Amplitude Modulation Systems

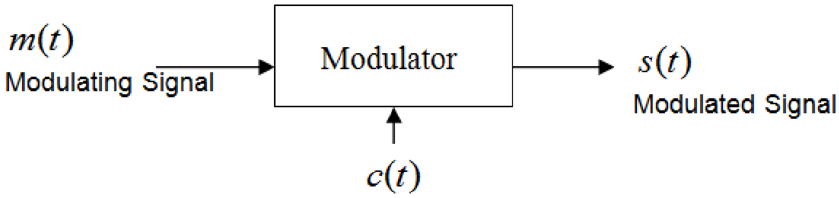
Modulation: is the process by which some characteristic of a carrier $c(t)$ is varied in accordance with a message signal $m(t)$.

Amplitude modulation is defined as the process in which the amplitude of the carrier $c(t)$ is varied linearly with $m(t)$. Four types of amplitude modulation will be considered in this chapter. These are normal amplitude modulation, double sideband suppressed carrier modulation, single sideband modulation, and vestigial sideband modulation.

A common form of the *carrier*, in the case of continuous wave modulation, is a sinusoidal signal of the form

$$c(t) = A_C \cos(2\pi f_C t + \phi)$$

The baseband (message) signal $m(t)$ is referred to as the *modulating signal* and the result of the modulation process is referred to as the *modulated signal* $s(t)$. The following block diagram illustrates the modulation process.



We should point out that modulation is performed at the transmitter and demodulation, which is the process of extracting $m(t)$ from $s(t)$, is performed at the receiver.

Normal Amplitude Modulation

A *normal AM* signal is defined as:

$$s(t) = A_C\left(1 + k_a m(t)\right)\cos 2\pi f_C t$$

where, k_a is the sensitivity of the AM modulator (units in 1/volt). $s(t)$ can be also be written in the form:

$$s(t) = A(t)\cos 2\pi f_C t$$

where, $A(t) = A_C + A_C k_a\, m(t)$. In this representation, we observe that $A(t)$ is related to $m(t)$ in a linear relationship of the form $y = a + bx$.

The *envelope* of $s(t)$ is defined as

$$|A(t)| = A_C|1 + k_a m(t)|$$

Notice that the envelope of $s(t)$ has the same shape as $m(t)$ provided that:

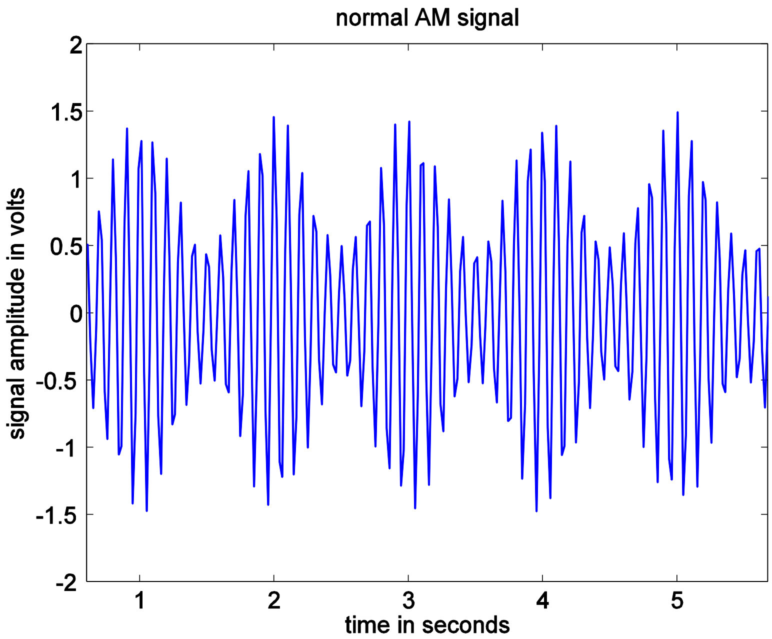
1. $|1 + k_a m(t)| \geq 0$ or, equivalently, $|k_a m(t)| \leq 1$. Over-modulation occurs when $|k_a m(t)| > 1$, resulting in envelope distortion.
2. $f_C >> w$, where w is bandwidth of $m(t)$. f_C has to be at least 10 w. This ensures the formation of an envelope, whose shape resembles the message signal.

Matlab Demonstration

The figure below shows the normal AM signal $s(t) = (1 + 0.5\cos 2\pi t)\cos 2\pi(10)t$

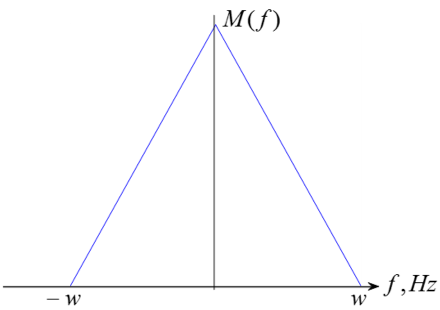
a. Make similar plots for the cases (μ = 0.5, 1, and 1.5)

b. Show the effect of f_C on the envelope. (Take f_C = 4 Hz, and f_C = 25Hz)



Spectrum of the Normal AM Signal

Let the Fourier transform of $m(t)$ be as shown (The B.W of $m(t)$ = w Hz).



