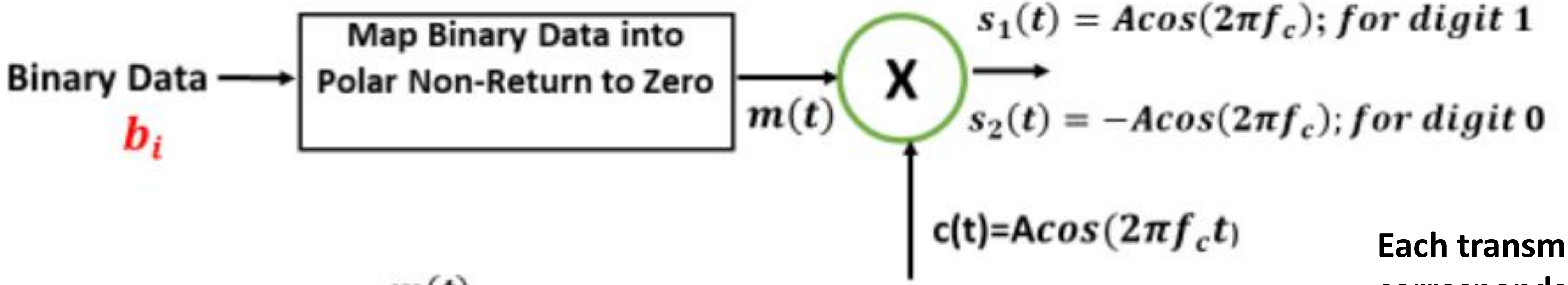
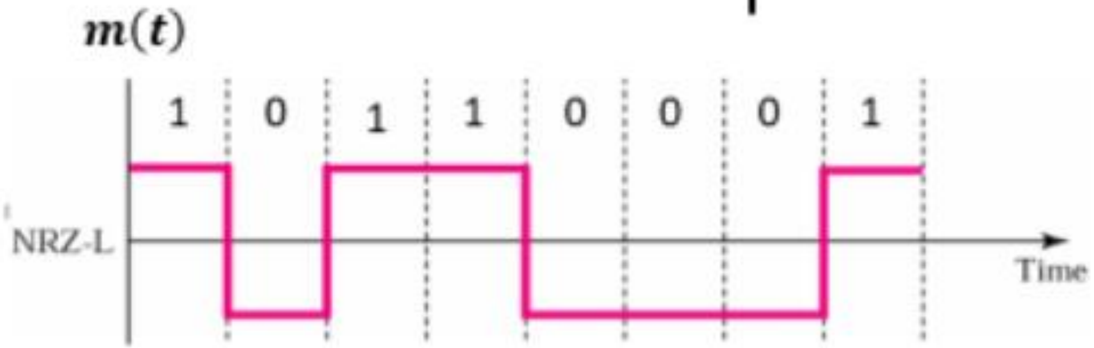
Quadri-phase Shift Keying (QPSK)

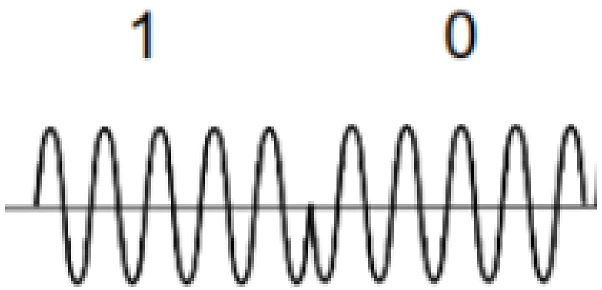
Review: Binary Phase Shift Keying Modulation and Demodulation



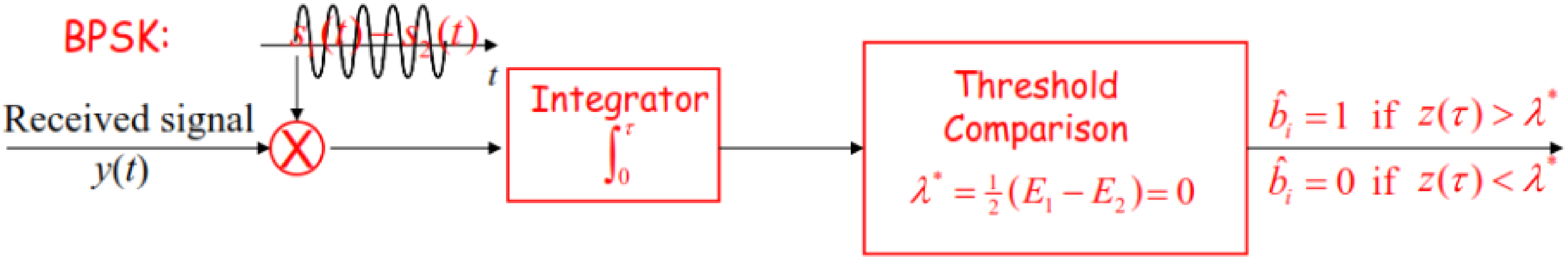
Each transmitted signal corresponds to one binary digit



