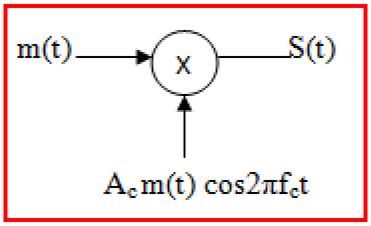
Double Sideband Suppressed Carrier Modulation (DSB-SC)

A DSB-SC signals is an AM signals that has the form:

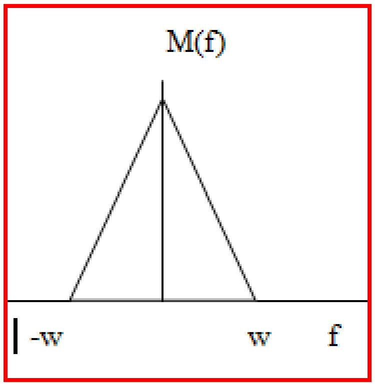
$$s(t) = A_c m(t) \cos(2\pi f_c t)$$

where $f_c >> w$, w is the baseband signal's bandwidth .

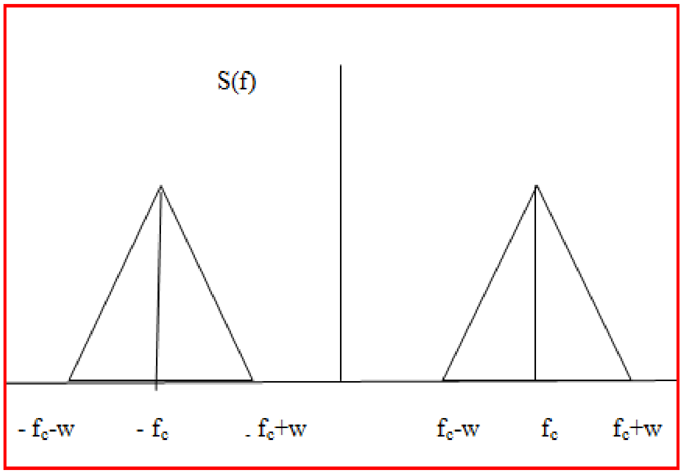


The spectrum of $s(t)$ is:

$$S(f) = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]$$



Remarks*:*



1. No impulses are present in the spectrum at ± f_c and, hence, no carriers is transmitted.

2. The transmission B.W of s(t)=2w. (same of AM).

3. power efficiency

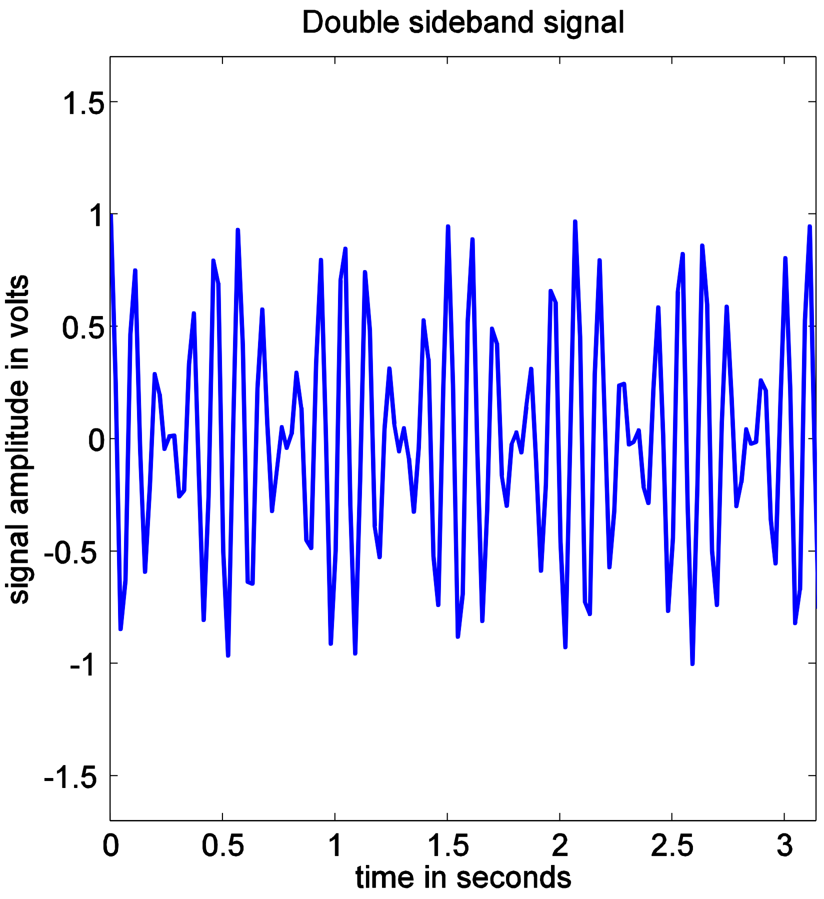
$$= \frac{\textit{power in the side bands}}{\textit{total transmitted power}} = 100\%.$$

This is a power efficient modulation scheme.

4. Coherent detector is required to extract m(t) from s(t), as we shall demonstrate shortly.

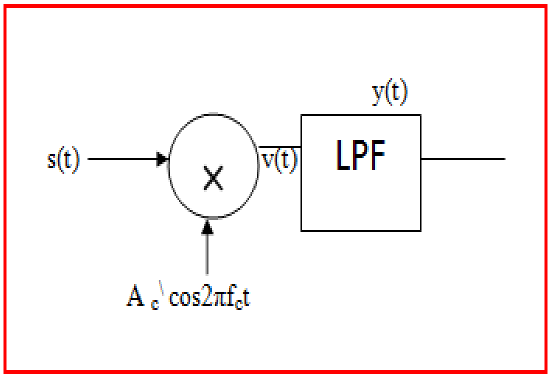
5. Envelope detection cannot be used.

Computer simulation: The next figure shows a DSB-SC signal when $m(t) = \cos(2\pi t)$ and $c(t) = \cos(2\pi(10)t)$. You can easily see that m(t) cannot be recovered using envelope detection.



Demodulation of a DSB-SC signals

A DSB-SC signal is demodulated using what is known as ***coherent demodulation***. This means that the modulated signal s(t) is multiplied by a locally generated signal at the receiver which has the same frequency and phase as the carrier c(t) at transmitting side.



a. **Perfect coherent demodulation**.

Let $c(t)= A_c\cos 2\pi f_c t$, $c^{\backslash}(t)= A_c^{\backslash}\cos(2\pi f_c t)$

Mixing the received signal with the version of the carrier at the receiving side, we get

$$v(t)=s(t)\ A_c^{\backslash}\cos 2\pi f_c t =A_c A_c^{\backslash} m(t)\cos^2 2\pi f_c t$$

$$=\frac{Ac\ A\ c\backslash}{2}m(t)[1+\cos 2\ (2\pi f_c t)]$$

$$=\frac{Ac\ A\ c\backslash}{2}m(t)+\frac{Ac\ A\ c\backslash}{2}m(t)\cos 2(2\pi f_c t)$$

The first term on the RHS is proportional to $m(t)$, while the second term is a DSB signal modulated on a carrier with frequency $2f_c$. The high frequency component can be eliminated using the LPF. The output is

$$y(t)=\frac{A_c A_c'}{2}m(t)$$

Therefore, $m(t)$ has been recovered from $s(t)$ without distortion, i.e., a distortion less system.

b. Effect of carrier noncoherence on demodulated signal (to be covered in class)

Here we consider two cases.

Case 1: A constant phase difference between c(t) and $c^{\backslash}(t)$

Let $c(t) = A_c\cos 2\pi f_c t$, $c^{\backslash}(t) = A\ _c{}^{\backslash}\cos(2\pi f_c t+\varnothing)$

We use the demodulator considered above

$$v(t)= A_c m(t)\cos 2\pi f_c t\ .\ A\ _c{}^{\backslash}\cos(2\pi f_c t+\varnothing)$$

$$=\frac{Ac\ A\ c\backslash}{2}m(t)[\ \cos(4\pi f_c t+\varnothing)+\cos\varnothing]$$

$$=\underbrace{\frac{Ac\ A\ c\backslash}{2}m(t)\cos(4\pi f_c t+\varnothing)}_{\text{high frequency term}}+\underbrace{\frac{Ac\ A\ c\backslash}{2}m(t)\cos\varnothing}_{\text{low frequency term}}$$

The output of the low pass filter is:

$$y(t)=\frac{Ac\ A\ c\backslash}{2}m(t)\cos\varnothing$$

For $0 < \varnothing < \frac{\pi}{2}$, $0 < \cos\varnothing < 1$, y(t) suffer from an attenuation due to Ø.

However, for $\varnothing=\frac{\pi}{2}$, $\cos\varnothing = 0$ and $y(t) = 0$, signal disappears. The disappearance of a message component at the demodulator output is called *quadrature null effect.*

This highlights the importance of maintaining synchronism between the transmitting and receiving carrier signals $c^{\backslash}(t)$ and c(t).

Case 2: Constant frequency difference between c(t) and $c^{\backslash}(t)$ (to be done at home as an exercise)

Let $c(t)= A_c\cos2\pi f_c t$ $\qquad$,$C^{\backslash}(t)= A_c^{\backslash}\cos(2\pi f_c+\Delta f)t$

In an analysis similar to case a, we get

$$v(t)= A_c m(t)\cos2\pi f_c t\ .\ A_c^{\backslash}\cos(2\pi f_c+\Delta f)t$$

$$=\frac{Ac\ A\ c\backslash}{2}m(t)[\ \cos\ (4\pi f_c t+2\pi\Delta ft)+\cos\ 2\pi\Delta ft]$$

After low pass filtering,

$$y(t)= \frac{Ac\ A\ c\backslash}{2}m(t)\ \cos\ 2\pi\Delta ft$$

So the demodulated signals appears as if double side band modulated on a carrier with magnitude Δf. As can be observed, this is not a distortionless transmission.

Example: Let $m(t)=\cos2\pi(1000)t$ and let $\Delta f=100Hz$ (**to be done at home as an exercise**)

From the analysis in case 2 above,

$$y(t)= \frac{Ac\ A\ c\backslash}{2}\ \cos2\pi(1000)t\ \cos2\pi(100)t$$

$$=\frac{Ac\ A\ c\backslash}{4}[\ \cos\ 2\pi(1100)t+\ \cos\ 2\pi(900)t]$$

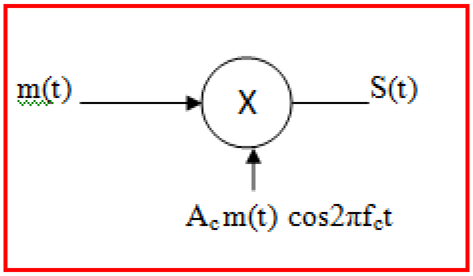
The original message was a signal with a frequency of =1000Hz, while the output consists of a signal two frequencies at f_1=1100Hz and f_2=900Hz.