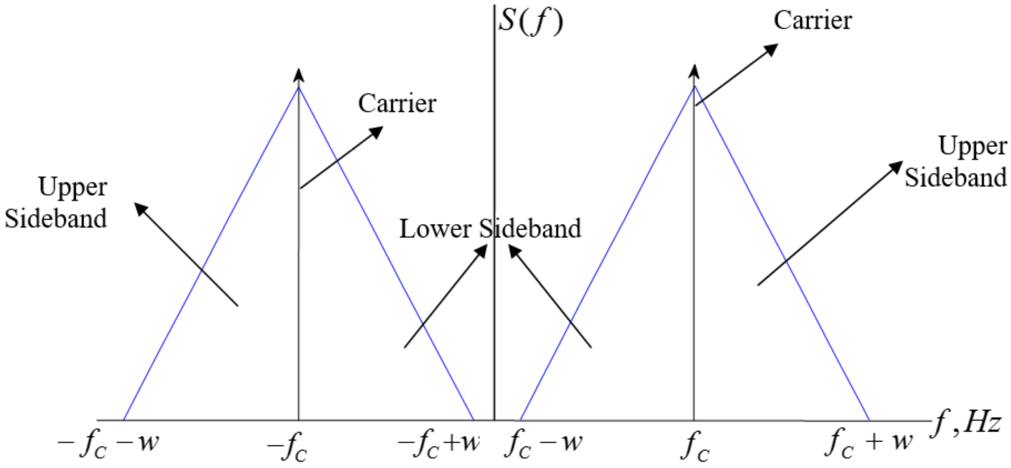
$$s(t) = A_C(1 + k_a m(t))\cos 2\pi f_C t \qquad \text{(dc + message)*carrier}$$

$$s(t) = A_C \cos 2\pi f_C t + A_C k_a m(t)\cos 2\pi f_C t \qquad \text{(carrier + message*carrier)}$$

Taking the Fourier transform, we get

$$S(f)=\frac{A_C}{2}\delta(f-f_C)+\frac{A_C}{2}\delta(f+f_C)+\frac{A_C k_a}{2}M(f-f_C)+\frac{A_C k_a}{2}M(f+f_C)$$

The spectrum of $s(t)$ is shown below



Remarks

a. The baseband spectrum $M(f)$, of the message has been shifted to the bandpass region centered around the carrier frequency f_C.

b. The spectrum $S(f)$ consists of two sidebands (upper sideband and lower sideband) and a carrier.

c. The transmission bandwidth of $s(t)$ is:

$$B.W=(f_C+w)-(f_C-w)=2w$$

Which is twice the message bandwidth.

Power Efficiency

The *power efficiency* of a normal AM signal is defined as:

$$\eta=\frac{power\ in\ the\ sidebands}{power\ in\ the\ sidebands+power\ in\ the\ carrier}$$

Now, we find the power efficiency of the AM signal for the single tone modulating signal $m(t)=A_m\cos(2\pi f_m t)$. Let $\mu=A_m k_a$, then $s(t)$ can be expressed as

$$s(t)=A_C(1+\mu\cos 2\pi f_m t)\cos 2\pi f_C t$$

$$s(t)=A_C\cos 2\pi f_C t+A_C\mu\cos 2\pi f_C t\cos 2\pi f_m t$$

$$s(t)=A_C\cos 2\pi f_C t+\frac{A_C\mu}{2}\cos 2\pi(f_C+f_m)t+\frac{A_C\mu}{2}\cos 2\pi(f_C-f_m)t$$