Polar Non-return to Zero



$$s_1(t) - s_2(t) = 2A\cos(2\pi f_c)t$$



Quadri-phase Shift Keying (QPSK): Signal Representation

"1 1" $s_1(t) = A\cos(2\pi f_c t - \pi/4) = +\frac{A}{\sqrt{2}}\cos(2\pi f_c t) + \frac{A}{\sqrt{2}}\sin(2\pi f_c t)$

"1 0" $s_2(t) = A\cos(2\pi f_c t + \pi/4) = +\frac{A}{\sqrt{2}}\cos(2\pi f_c t) - \frac{A}{\sqrt{2}}\sin(2\pi f_c t)$

"0 0" $s_3(t) = A\cos(2\pi f_c t + 3\pi/4) = -\frac{A}{\sqrt{2}}\cos(2\pi f_c t) - \frac{A}{\sqrt{2}}\sin(2\pi f_c t)$

"0 1" $s_4(t) = A\cos(2\pi f_c t + 5\pi/4) = -\frac{A}{\sqrt{2}}\cos(2\pi f_c t) + \frac{A}{\sqrt{2}}\sin(2\pi f_c t)$

$$s_{QPSK}(t) = d_I \frac{A}{\sqrt{2}}\cos(2\pi f_c t) + d_Q \frac{A}{\sqrt{2}}\sin(2\pi f_c t)$$

$$d_I = \begin{cases} 1 & \text{if } b_{2i-1} = 1 \\ -1 & \text{if } b_{2i-1} = 0 \end{cases}$$

Odd bits

Even bits

$$d_Q = \begin{cases} 1 & \text{if } b_{2i} = 1 \\ -1 & \text{if } b_{2i} = 0 \end{cases}$$

$$s_{QPSK}(t) = A_k\cos(2\pi f_c t) + B_k\sin(2\pi f_c t)$$

$$s_{QPSK}(t) = Binary\ PSK\ on\ \cos(2\pi f_c t) + Binary\ PSK\ on\ \sin(2\pi f_c t)$$

- **In this type of modulation two binary digits are grouped together to form one message that phase modulates the carrier $A\cos(2\pi f_c t)$.**
- **The transmitted signal assumes one of four possible phases (+ 45, -45, +135, -135) $A\cos(2\pi f_c t + \theta_i)$**
- **A QPSK signal can be decomposed into a sum of two PSK signals; an in-phase component and a quadrature component. The serial to parallel converter splits the incoming data sequence into two sequences that consist of the odd and even bits of the main sequence. The odd bit stream sequence modulates the in-phase carrier, while the even bit stream sequence modulates the quadrature carrier.**

Quadri-phase Shift Keying (QPSK): Signal Representation

$$s_{QPSK}(t) = d_I \frac{A}{\sqrt{2}} \cos(2\pi f_c t) + d_Q \frac{A}{\sqrt{2}} \sin(2\pi f_c t)$$