⇨ Distortion

Exercise: Use Matlab to plot both m(t) and y(t) and see the distortion caused by the lack of synchronization between the transmitting and receiving oscillators.

Generation of DSB-SC

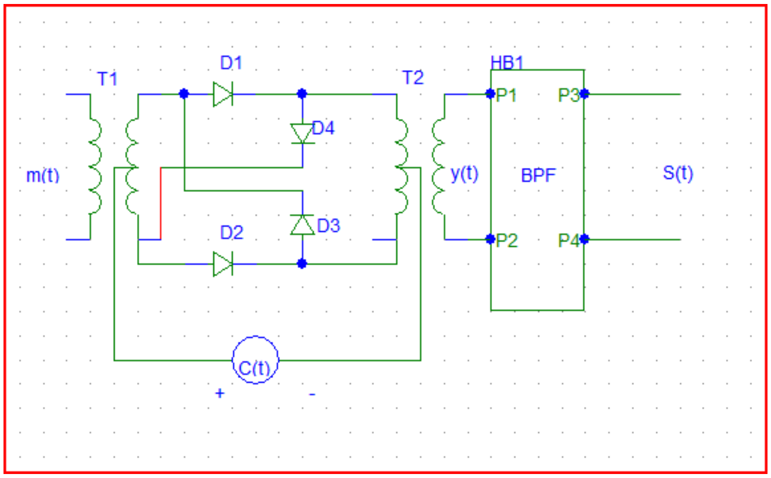
a. **Product modulator** : It multiplies the message signal m(t) with the carrier c(t). This technique is usually applicable when low power levels are possible and over a limited carrier frequency range.



b .**Ring modulator**: **(to be covered in class)**

consider the scheme shown in the figure.

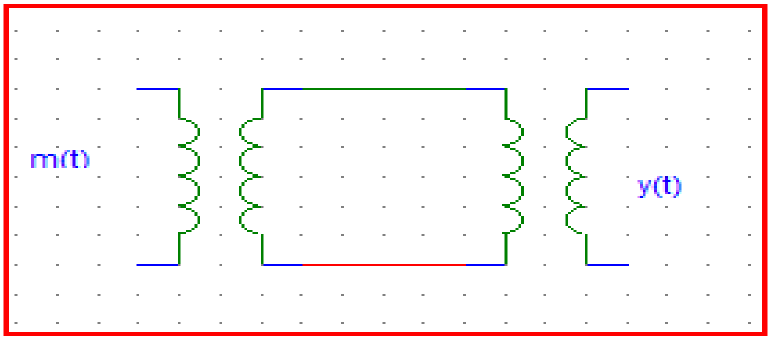
Let c(t)>> m(t). Here the carrier c(t) control the behavior of the diodes .



During the positive half cycle of c(t), c(t) > 0, and D1 and D2 are ON while D3 and D4 are OFF.

y(t) = m(t)

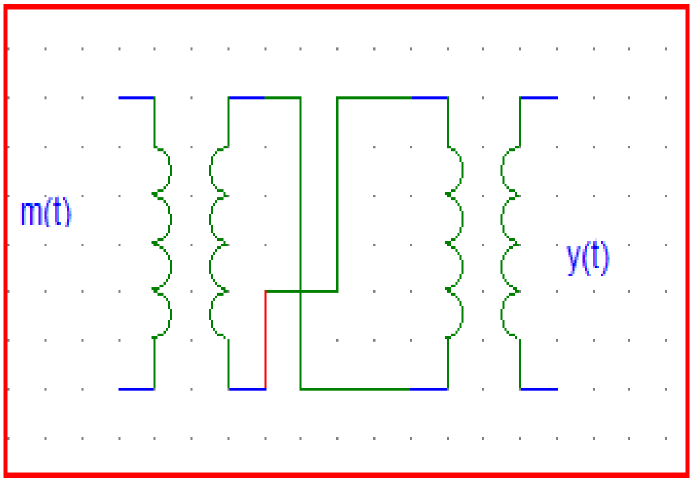
and the circuit appears like this



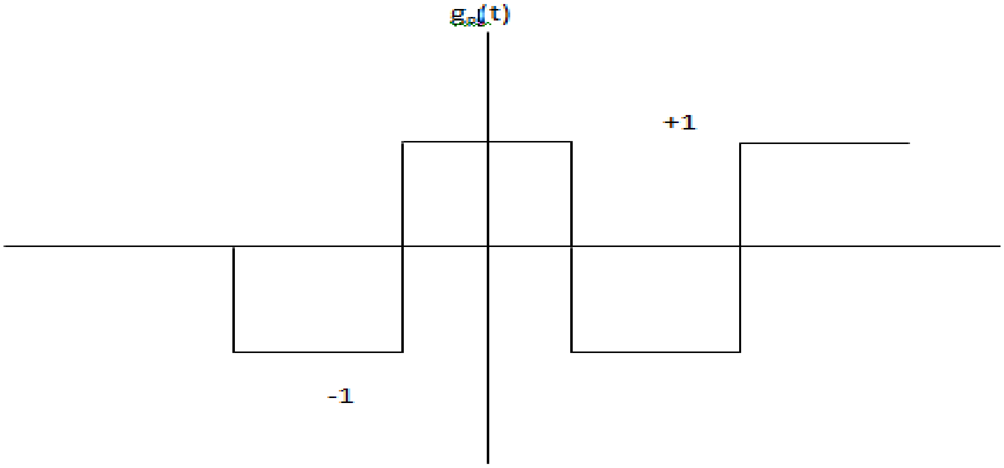
During the negative half cycle of c(t), c(t) < 0 and D3 and D4 are ON while and D1 and D2 are OFF

y(t) = -m(t)

and the circuit appears like this



So m(t) is multiplied by +1 during the +ve half cycle of c(t) and m(t) multiplied by -1 during the -ve half cycle of c(t). Mathematically, y(t) behaves as if multiplied by the switching function $g_p(t)$ where $g_p(t)$ is the square periodic function with period $T_c=\frac{1}{fc}$, where f_c the period of c(t). By expanding $g_p(t)$ in a Fourier series, we get



$$y(t) = m(t)[\frac{4}{\pi} \cos 2\pi f_c t \ -\frac{4}{3\pi} \cos 3(2\pi f_c t)+ \frac{4}{5\pi} \ \cos 5(2\pi f_c t)]$$

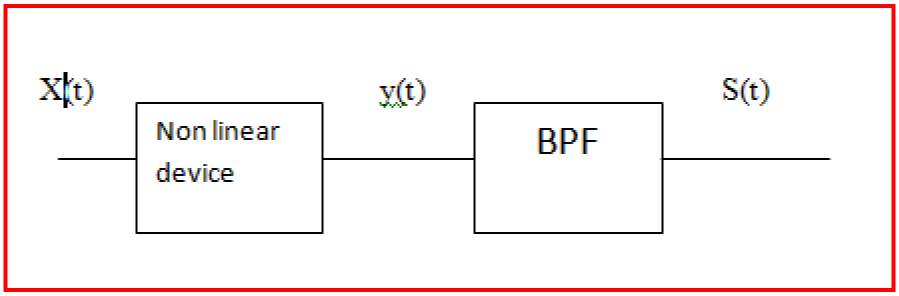
$$=m(t)\frac{4}{\pi} \cos 2\pi f_c t \ -m(t)\frac{4}{3\pi} \cos 3(2\pi f_c t)+ m(t) \frac{4}{5\pi} \ \cos 5(2\pi f_c t)$$

When y(t) passes through the BPF, the only component that appears at the output is the desired DSB-SC signal, which is

$$s(t)= m(t)\frac{4}{\pi} \cos 2\pi f_c t$$

C. **Nonlinear characteristic (to be done at home as an excecise)**

Consider the scheme shown in the figure



Let the non linear characteristic be of the form

$$y(t)=a_0x(t)+a_1x^3(t)$$

Let x(t)= $A\cos 2\pi f_c t+m(t)$, (m(t) is the message signals)

$$y= a_0(A\cos 2\pi f_c t+m(t))+a_1 (A\cos 2\pi f_c t+m(t))^3$$

$$= a_0 A\cos 2\pi f_c t+ a_0 m(t)+ a_1 A^3\cos^3 2\pi f_c t+ a_1 m(t)^3+3 a_1 A^2 m(t)\cos^2 2\pi f_c t$$

$$+3A a_1\cos 2\pi f_c t$$

After some algebraic manipulations, a DSB-SC term appear in x(t) along with other undesirable terms. The band pass filter will admit the desired signal, which is

$$s(t)=\frac{3(A)^2 a1}{2} m(t) \cos(2)2\pi f_c t,$$

Note that the carrier frequency=$2f_c$ in this case.