Carrier | Upper Sideband | Lower Sideband

Power in carrier $= \dfrac{A_C^{\ 2}}{2}$

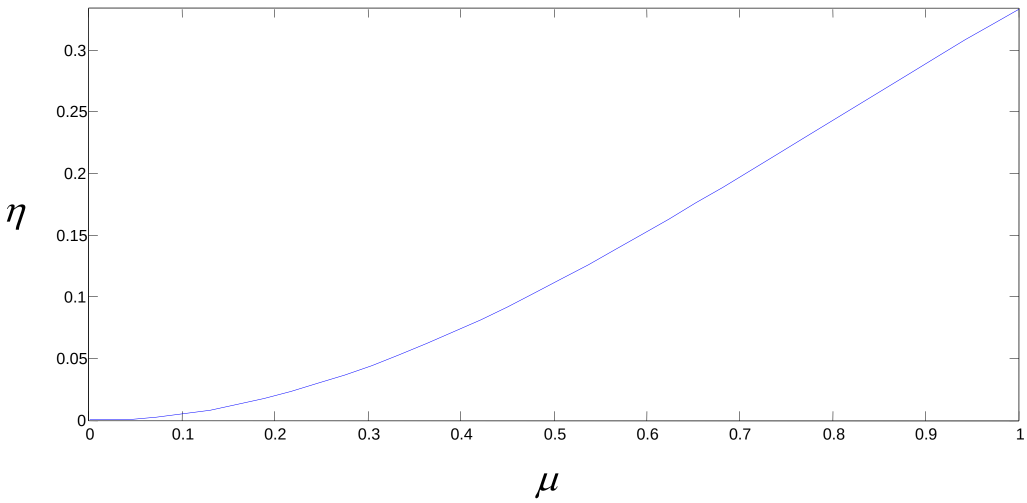
Power in sidebands $= \dfrac{1}{2}\left(\dfrac{A_C\mu}{2}\right)^2 + \dfrac{1}{2}\left(\dfrac{A_C\mu}{2}\right)^2$

$$= \frac{1}{8}A_C^{\ 2}\mu^2 + \frac{1}{8}A_C^{\ 2}\mu^2 = \frac{1}{4}A_C^{\ 2}\mu^2$$

Therefore,

$$\eta = \frac{\frac{1}{4}A_C^{\ 2}\mu^2}{\frac{A_C^{\ 2}}{2} + \frac{1}{4}A_C^{\ 2}\mu^2} = \frac{\mu^2}{2+\mu^2} \qquad ; \quad 1 \ge \mu \ge 0$$

The following figure shows the relationship between η and μ



The maximum efficiency occurs when μ = 1, i.e. for a 100% modulation index. The corresponding maximum efficiency is only η = 1/3. As a result, 2/3 of the transmitted power is wasted in the carrier.

<u>Remark:</u> Normal AM is not an efficient modulation scheme in terms of the utilization of the transmitted power.

<u>Exercise</u>:

a. Show that for the general AM signal $s(t) = A_C\ \ 1 + k_a m(t)\ \ \cos(2\pi f_c t)$, the power efficiency is given by $\eta = \dfrac{\frac{1}{2}A_C^{\ 2}\left\langle k_a^{\ 2} m(t)^2\right\rangle}{\frac{A_C^{\ 2}}{2} + \frac{1}{2}A_C^{\ 2}\left\langle k_a^{\ 2} m(t)^2\right\rangle} = \dfrac{\left\langle k_a^{\ 2} m(t)^2\right\rangle}{1+\left\langle k_a^{\ 2} m(t)^2\right\rangle}$, where $\left\langle k_a^{\ 2} m(t)^2\right\rangle$ is the average power in $k_a m(t)$

b. Apply the above formula for the single tone modulated signal $s(t) = A_C(1+\mu\cos 2\pi f_m t)\cos 2\pi f_C t$

AM Modulation Index

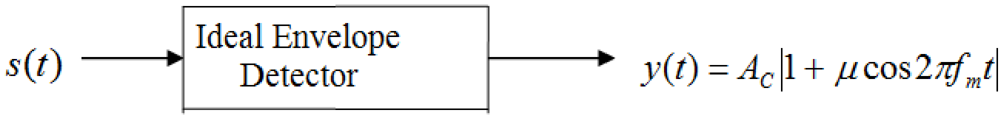
Consider the AM signal

$$s(t) = A_C\left(1 + k_a m(t)\right)\cos 2\pi f_C t = A(t)\cos 2\pi f_C t$$

The envelope of $s(t)$ is defined as:

$$|A(t)| = A_C|1 + k_a m(t)|$$

The following block diagram illustrate the envelope detection process for a sinusoidal message signal.



To avoid distortion, the following condition must hold

$$|1 + k_a m(t)| \geq 0 \qquad \text{or} \qquad |k_a m(t)| \leq 1$$

The modulation index of an AM signal is defined as:

$$Modulation\ \ Index\ (M.I) = \frac{|A(t)|_{\max} - |A(t)|_{\min}}{|A(t)|_{\max} + |A(t)|_{\min}}$$

Example: (single tone modulation)