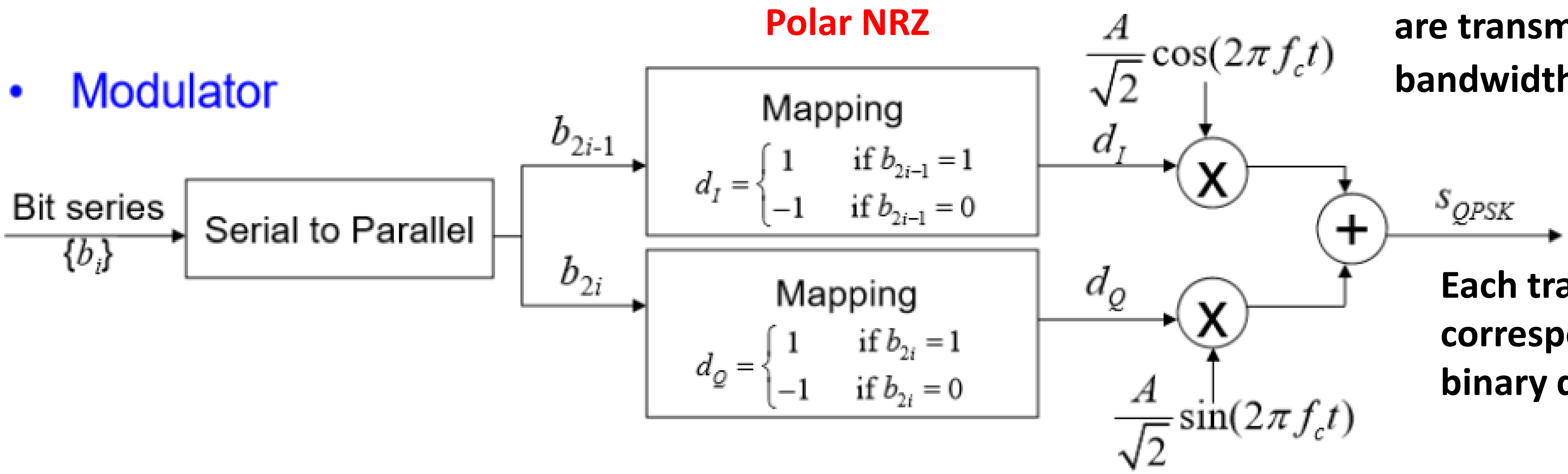
$$d_I = \begin{cases} 1 & \text{if } b_{2i-1} = 1 \\ -1 & \text{if } b_{2i-1} = 0 \end{cases} \qquad d_Q = \begin{cases} 1 & \text{if } b_{2i} = 1 \\ -1 & \text{if } b_{2i} = 0 \end{cases}$$

$$s_{QPSK}(t) = A_k cos(2\pi f_c t) + B_k sin(2\pi f_c t)$$

$$s_{QPSK}(t) = Binary\ PSK\ on\ cos(2\pi f_c t) + Binary\ PSK\ on\ sin(2\pi f_c t)$$

- **The serial to parallel converter splits the incoming data sequence into two sequences that consist of the odd (A_k) and even bits (B_k) of the main sequence. The odd bit stream sequence modulates the in-phase carrier, while the even bit stream sequence modulates the quadrature carrier.**
- **Both in phase and quadrature BPSK are transmitted over the same bandwidth.**

- Modulator



Each transmitted signal corresponds to two binary digits

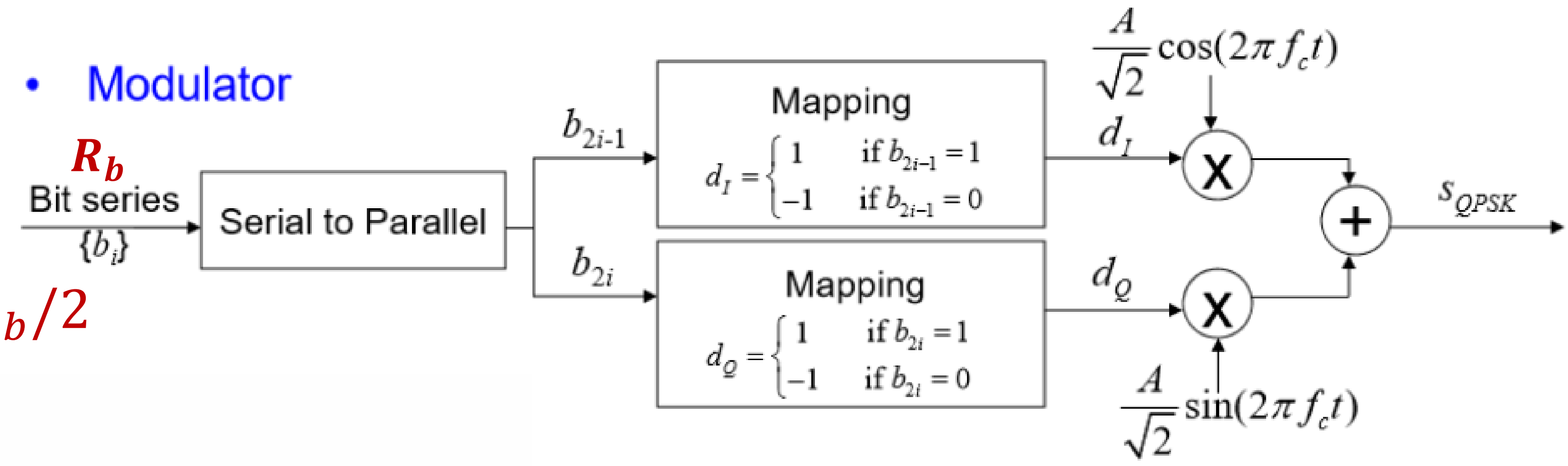
Quadri-phase Shift Keying (QPSK): Modulation

