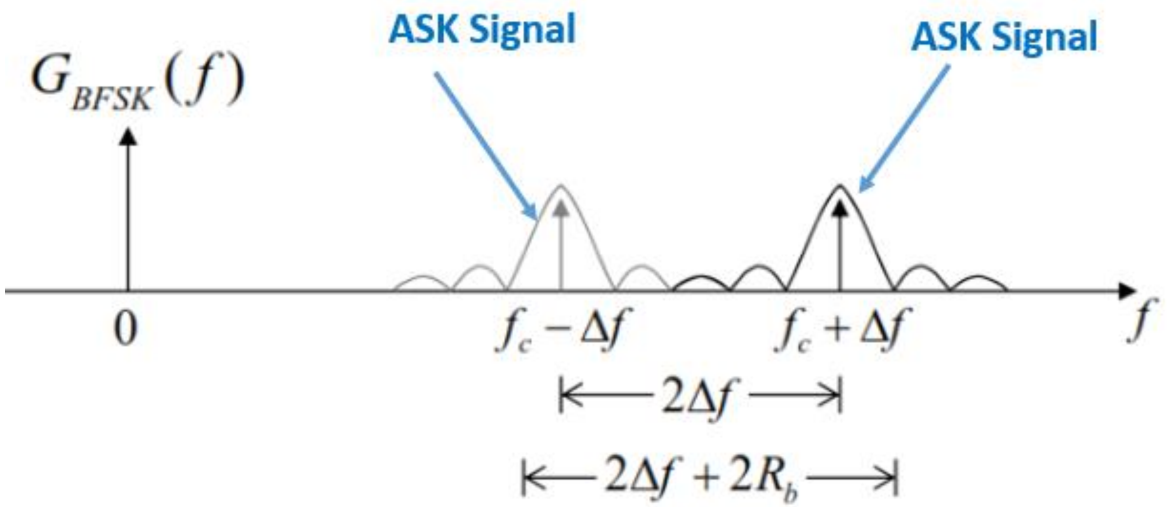


The required channel bandwidth for 90% in-band power

$$B_{h\_90\%} = 2\Delta f + 2R_b$$

$$\boldsymbol{B.W = (f_1 - f_2) + 2R_b = \frac{R_b}{2} + 2R_b}$$

Binary FSK : Non-coherent Demodulation



$$\mathbf{r(t)} = Acos(2\pi(f_c + \Delta f\,)t) + n(t)$$

$$\mathbf{r(t)} = Acos(2\pi(f_c - \Delta f\,)t) + n(t)$$