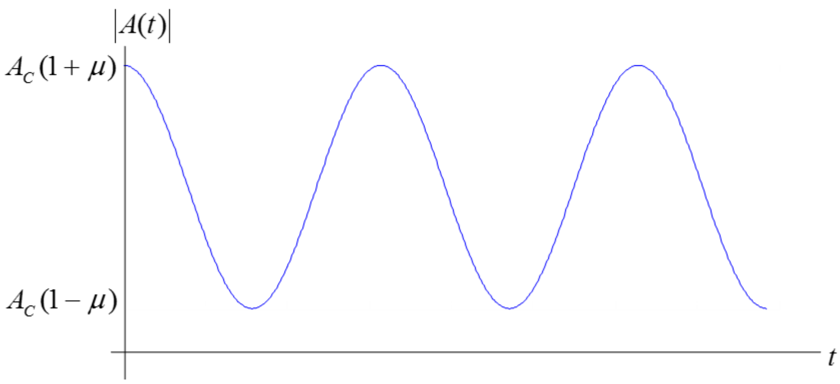
Let $m(t) = A_m \cos 2\pi f_m t$

$$then,\ s(t) = A_C(1 + k_a A_m \cos 2\pi f_m t)\cos 2\pi f_C t$$

$$= A_C(1 + \mu\cos 2\pi f_m t)\cos 2\pi f_C t \qquad where,\ \mu = k_a A_m$$

To avoid distortion $k_a A_m = \mu < 1$

The envelope $|A(t)| = A_C|(1 + \mu\cos 2\pi f_m t)|$ is plotted below



$$|A(t)|_{\max} = A_C(1 + \mu), \qquad |A(t)|_{\min} = A_C(1 - \mu)$$

$$M.I = \frac{A_C(1 + \mu) - A_C(1 - \mu)}{A_C(1 + \mu) + A_C(1 - \mu)} = \frac{2A_C\mu}{2A_C} = \mu$$

Therefore, the modulation index is μ.

Over-modulation

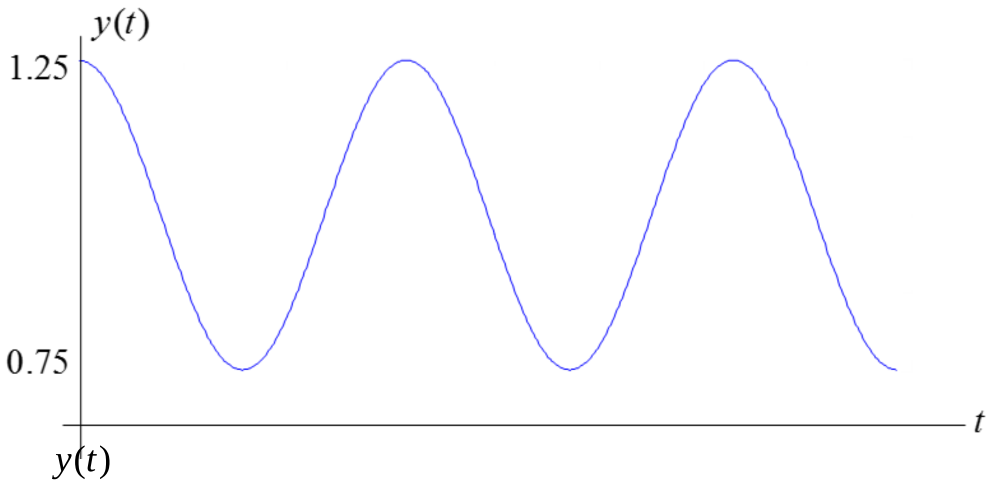
When the modulation index $\mu > 1$, an ideal envelope detector cannot be used to extract $m(t)$ and *distortio*n takes place.

Example: Let $s(t) = A_C(1+\mu\cos 2\pi f_m t)\cos 2\pi f_C t$ be applied to an ideal envelope detector, sketch the demodulated signal for $\mu = 0.25, 1.0, \textit{and } 1.25$.

As was mentioned before, the output of the envelope detector is $y(t) = A_C\left|1+\mu\cos 2\pi f_m t\right|$

Case1 : (μ = 0.25)

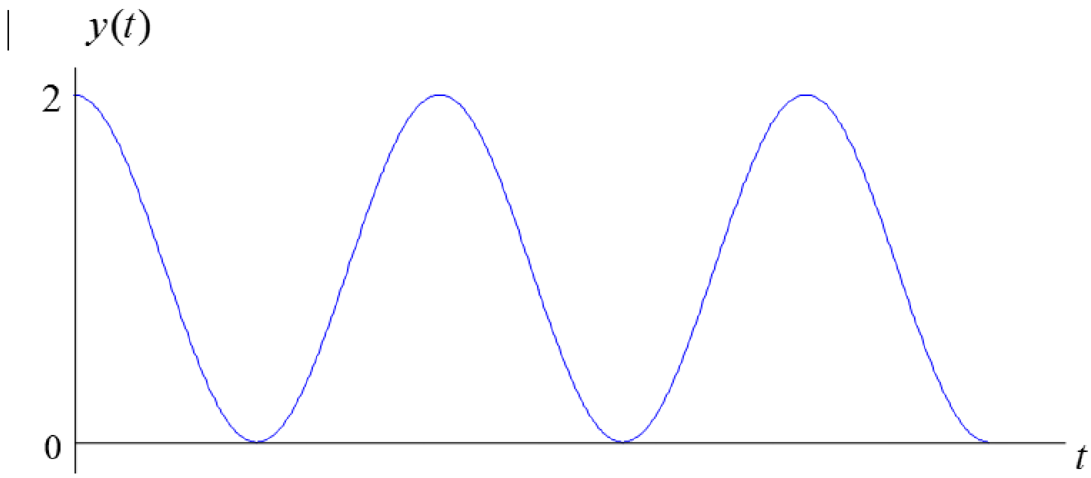
$$y(t) = A_C\left|1+0.25\cos 2\pi f_m t\right|$$



Here, $m(t)$ can be extracted without distortion.