Carriers recovery for coherent demodulation (will not be covered)

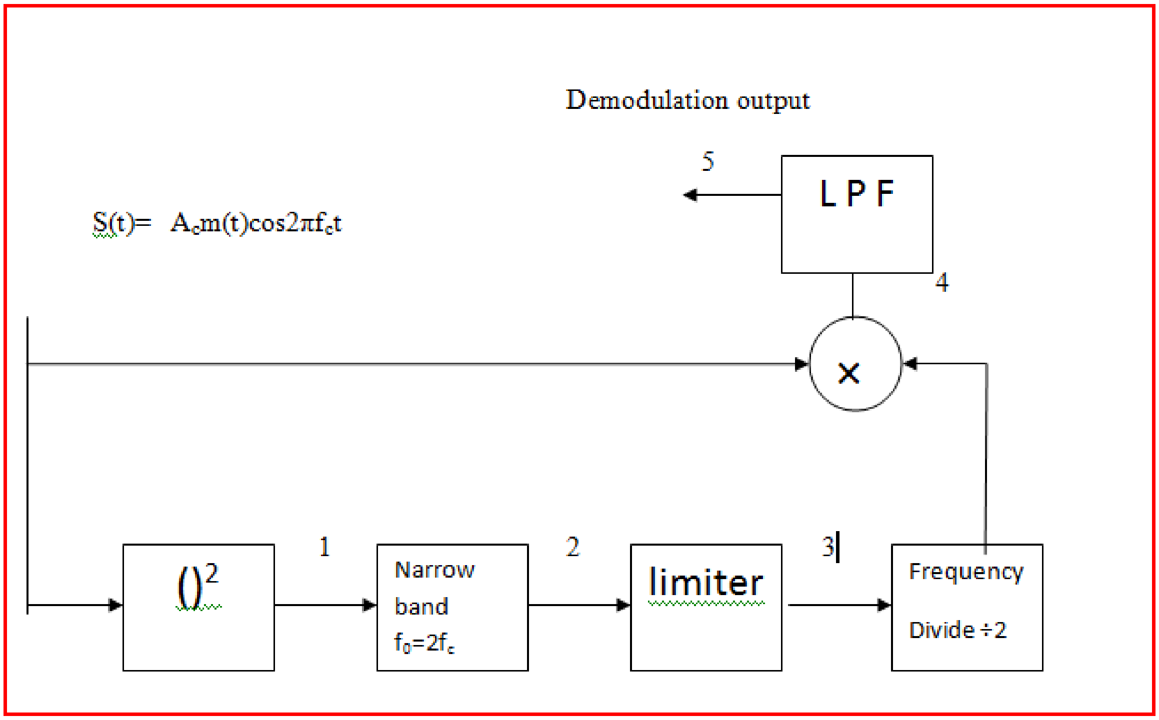
We consider briefly two circuits that are used to extract the carriers f_c from the incoming DSB-SC signal. we recall that demodulation of DSB-SC signal requires the availability of a signal with the same frequency and phase as the carrier c(t) at the transmitter

a. Squaring loop :

The basic elements of squaring loop are shown in the figure below. The incoming signal is the DSB-SC signal:

$$s(t)=A_c m(t)\cos 2\pi f_c t.$$

In the figure, we mark five signals that appear at the output of the five blocks. In summary these signals are:



1- $(A_c m(t)\cos 2\pi f_c t)^2 = (\frac{Ac}{2} m(t))^2 (1+\cos 2w_c t)$

$= (\frac{Ac}{2} m(t))^2 + (\frac{Ac}{2} m(t))^2 (\cos 2w_c t)$

$$\underset{\text{Low pass term}}{\uparrow} \qquad \underset{\text{band pass around } 2f_c}{\uparrow}$$

2- $(\frac{Ac}{2} m(t))^2 (\cos 2w_c t) = K\cos 2w_c t$ (when the BPF is of narrow B.W)

3- $K\cos 2w_c t$ (The limiter removes any variation in amplitude but keeps the frequency unchanged). The frequency divider produces a signal $K\cos w_c t$.

4- $(A_c m(t)\cos 2\pi f_c t)\, K\cos w_c t$

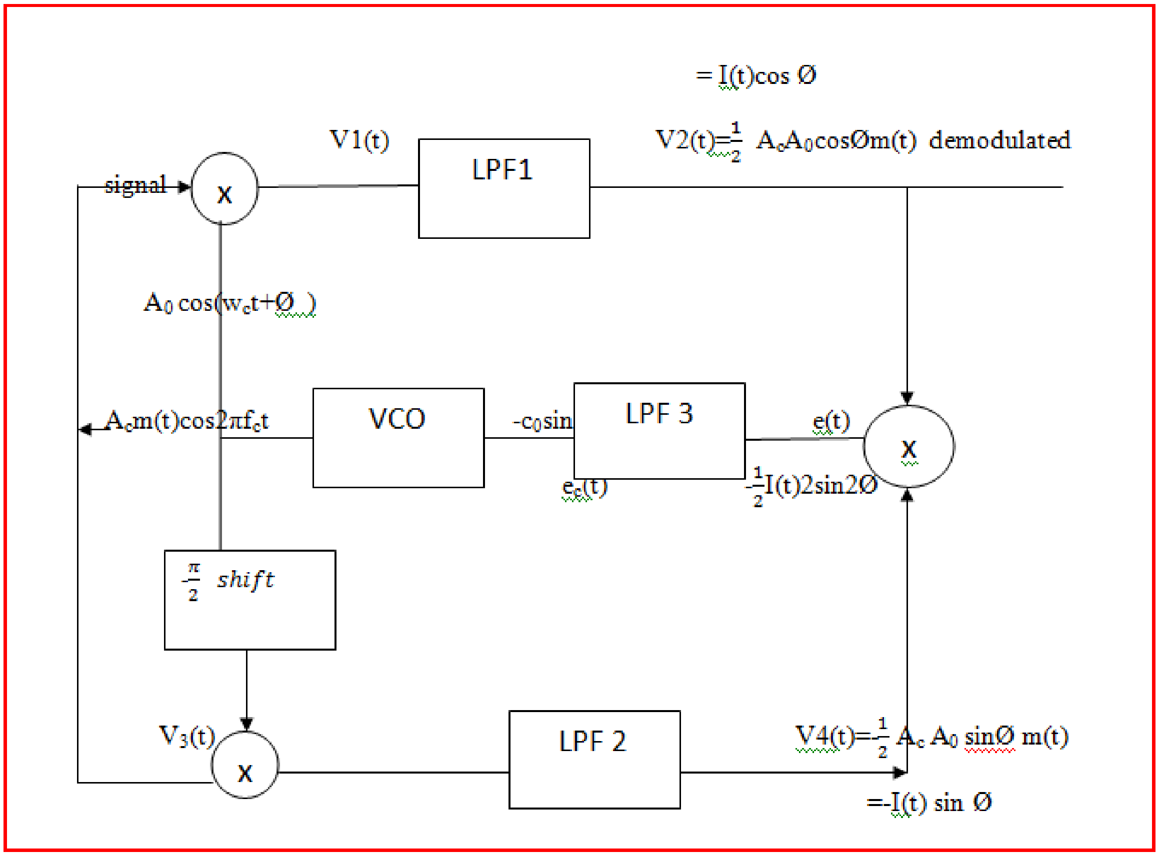
$= A_c K m(t)\cos^2 w_c t$

$\frac{A_c(K)}{2} m(t) + \frac{A_c(K)}{2} m(t) \cos 2w_c t$

5- $A_c \frac{(K)}{2} m(t)$

Therefore, demodulation has been achieved, even though the receiver does not have a copy of the carrier, but was able to generate its own version of the carrier via this loop.

Costas Loop:



The VCO: is an oscillator that produces a signal whose frequency is proportional to the voltage $e_c(t)$.

When $e_c(t) = 0$, the frequency of the oscillator is called the free running frequency. Let this frequency = f_c (the incoming carrier frequency)

When there is a phase difference Ø, we have

$$V_1(t)= A_cA_0m(t) \cos (w_ct) \cos(w_ct+ Ø)$$

$$V_2(t)=\frac{AcA0}{2}m(t)\cos Ø$$

$$V_4(t)=\text{Low pass } \{A_0A_cm(t)\cos(w_ct)\sin(w_ct+ Ø)\}$$

$$V_4(t)= \frac{AcA0}{2}m(t)\sin Ø \text{ after LPF 2}$$

$$e(t)= \frac{AcA0}{2}m(t)^2 (\sin Ø)(\cos Ø)$$

$=\frac{AcA0}{24}m(t)^2 \sin 2Ø$

When the B.W of LPF3 is very narrow, the output can be approximated as:

$e_c(t)= c_0\sin 2Ø$

This is the feedback signal that is applied to the VCO. Ideally, when Ø=0, $e_c(t)=0$ and VCO frequency (and phase) are equal to the frequency of the input signal s(t).

If the phase difference Ø between the incoming s(t) and the VCO output increases, then $e_c(t)$ increases forcing the frequency of the VCO to decrease so that it remains in synchronism with the input phase. (Recall that the frequency of the VCO decreases if its input voltage increases; the slope of the VCO characteristics is negative).

Single Sideband Modulation

In this type of modulation, only one of the two sidebands of DSB-SC is retained while the other sideband is suppressed. This mean that B.W of the SSB signal is one half that of DSB-SC. The saving in the bandwidth comes at the expense of increasing modulation complexity.

The time-domain representation of a SSB signal is

$$s(t) = A_c m(t) \cos w_c t \pm A_c \hat{m}(t) \sin w_c t$$

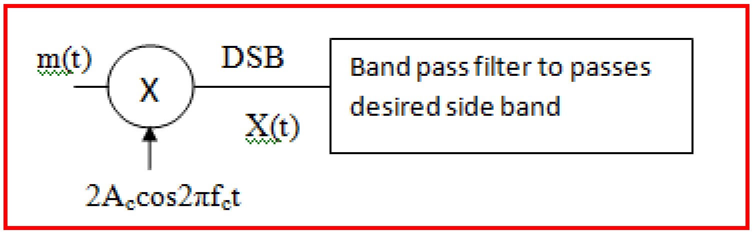
$\hat{m}(t)$: Hilbert transform of m(t) obtained by passing m(t) through a 90-degree phase shifter.