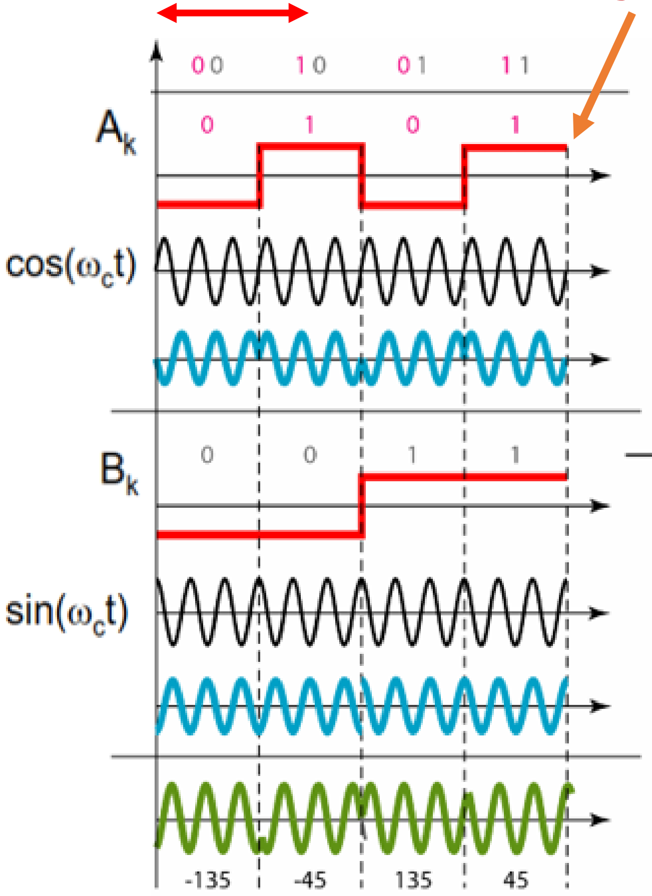
One message (or symbol) consists of two bits