Case2: (μ = 1.0)

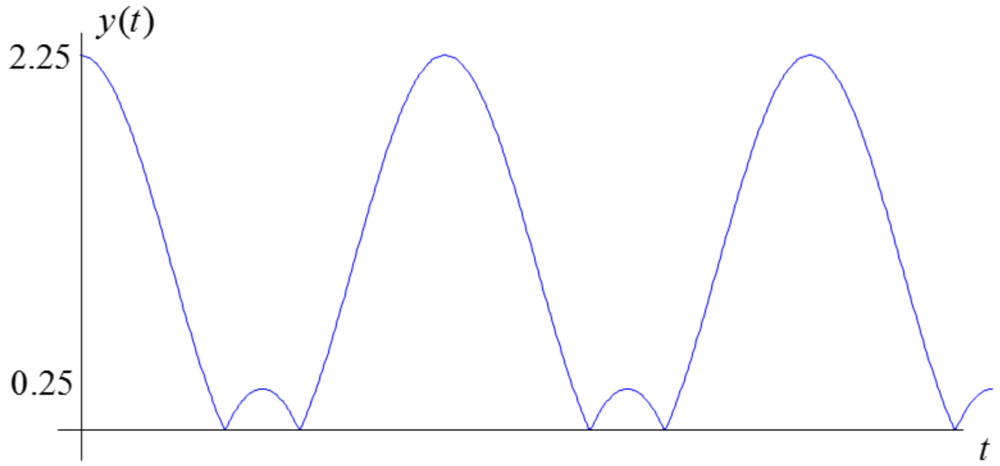
$$y(t) = A_C\left|1+\cos 2\pi f_m t\right|$$



Here again, $m(t)$ can be extracted without distortion.

Case3: (μ = 1.25)

$$y(t) = A_C\left|1 + 1.25\cos 2\pi f_m t\right|$$

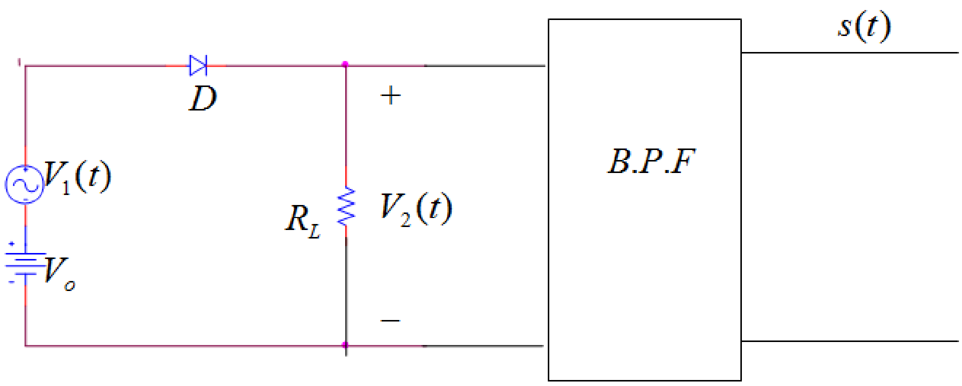


Here, $m(t)$ cannot be recovered without distortion.

<u>Generation of Normal AM</u>:

Square Law Modulator (will not be covered for ENCS students)

Consider the following circuit



For small variations of $V_1(t)$ around a suitable operating point, $V_2(t)$ can be expressed as:

$$V_2 = \alpha_1 V_1 + \alpha_2 V_1^{\,2} \; ; \qquad \text{Where } \alpha_1 \text{ and } \alpha_2 \text{ are constants.}$$