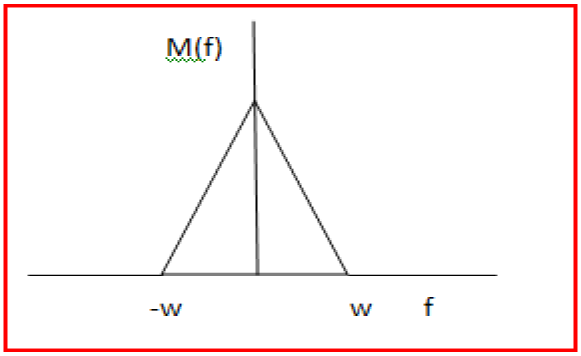
-sign: upper sideband is retained

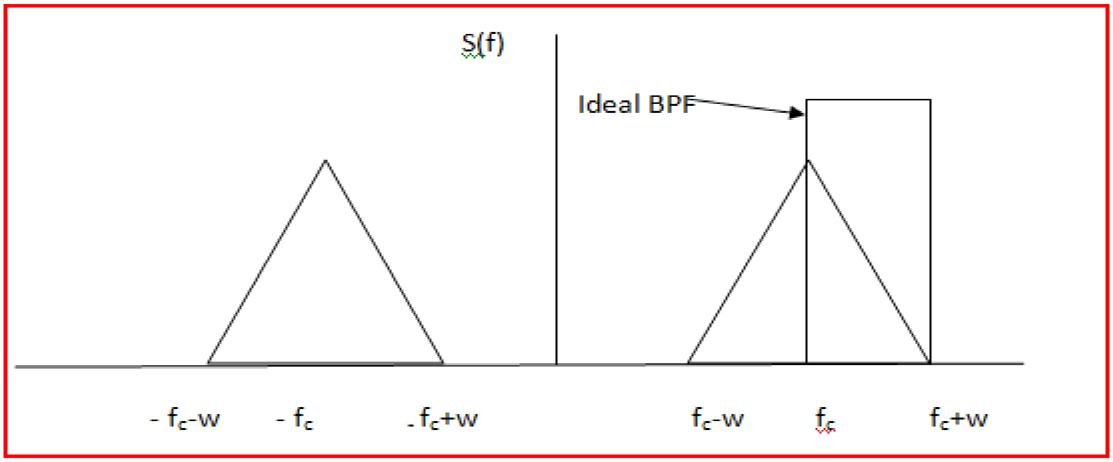
+sign: lower sideband is retained

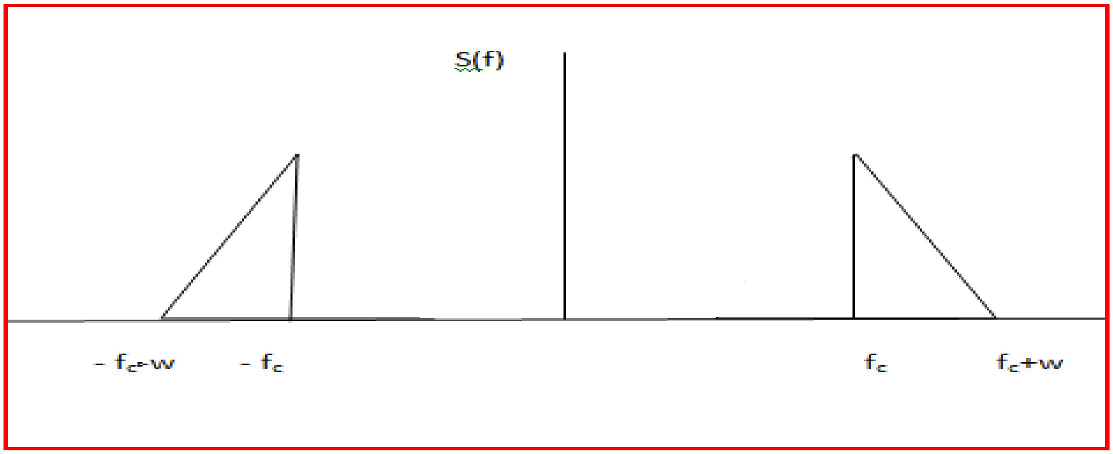
Generation of SSB: Filtering Method (Frequency Discrimination Method)

A DSB-SC signal $X(t)=2A_c m(t) \cos w_c t$ is generated first. A band pass filter with appropriate B.W and center frequency is used to pass the desired side band only



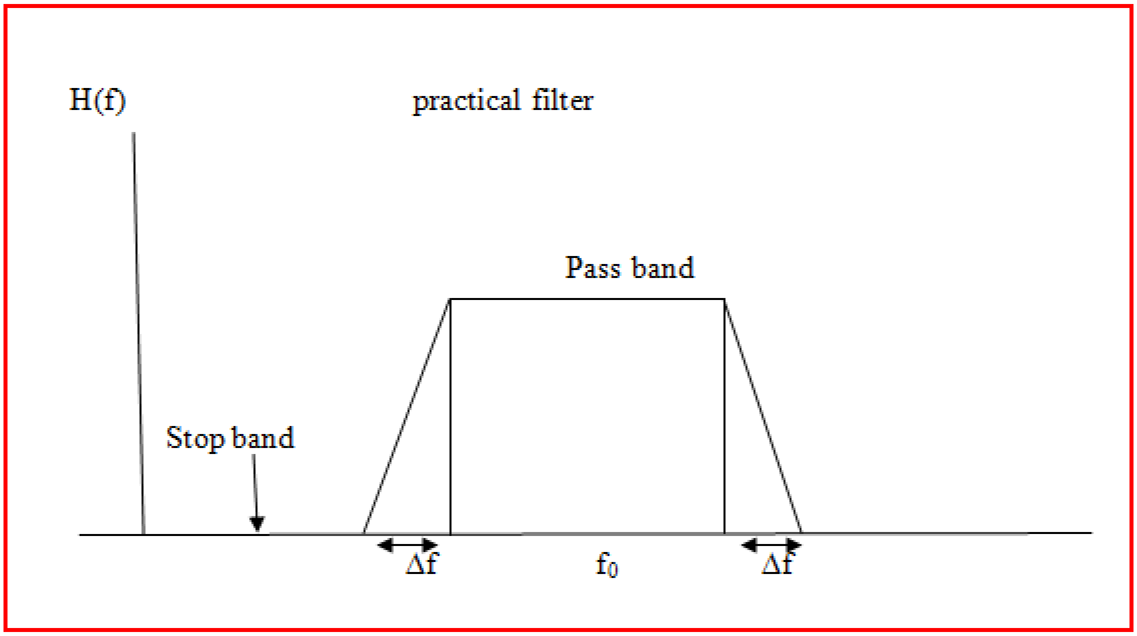






The band pass filter must satisfy two conditions:

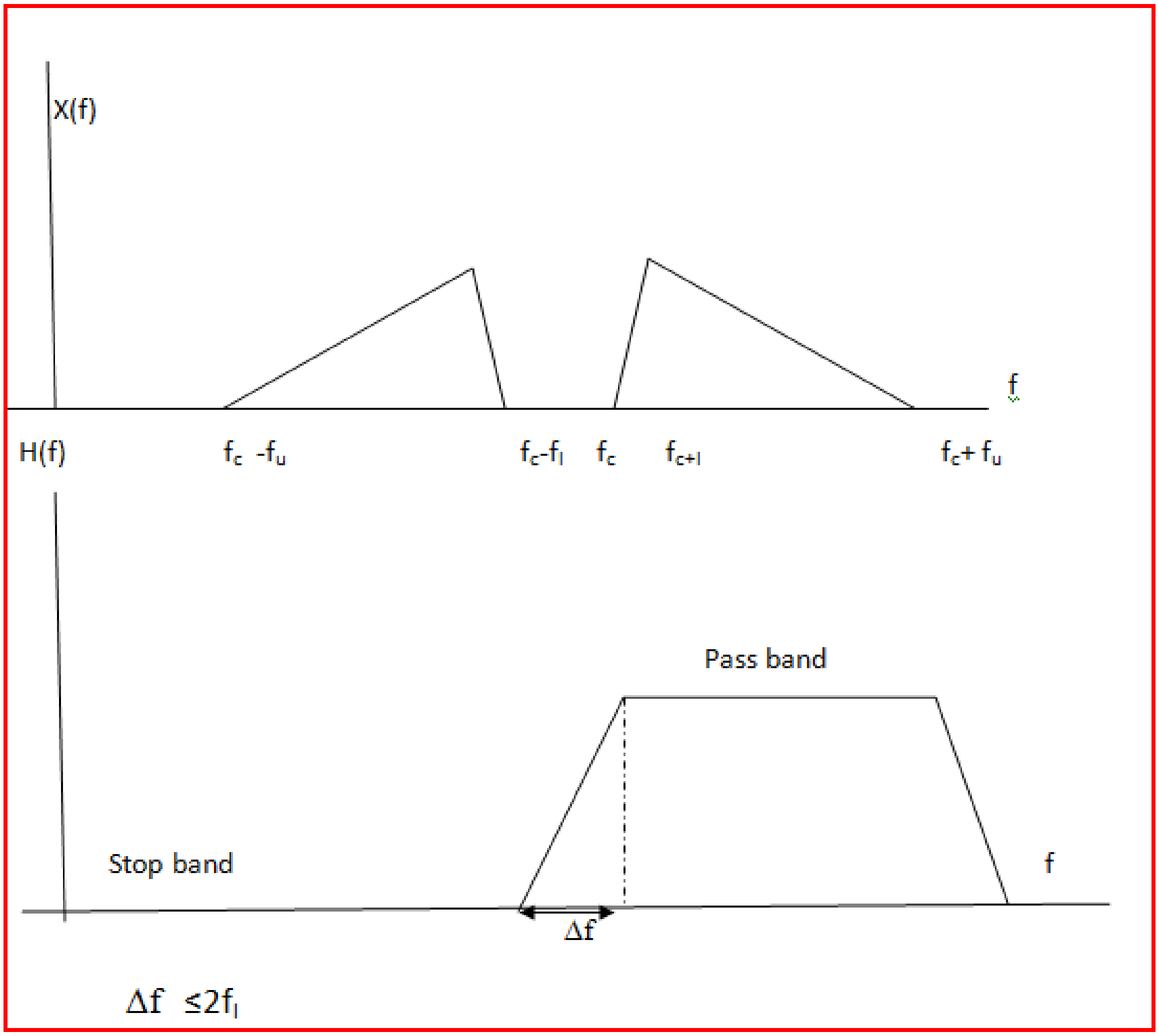
a. The pass band of the filter must occupy the same frequency range as the desired sideband.
b. The width of the transition band of the filter separating the pass band and the stop band must be at least 1% of the center frequency of the filter. i.e., $0.01f_0 \leq \Delta f$. This is sort of a rule of thumb for realizable filters on the relationship between the transition band and the center frequency.



Two remarks should be considered when generating a SSB signal.

1. Ideal filter do not exist in practice meaning that a complete elimination of the undesired side band is not possible. The consequence of this is that either part of the undesired side band is passed or the desired one will be highly attenuated. SSB modulation is suitable for signals with low frequency components that are not rich in terms of their power content.