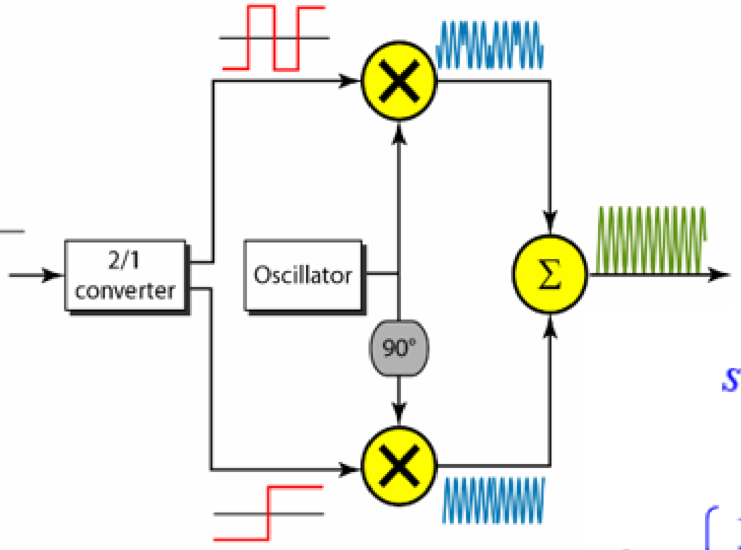
$$T_s = 2T_b$$

$$R_s = R_b/2$$

- Modulator







Message duration $T_s = 2T_b$

Message Rate $R_s = R_b/2$

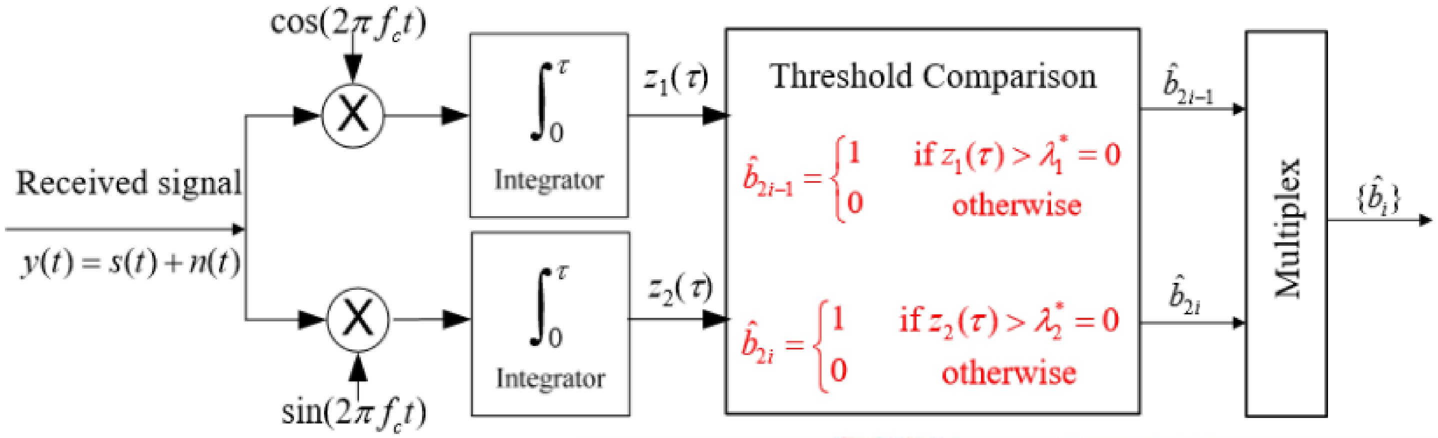
$$s_{QPSK}(t) = d_I \frac{A}{\sqrt{2}}\cos(2\pi f_c t) + d_Q \frac{A}{\sqrt{2}}\sin(2\pi f_c t)$$

$$d_I = \begin{cases} 1 & \text{if } b_{2i-1} = 1 \\ -1 & \text{if } b_{2i-1} = 0 \end{cases} \qquad d_Q = \begin{cases} 1 & \text{if } b_{2i} = 1 \\ -1 & \text{if } b_{2i} = 0 \end{cases}$$

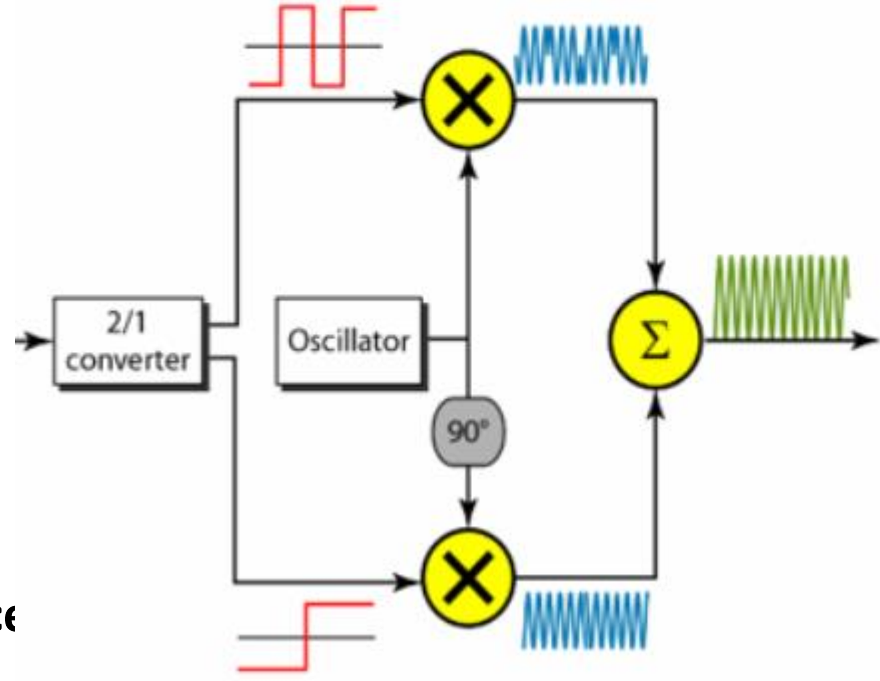
Quadri-phase Shift Keying (QPSK): Demodulation