Let $V_1(t) = m(t) + A_C' \cos 2\pi f_C t$

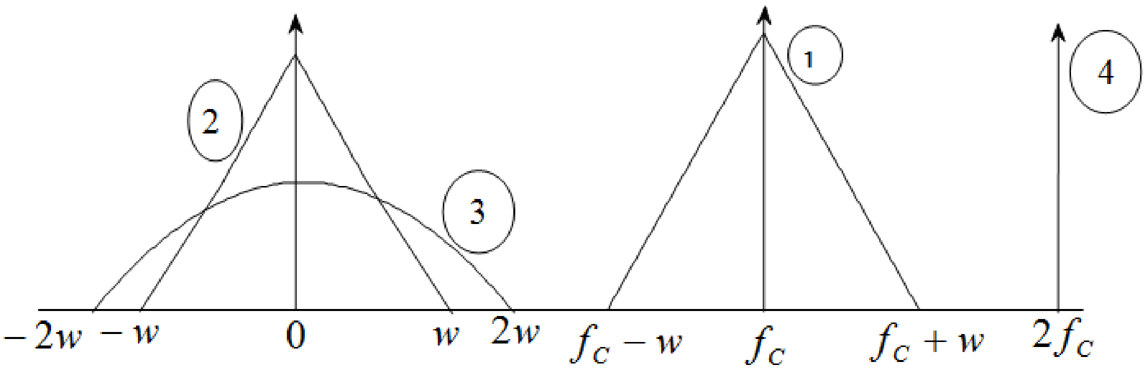
Substituting $V_1(t)$ into the nonlinear characteristics and arranging terms, we get

$$V_2(t) = \alpha_1 A_C'\left[1 + \frac{2\alpha_2}{\alpha_1} m(t)\right]\cos 2\pi f_C t + \alpha_1 m(t) + \alpha_2 m(t)^2 + \alpha_2 A_C'^2 \cos^2(2\pi f_C t)$$

$$V_2(t) = (1) + (2) + (3) + (4)$$

The first term is the desired AM signal obtained by passing $V_2(t)$ through a bandpass filter.

$$s(t) = \alpha_1 A_C'\left[1 + \frac{2\alpha_2}{\alpha_1} m(t)\right]\cos 2\pi f_C t$$



Note: the numbers shown in above figure represent the number of term in $V_2(f)$.

(1) = The desired normal AM signal

(2) = $M(f)$

(3) = $M(f) * M(f)$

(4) = The cosine square term amounts to a term at $2f_c$ and a DC term.

Limitations of this technique:

a. Variations of $V_1(t)$ should be small to justify the second order approximation of the nonlinear characteristic.

b. The bandwidth of the filter should be such that $f_C - w > 2w \Rightarrow f_C \geq 3w$

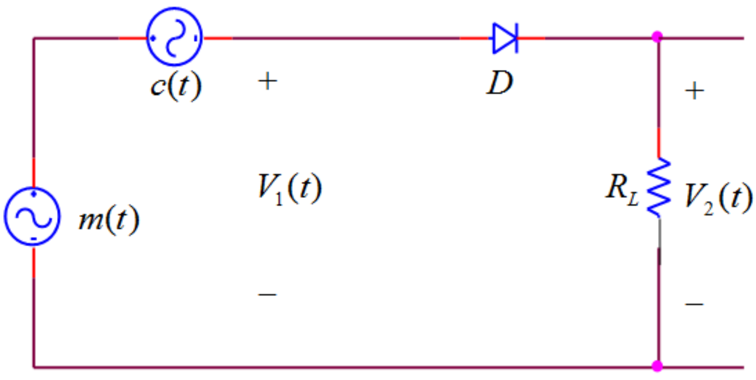
When $f_C >> w$, a bandpass filter with reasonable edge could be used.

When f_C is of the order $3w$, a filter with sharp edges should be used.

<u>Generation of Normal AM</u>:

The switching Modulator (will be covered)

Assume that the carrier $c(t)$ is large in amplitude so that the diode –shown in the figure below- acts like an ideal switch.



When $m(t)$ is small compared to $|c(t)|$,

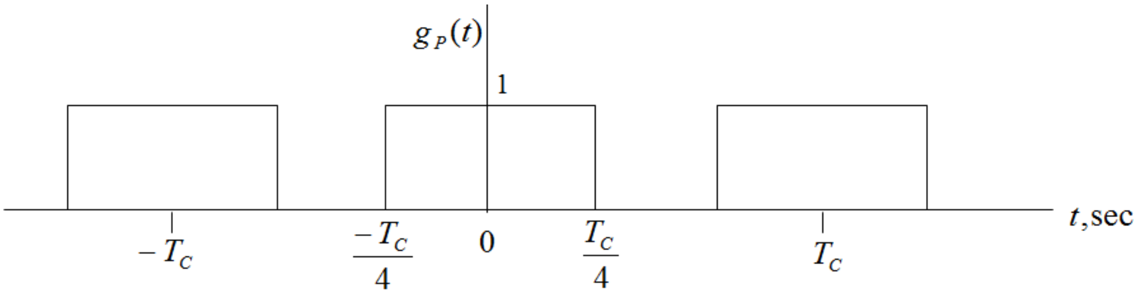
$$V_2(t) = \begin{cases} m(t) + A_C \cos\omega_C t & ; \quad c(t) > 0 \\ 0 & ; \quad c(t) < 0 \end{cases}$$