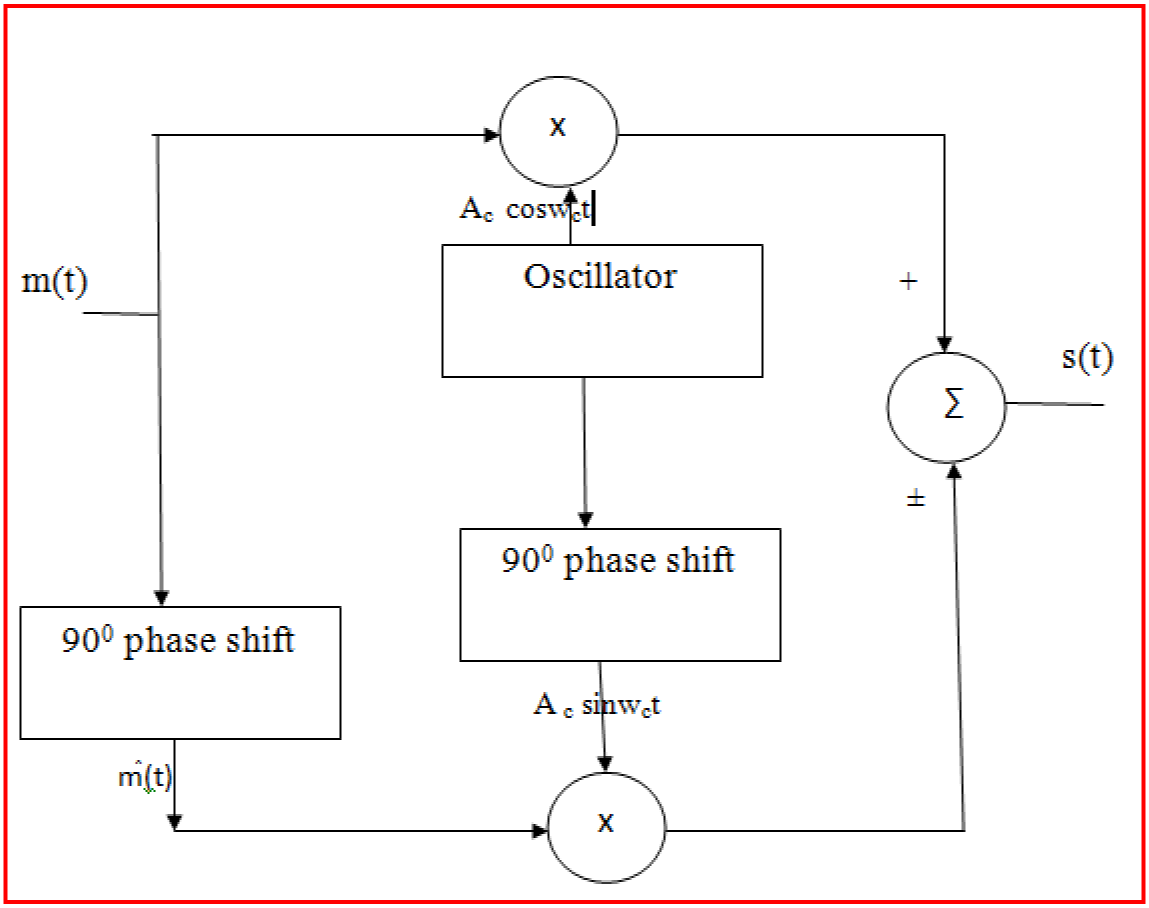
2. The width of the transition band of the filter should be at most twice the lowest frequency components of the message signal so that a reasonable separation of the two side band is possible.



Generation of SSB Signal: Phase Discrimination Method .

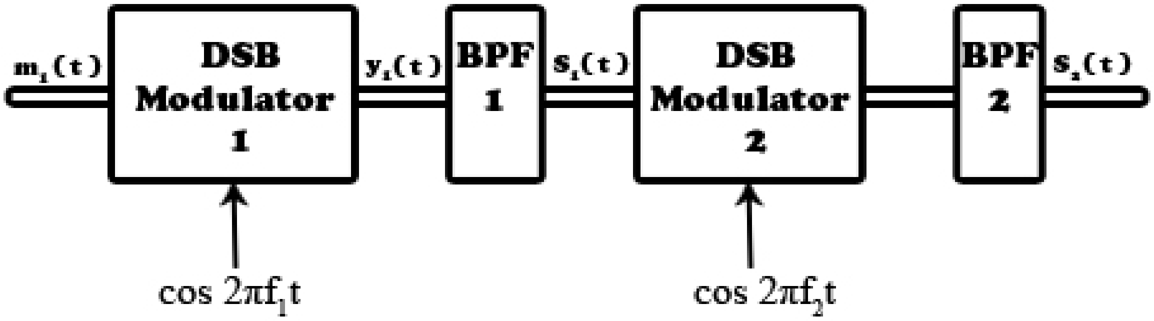
The method is based on the time –domain representation of the SSB signal

$$s\ (t)= A_c m(t)\cos w_c t \pm A_c\, \hat{m}\ (t)\ \sin w_c t$$



Two- stage generation of SSB signal (will not be covered in class)

When the conditions on the filter cannot be met in a single-stage SSB system, a two-stage scheme is used instead where less stringent conditions on the filters can be imposed. The block diagram illustrates this procedure.



$m_1(t)$ is the base band signal with a gap in its spectrum extending over (0, f_l).

$y_1(t)$: is a DSB-SC signal on a carrier frequency f_1.

BPF_1 selects the upper side band of $y_1(t)$. The parameters of the filter are f_{01} (center frequency) and the transition band length $\Delta f_{1.}$

We must maintain that

$\Delta f_1 \geq 0.01\ f_{01}$ and $\Delta f_1 \leq 2\ f_I$

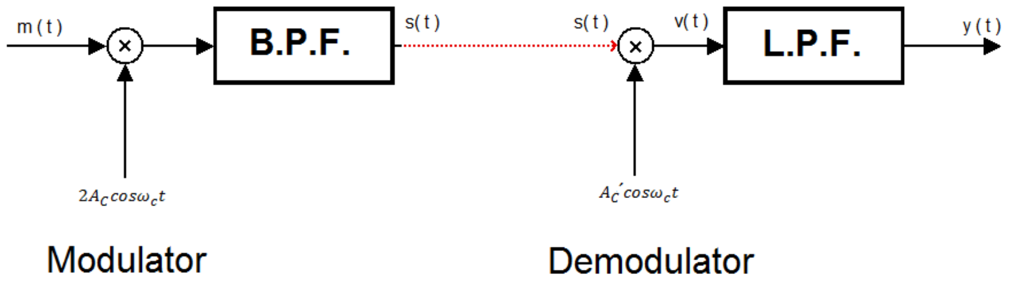
$s_1(t)$ is a single side band signal. The frequency gap of this signal extends over $(0, f_{1+}\ f_L)$. The second modulator views this signal as the baseband signal to be modulated on a carrier with frequency f_2.

The second modulator generates a DSB signal. The second BPF with center frequency f_{02} and transition band Δf_2 selects the upper side band. Again, we maintain that

$\Delta f_2 \geq 0.01\ f_{02}$ and $\Delta f_2 \leq\ 2\ (f_{1+}f_I)$

Demodulation of SSB: Time-Domain Analysis

A SSB signal can be demodulated using coherent demodulation (oscillator at the receiver should have the same frequency and phase as those of transmitter carrier) as shown in the figure:



Let the received signal be the upper single sideband

$$s(t) = A_C m(t)cos\omega_c t - A_C\widehat{m}(t)sin\omega_c t$$

At the receiver, $s(t)$ is mixed with the carrier signal. The result is

$$v(t) = s(t)A_C' cos\omega_c t$$

$$= A_C' \left[A_C m(t)cos\omega_c t - A_C\widehat{m}(t)sin\omega_c t\right]cos\omega_c t$$

$$= A_C A_C' m(t)cos2\omega_c t - A_C A_C'\widehat{m}(t)\, sin\omega_c t\, cos\omega_c t$$

$$= \frac{A_C A_C'}{2}m(t) + \frac{A_C A_C'}{2}m(t)cos2\omega_c t - \frac{A_C A_C'}{2}\widehat{m}(t)\, sin2\omega_c t$$

The low pass filter admits only the first terms. The output is:

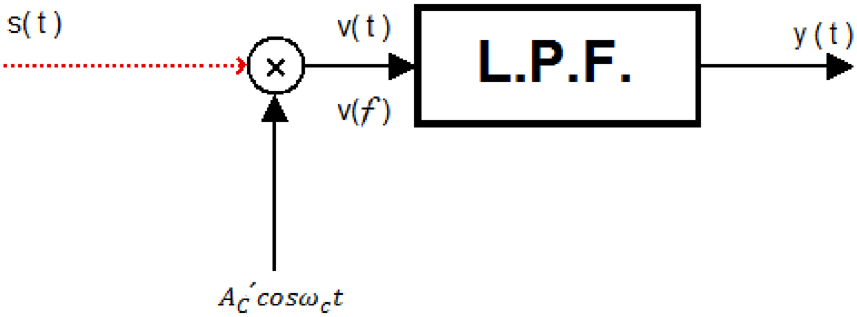
$$y(t) = \frac{A_C A_C'}{2}m(t)$$

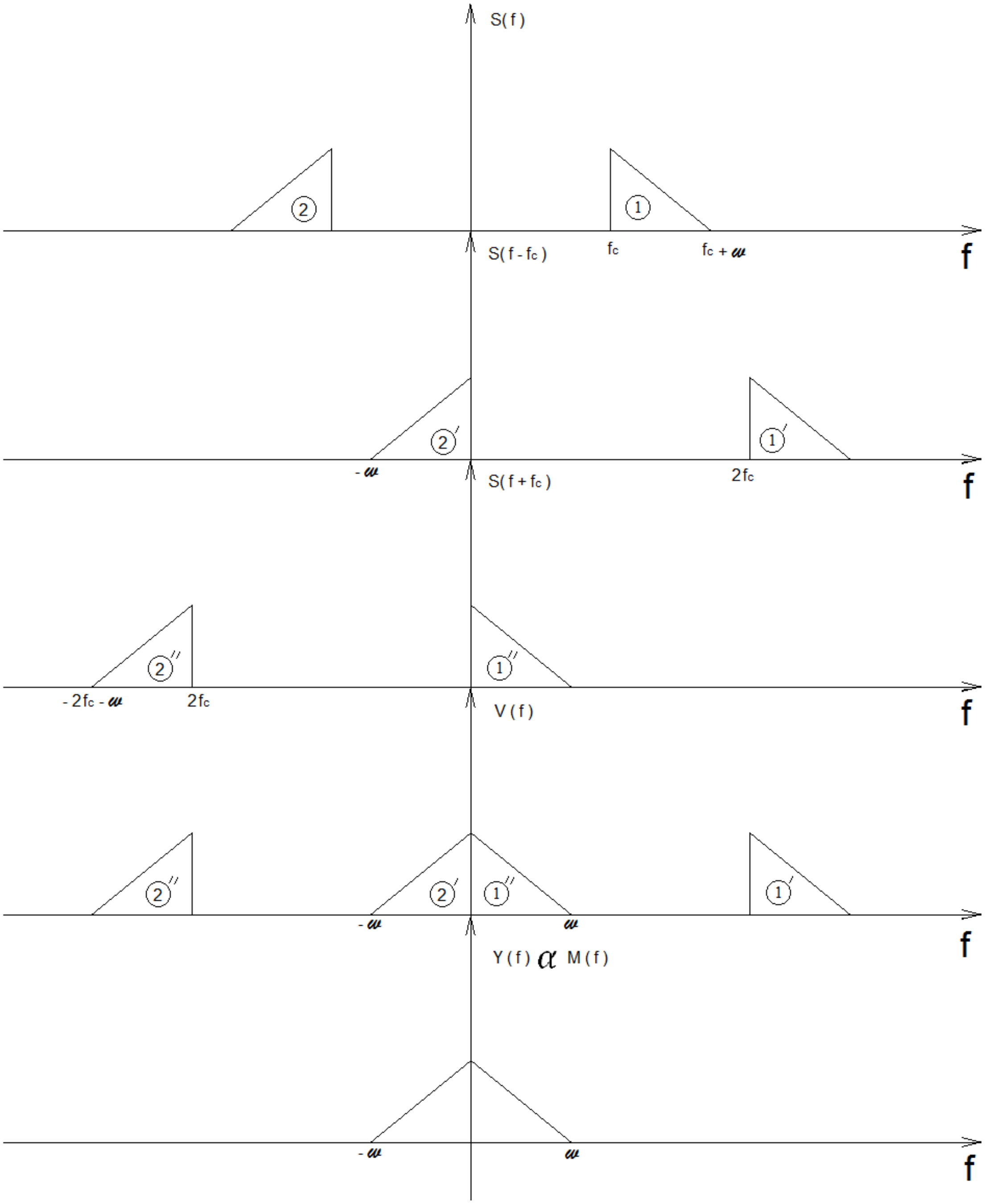
The following steps demonstrate the demodulation process viewed in the frequency domain .