$$z_1(\tau) = \int_0^{\tau} [s_{QPSK}(t) + n(t)] cos(2\pi f_c t) dt$$

- $z_2(\tau) = \int_0^{\tau} [s_{QPSK}(t) + n(t)] sin(2\pi f_c t) dt$



- $z_1(\tau) = \int_0^{\tau} [A_k cos(2\pi f_c t) + B_k sin(2\pi f_c t) + n(t)] cos(2\pi f_c t) dt$
- $z_2(\tau) = \int_0^{\tau} [A_k cos(2\pi f_c t) + B_k sin(2\pi f_c t) + n(t)] sin(2\pi f_c t) dt$
- $z_1(\tau) = \frac{A_k}{2}\tau + N_1 = \frac{Ad_I}{2\sqrt{2}}\tau + N_1$ signal component proportional to d_i
- $z_2(\tau) = \frac{B_k}{2}\tau + N_2; = \frac{Ad_Q}{2\sqrt{2}}\tau + N_2$ signal component proportional to d_q
- The odd and even bits can be recovered using the decision rules
- $d_I = \begin{cases} 1, & z_1(\tau) \geq 0 \\ -1, & z_1(\tau) < 0 \end{cases} \Rightarrow b_{2i-1} = \begin{cases} 1, & d_I = 1 \\ 0, & d_I = -1 \end{cases};$ Odd sequence
- $d_Q = \begin{cases} 1, & z_2(\tau) \geq 0 \\ -1, & z_2(\tau) < 0 \end{cases} \Rightarrow b_{2i} = \begin{cases} 1, & d_Q = 1 \\ 0, & d_Q = -1 \end{cases};$ Even sequence



QPSK: Probability of Error

- The symbol error probability is twice the bit error probability, and is given as (will also be derived in the next chapter when we consider M-ary PSK).

$$P_b^* = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{\frac{E_s}{N_0}}\right)$$

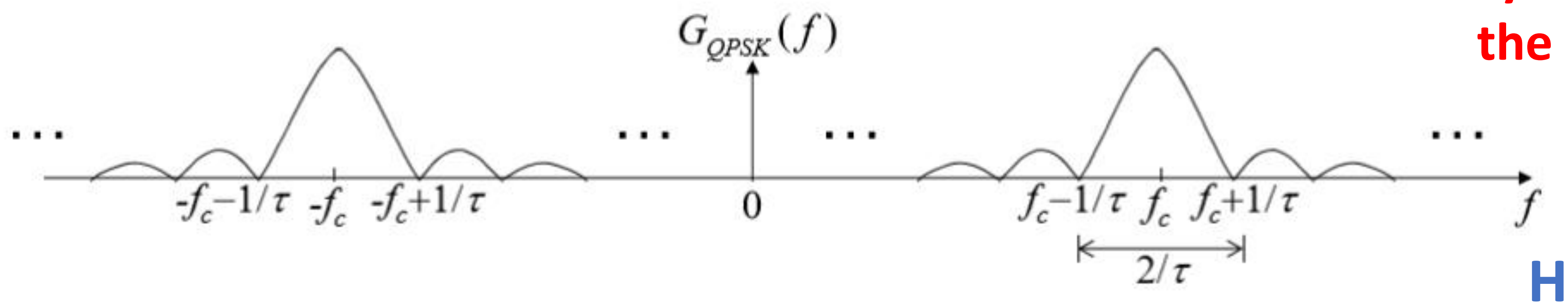
- This is the same as that for binary PSK, provided that both message bits have the same energy. The advantage of QPSK is that it is more bandwidth efficient than BPSK (**can transmit twice the data rate within the same bandwidth)**

QPSK: Power Spectral Density and Bandwidth

The power spectral density has the same shape as that for BPSK (the QPSK is the sum of two BPSK signals one modulated on $cos(2\pi f_c t)$ and the other on $sin(2\pi f_c t)$.

Remember that the symbol duration τ is twice the bit duration.

$$R_s = \frac{1}{\tau} = \frac{R_b}{2}.$$



$$s_{QPSK}(t) = A_k cos(2\pi f_c t) + B_k sin(2\pi f_c t)$$

Here, $\tau = T_s = 2T_b$

$$\text{B.W} = \frac{2}{\tau} = \frac{2}{2T_b} = R_b$$

$$s_{QPSK}(t) = Binary\ PSK\ on\ cos(2\pi f_c t) + Binary\ PSK\ on\ sin(2\pi f_c t)$$

Note that for QPSK, the data rate is R_b bits/sec and the B.W $= R_b$ Hz
Which means we can transmit $2R_b$ bits/sec and the B.W $= 2R_b$ Hz
While for regular BPSK, when the data rate is R_b bits/sec the B.W $= 2R_b$ Hz
Hence, QPSK is more bandwidth efficient than BPSK since in W Hz, we can transmit W bits/sec while in BPSK we can transmit half that value W/2 bits/sec for the same probability of error.