Here, the diode opens and closes at a rate equals to the carrier frequency f_C. This switching mechanism can be modeled as:

$$V_2(t) = [A_C \cos\omega_C t + m(t)]g_P(t)$$

where $g_P(t)$ is the periodic square function, expanded in a Fourier series as

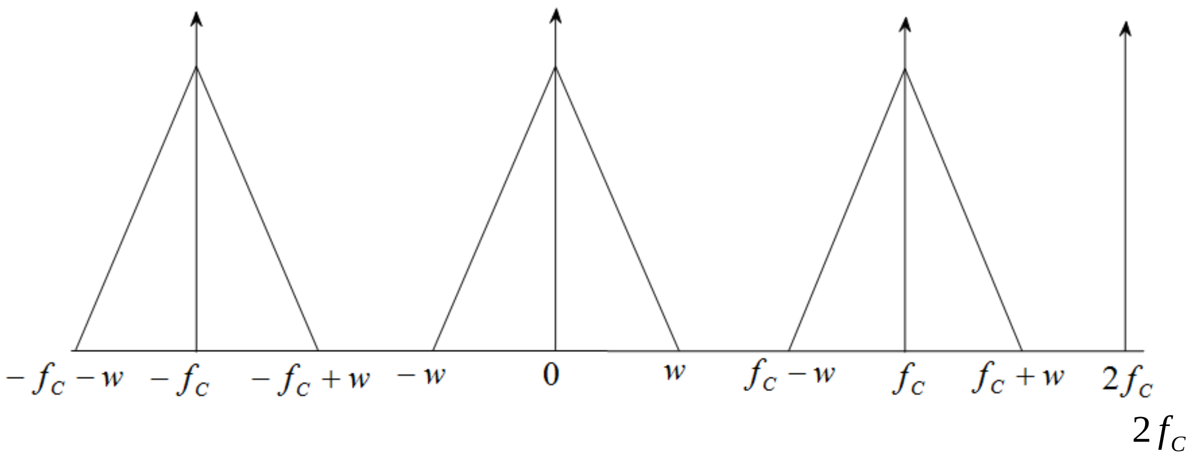
$$g_P(t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_C t - \frac{1}{3}\cos 3\omega_C t + \frac{1}{5}\cos 5\omega_C t + \ldots\right)$$



$$V_2(t) = [A_C \cos\omega_C t + m(t)]\left(\frac{1}{2}\right) + \left(\frac{2}{\pi}\cos\omega_C t\right)\left(A_C \cos\omega_C t + m(t)\right) - \left(\frac{2}{3\pi}\cos 3\omega_C t\right)$$
$$\left(m(t) + A_C \cos\omega_C t\right) + \ldots$$

$\Longrightarrow$

$$V_2(t) = \frac{m(t)}{2} + \frac{A_C}{2}\cos\omega_C t + \frac{2}{\pi}m(t)\cos\omega_C t + \frac{A_C}{\pi} + \frac{A_C}{\pi}\cos 2\omega_C t + \frac{2}{3\pi}m(t)\cos 3\omega_C t$$
$$+ \frac{2}{3\pi}A_C \cos 2\omega_C t + \ldots$$



A bandpass filter with a bandwidth $2w$, centered at f_C, passes the second term (a carrier) and the third term (a carrier multiplied by the message). The filtered signal is

$$s(t) = \frac{A_C}{2}\cos\omega_C t + \frac{2}{\pi}m(t)\cos\omega_C t$$

$$s(t) = \frac{A_C}{2}\left(1 + \frac{4}{\pi A_C}m(t)\right)\cos\omega_C t \quad ; \qquad \text{Desired AM signal.}$$

Modulation Index $= M.I = \frac{4}{\pi A_C}|m(t)|_{\max}$

Demodulation of AM signal: (The Ideal Envelope Detector)

The ideal envelope detector responds to the envelope of the signal, but is insensitive to phase variation. If

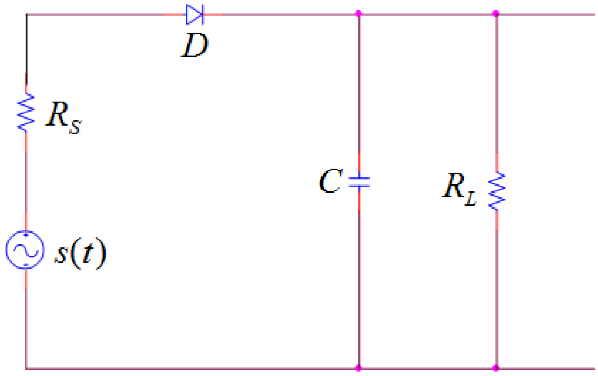
$$s(t) = A_C(1 + k_a m(t))\cos 2\pi f_C t$$