$$V(f) = \frac{A_C'}{2}S(f - f_C) + \frac{A_C'}{2}S(f + f_C)$$

$$Y(f) = Low\ pass\ \left\{\frac{A_C'}{2}S(f - f_C) + \frac{A_C'}{2}S(f + f_C)\right\}$$

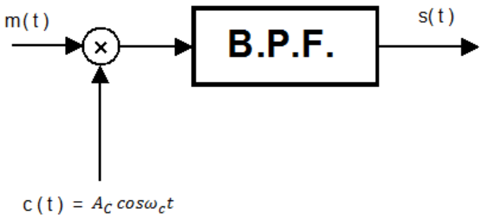
Demodulation of SSB signal : Why one side band is enough ?: A frequency-domain perspective

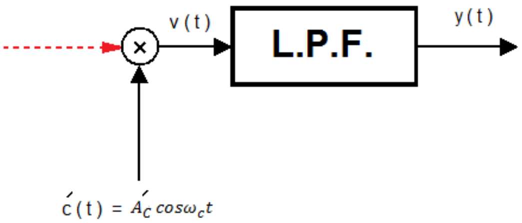




Demodulation of SSB : Coherent Demodulation

a. perfect coherent





when $c(t) = A_c \cos\omega_c t$, $\acute{c}(t) = \acute{A}_c \cos\omega_c t$,

we have perfect coherence and

$$y(t) = \frac{A_c\acute{A}_c}{2} m(t)$$

as was derived earlier

b. Constant phase difference

The local oscillator takes the form

$$\acute{c}(t) = \acute{A}_c \cos(\omega_c t + \emptyset);$$

$$v(t) = [A_c m(t)\cos\omega_c t - A_c\widehat{m}(t)\sin\omega_c t]\acute{A}_c\cos(\omega_c t + \emptyset)$$

$$= A_c\acute{A}_c m(t)\cos\omega_c t\cos(\omega_c t + \emptyset) - A_c\acute{A}_c\widehat{m}(t)\sin\omega_c t\cos(\omega_c t + \emptyset)$$

$$= \frac{A_c\acute{A}_c}{2} m(t)\cos(2\omega_c t + \emptyset) + \frac{A_c\acute{A}_c}{2} m(t)\cos(\emptyset)$$

$$-\frac{A_c\acute{A}_c}{2}\widehat{m}(t)\cos(2\omega_c t + \emptyset) - \frac{A_c\acute{A}_c}{2}\widehat{m}(t)\sin(\emptyset)$$

➔ $$y(t) = \frac{A_c\acute{A}_c}{2} m(t)\cos(\emptyset) - \frac{A_c\acute{A}_c}{2}\widehat{m}(t)\sin(\emptyset)$$

Note that there is a distortion due to the appearance of the Hilbert transform of the message signal at the output.

c. $\acute{c}(t) = \acute{A}_c \cos\ 2\pi(f_c + \Delta f)t;$ Constant frequency shift

$$v(t) = [A_c m(t)\cos\omega_c t - A_c\widehat{m}(t)\sin\omega_c t]\acute{A}_c\cos\ 2\pi(f_c + \Delta f)t$$

$$= \frac{A_c \acute{A}_c}{2} m(t)[\cos(2\omega_c + \Delta\omega)t + \cos 2\pi\Delta f t]$$

$$- \frac{A_c \acute{A}_c}{2} \widehat{m}(t)[\sin(2\omega_c + \Delta\omega)t + \sin 2\pi\Delta f t]$$

➔ $$y(t) = \frac{A_c \acute{\acute{A}}_c}{2} m(t) \cos 2\pi\Delta f t + \frac{A_c \acute{A}_c}{2} \widehat{m}(t) \sin 2\pi\Delta f t$$

Once again we have distortion and m(t) appears as if single sideband modulated on a carrier frequency = Δ f.

<u>Example :</u>

Let m(t) = $cos2\pi(1000)t$, $\Delta f =$ 100Hz and let s (t) be an upper sideband signal. Then ,

$$y(t) = \frac{A_c \acute{A}_c}{2} cos2\pi(1000)t\ cos2\pi(100)t + \frac{A_c \acute{A}_c}{2} sin2\pi(1000)t\ \ sin2\pi(100)t$$

$$\cos(a - b) = \cos a \cos b + \sin a \sin b$$

$$y(t) = \cos 2\pi(900)t \neq \cos 2\pi(1000)t$$

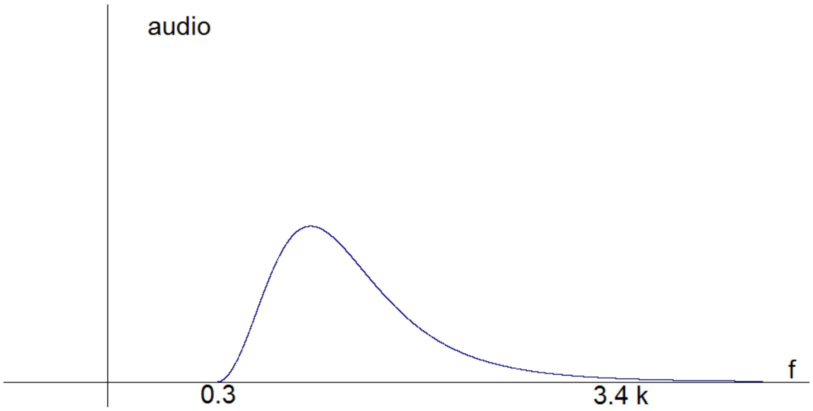
→ Distortion

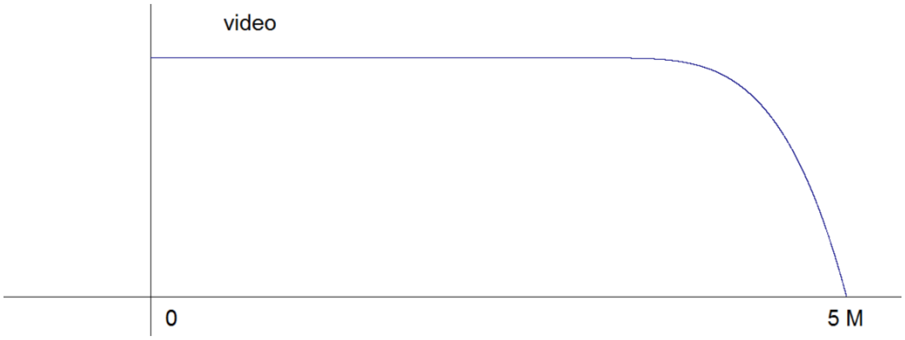
So, a message component with f = 1000Hz appears as a 900Hz component at the demodulator output.

Again , distortion results as a result of failing to synchronize the transmitter and receiver carrier frequencies.

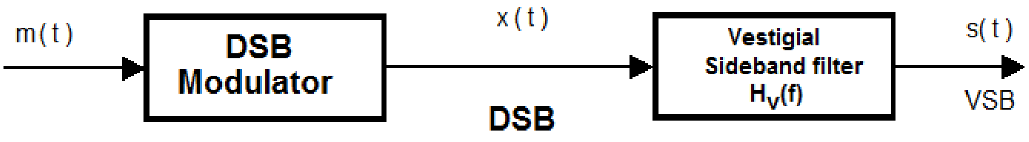
Vestigial sideband (VSB) modulation : (Not included and will not be covered)

- This type of modulation finds applications in the transmission of video signal .
- Unlike the audio signal , the video signal is rich in low frequency components around the zero frequency .
- The B.W of a video signal is about 5MHz.
- If a video signal is to be transmitted using DSB, it requires a 10 MHz B.W ; too large .
- If a video signal is to be transmitted using SSB (B.W = 5MHz) distortion will results due to the inability to suppress one of the sidebands completely using practical filters.
- A compromise between DSB and SSB was proposed called vestigial sideband modulation .
- Here, a DSB-SC signal is first generated The DSB is applied to a band pass filter (called a *vestigial filter*) that has an asymmetrical frequency response about (-+fc).
- The filter allows one of the sidebands to pass almost without attenuation , while a trace or a vestige of the second sideband is allowed to pass (most of the second sideband is attenuated)
- A typical spectral density of an audio and a video signal is shown below.





Generation of a VSB : Filtering method

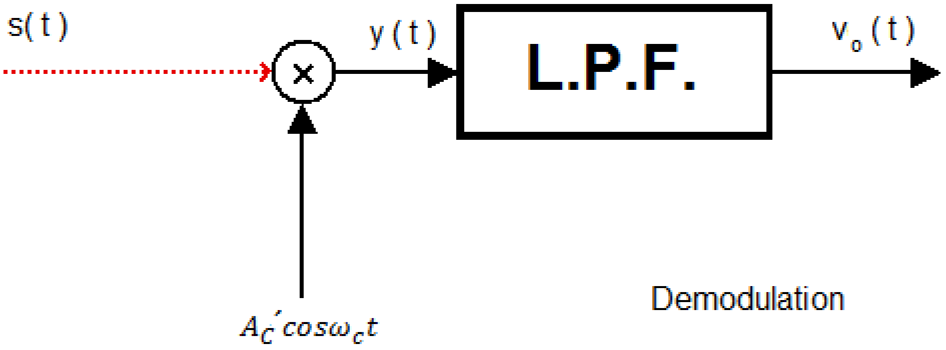


Let $H_v(f)$ be the transfer function of the vestigial filter. We need to find a condition on the characteristic of the filter such that the demodulated signal at the receiver is proportional to the message signal. Now we proceed to find such a condition.