then, the output of the ideal envelope detector is

$$y(t) = A_C|1 + k_a m(t)|$$

A simple practical envelope detector

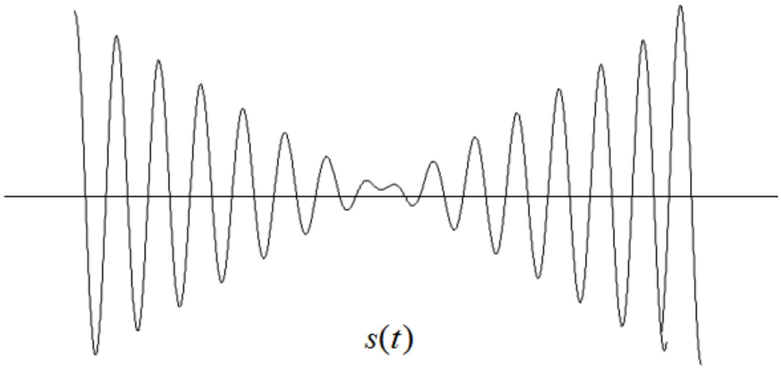
It consists of a diode followed by an RC circuit that forms a low pass filter.

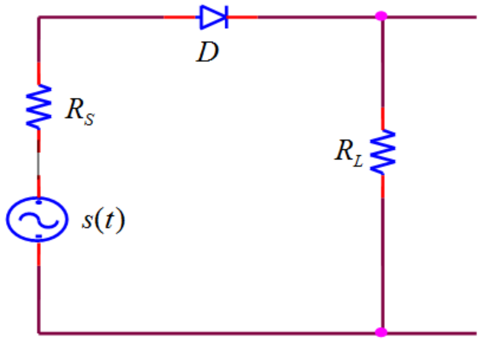


During the positive half cycle of the input, the diode is forward biased and C charges rapidly to the peak value of the input. When $s(t)$ falls below the maximum value, the diode becomes reverse biased and C discharges slowly through R_L. To follow the envelope of $s(t)$, the circuit time constant should be chosen such that :

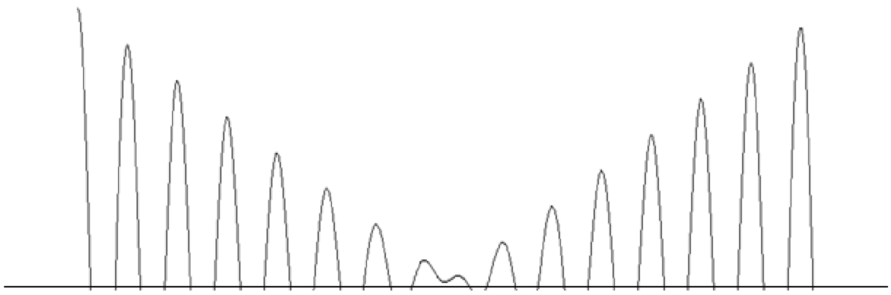
$$\frac{1}{f_C} << R_L C << \frac{1}{w}$$

Where w is the message B.W and f_C is the carrier frequency.





Half – Wave Rectifier

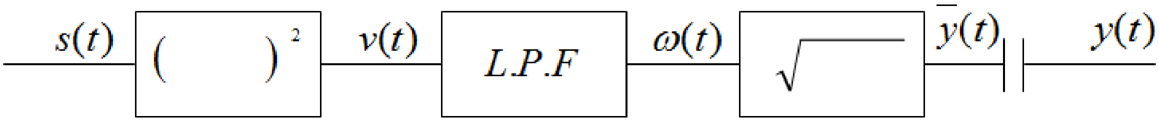


Output of half wave rectifier (without C)

When a capacitor C is added to a half wave rectifier circuit, the output follows the envelope of $s(t)$. The circuit output (with C connected) follows a curve that connects the tips of the positive half cycles, which is the envelope of the AM signal.

Example: (Demodulation of AM signal)

Let $s(t) = \left(1 + k_a m(t)\right)\cos\omega_C t$ be applied to the scheme shown below, find $y(t)$.



$$v(t) = s(t)^2 = \left(1 + k_a m(t)\right)^2 \cos^2\omega_C t$$

$$= \frac{1}{2}\left(1 + k_a m(t)\right)^2 + \frac{1}{2}\left(1 + k_a m(t)\right)^2 \cos 2\omega_C t$$

The filter suppresses the second term and passes only the first term. Hence,

$$\omega(t) = \frac{1}{2}\left(1 + k_a m(t)\right)^2$$

$$\bar{y}(t) = \sqrt{\omega(t)} = \frac{1}{\sqrt{2}}\left(1 + k_a m(t)\right)$$

$$y(t) = \frac{1}{\sqrt{2}} k_a m(t)$$

Note that the dc term is blocked by capacitor.

<u>Concluding remarks about AM</u>:

i. Modulation is accomplished using a nonlinear device.
ii. Demodulation is accomplished using a simple envelope detector.
iii. AM is wasteful of power; most power resides in the carrier (not in the sidebands).
iv. The transmission B.W = twice message B.W