$x(t) = A_c m(t) \cos \omega_c t$; A DSB-SC signal

$S(f) = X(f) H_v(f)$; The Fourier transform of the filter output.

$= \frac{A_c}{2}\{M(f-f_c) + M(f+f_c)\}H_v(f)$; VSB signal



The objective is to specify a condition on $H_v(f)$ such that $V_0(t)$ is an exact replica of m(t) .

$$y(t) = \acute{A}_c s(t) \cos \omega_c t$$

$$Y(f) = \frac{\acute{A}_c}{2}\{S(f-f_c) + S(f+f_c)\}$$

$$= \frac{A_c\acute{A}_c}{4}\{M(f-2f_c) + M(f)\}H_v(f-f_c)$$

$$+ \frac{A_c\acute{A}_c}{4}\{M(f+2f_c) + M(f)\}H_v(f+f_c)$$

The LPF will eliminate the high frequency component and retains only the low frequency terms.

$$Vo(f) = \frac{A_c \acute{A}_c}{4}\{H_v(f - f_c) + H_v(f + f_c)\}M(f)$$

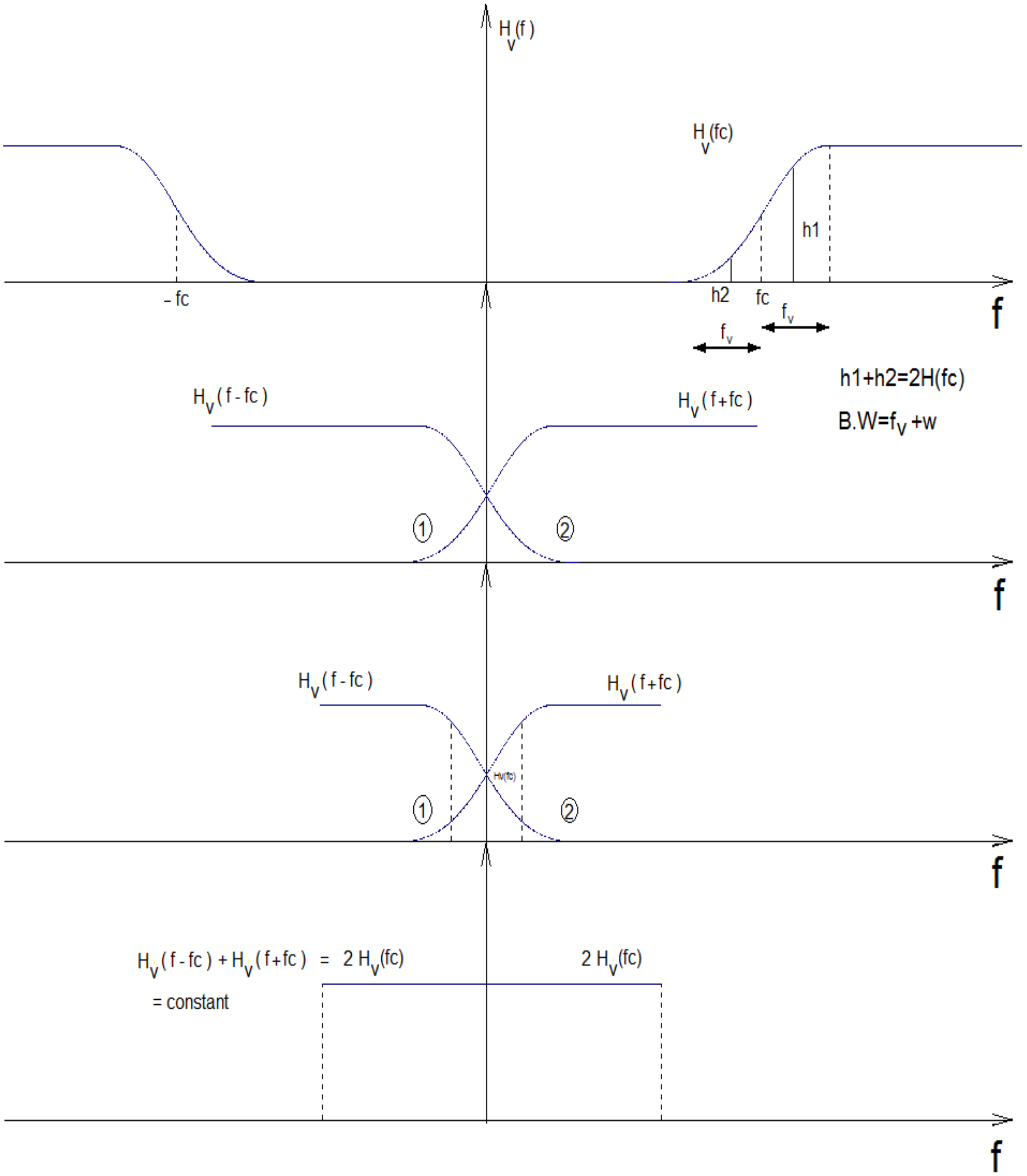
In order for $V_0(f)$ to be proportional to M(f) , we require that

$$H_v(f - f_c) + H_v(f + f_c) = constant = 2H_v(f_c)$$

When this condition is imposed on the filter, the output becomes

$$V_o(f) = \frac{A_c \acute{A}_c}{2} H_v(f_c) M(f)$$

$$v_o(t) = \frac{A_c \acute{A}_c}{2} H_v(f_c) m(t)$$



Two remarks :

1. B.W = $W + f_v$; f_v is the size of the vestige .
2. VSB can be demodulated using coherent demodulation .

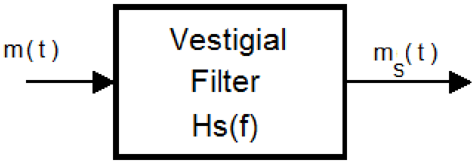
Generation of VSB: phase discrimination method

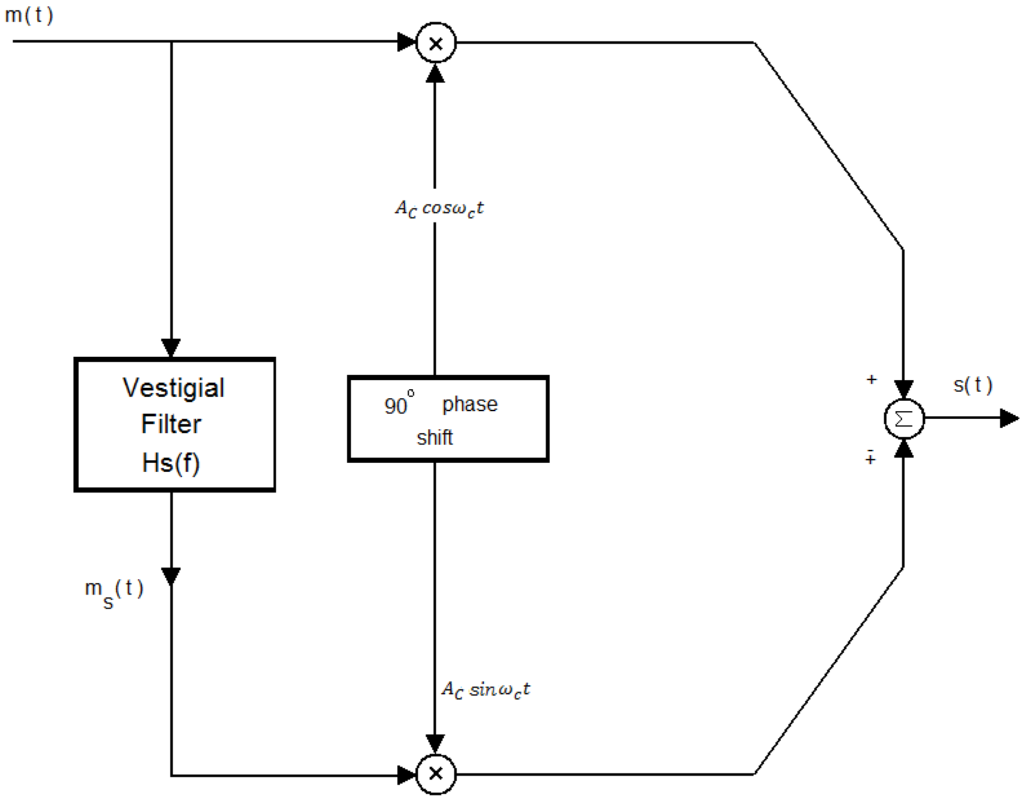
The time-domain representation of a VSB signal is

$$s(t) = \ A_c m(t) \cos \omega_c t \mp A_c m_s(t) \sin \omega_c t$$

Where $m_s(t)$ is the response of a vestigial filter (in the base band spectrum) to the message m(t). Using the time-domain representation ,

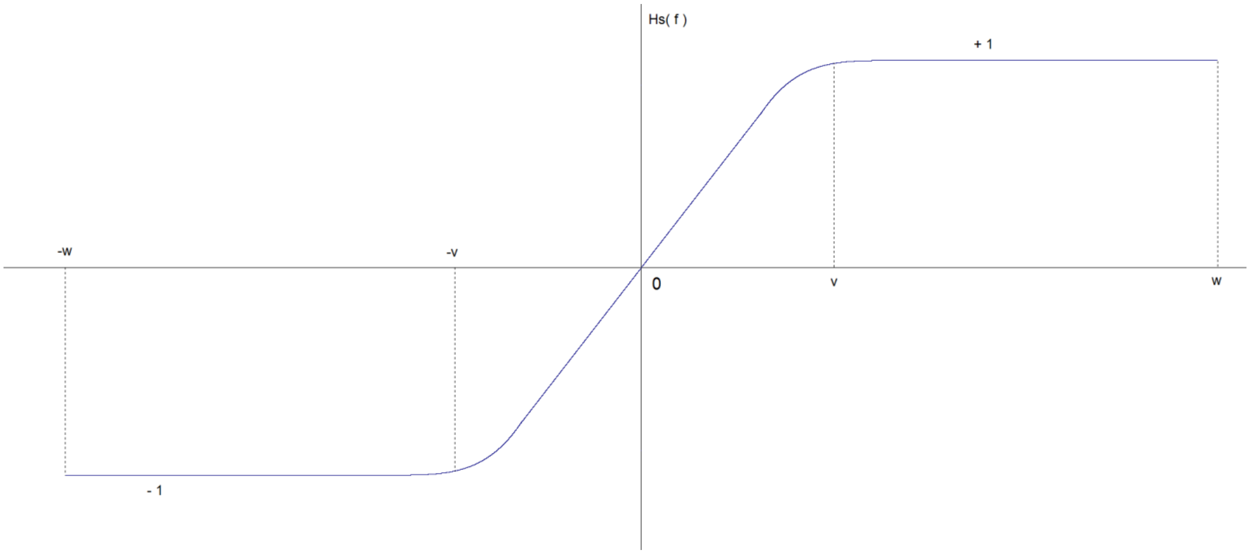
the following scheme may be used to generate a VSB signal





The – sign means that most of the upper sideband is admitted

+ sign means that most of the lower sideband is admitted

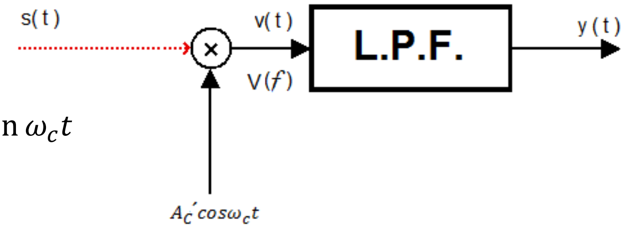


The transfer function HS(f) of the low pass filter is related to the band pass characteristic by:

$$H_s(f) = Low\ pass\ \{H_v(f+f_c) - H_v(f-f_c)\}$$

Coherent Detection of VSB : Time Domain Analysis

Let the received VSB signal be given as:



$$s(t) = A_c m(t)\cos\omega_c t - A_c m_s(t)\sin\omega_c t$$

This signal is mixed with a version of the transmitted carrier of the same phase and frequency.

$$v(t) = s(t)\acute{A}_c \cos 2\pi f_c t$$

$$= A_c\acute{A}_c[m(t)\cos\omega_c t - m_s(t)\sin\omega_c t]\cos\omega_c t$$

$$= A_c\acute{A}_c m(t)\cos^2\omega_c t - A_c\acute{A}_c m_s(t)\sin\omega_c t\cos\omega_c t$$

$$= \frac{A_c\acute{A}_c}{2}m(t) + \frac{A_c\acute{A}_c}{2}m(t)\cos 2\omega_c t - \frac{A_c\acute{A}_c}{2}m_s(t)\sin 2\omega_c t$$

The low pass filter admits only the low pass component, which is nothing but a scaled version of the message signal.

$$y(t) = \frac{A_c \acute{A}_c}{2} m(t)$$

Envelope Detection of VSB + Carrier :

This type of modulation takes the form :

$$s(t) = carrier + VSB$$

$$s(t) = A_c \cos \omega_c t + A_c \beta m(t) \cos \omega_c t \mp A_c \beta m_s(t) \sin \omega_c t$$

β is a scaling factor chosen to minimize envelope distortion. The addition of the carrier is meant to simplify the demodulation of the video signal in practical TV systems and avoids the complexity of coherent demodulation. It is also less expensive since a simple envelope detector, of the type described in demodulating a normal AM signal, can be used.

$$s(t) = \left(A_c + A_c \beta m(t)\right) \cos \omega_c t \mp A_c \beta m_s(t) \sin \omega_c t$$

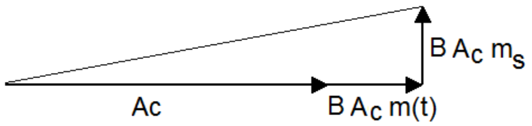
$$s(t) = \sqrt{\left(A_c + A_c \beta m(t)\right)^2 + {A_c}^2 \beta^2 {m_s}^2(t)} \;\; \cos(\omega_c t + \emptyset)$$

If s(t) is applied to an envelope detector (which is insensitive to phase variations), the output is

$$y(t) = \sqrt{(A_c\left(1 + \beta m(t)\right)^2 + {A_c}^2 \beta^2 {m_s}^2(t)}$$

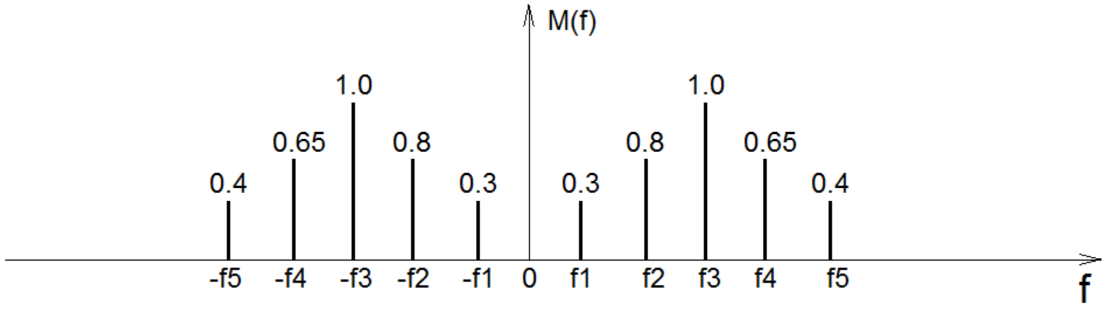
If $A_c \gg \beta m(t)$, then

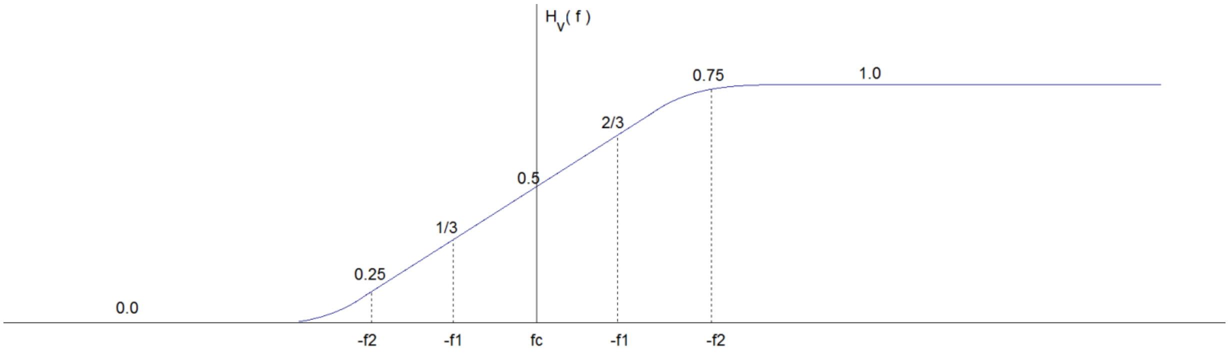
$$y(t) \cong A_c(1 + \beta m(t))$$

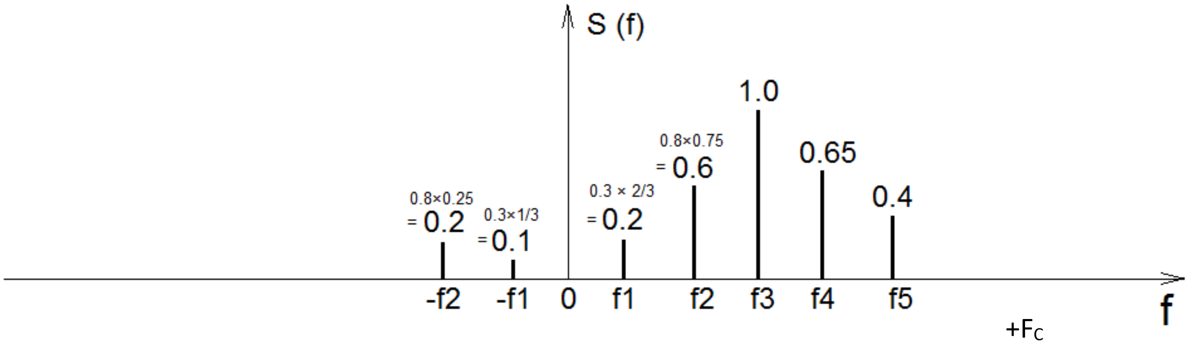


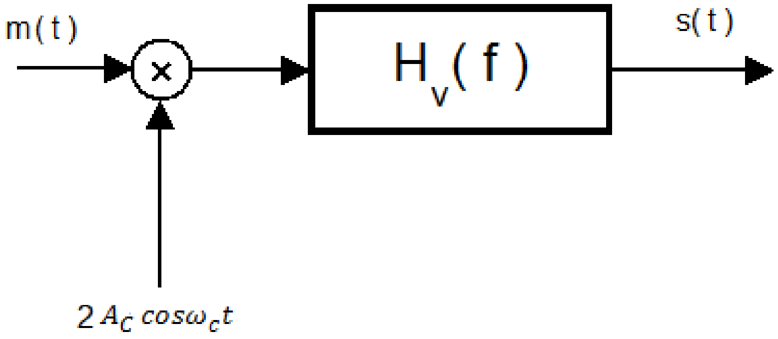
Hence , m(t) can be demodulated, almost without distortion, using simple envelope detection techniques if the above condition is satisfied .

Example: A VSB is generated from the DSB-SC signal $2m(t)\cos\omega_c t$. M(f) and H_v(f) are shown below . Find the spectrum of the transmitted signal s(t).









Baseband Signal :

The input signal consists of five frequency components. It is represented as:

$$m(t) = 0.6 \cos 2\pi f_1 t + 1.6 \cos 2\pi f_2 t$$

$$+2 \cos 2\pi f_3 t + 1.3 \cos 2\pi f_4 t + 0.8 \cos 2\pi f_5 t$$

Transmitted signal :

The spectrum of the transmitted signal is:

$$S(f) = H_v(f)M(f - f_c) + H_v(f)M(f + f_c)$$

If we perform the multiplication in the frequency domain and take the inverse Fourier transform, we get the time domain representation of the transmitted signal.

$$s(t) = 0.4 \cos 2\pi(f_c - f_2)t + 0.2 \cos 2\pi(f_c + f_1)t$$

$$+0.4 \cos 2\pi(f_c + f_1)t + 1.2 \cos 2\pi(f_c + f_2)t + 2 \cos 2\pi(f_c + f_3)t$$

$$+ 1.3 \cos 2\pi(f_c + f_4)t + 0.8 \cos 2\pi(f_c + f_5)t$$