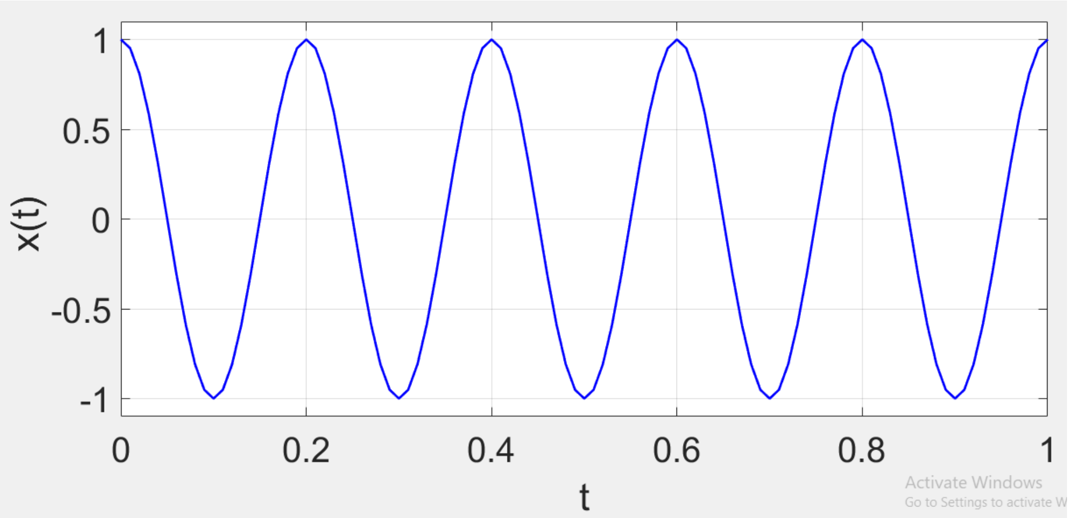
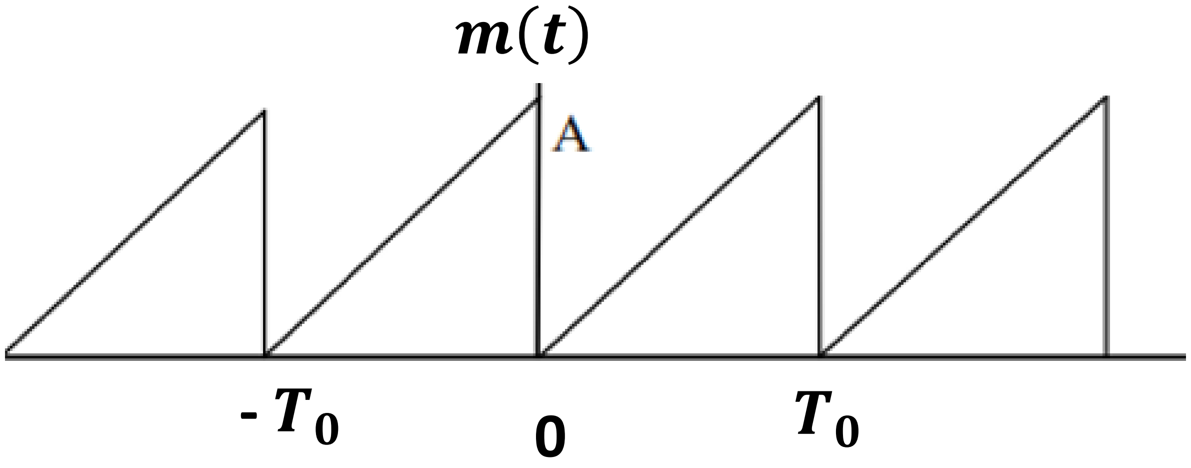
Signal Classifications

- **Definition:** A signal may be defined as a single valued function of time that conveys information.
- Depending on the feature of interest, we may distinguish four different classes of signals:
  - Periodic and Non-periodic Signals
  - Deterministic and Random Signals
  - Analog and Digital Signals
  - Energy and Power Signals

Classification of Signals: Periodic and Non-periodic

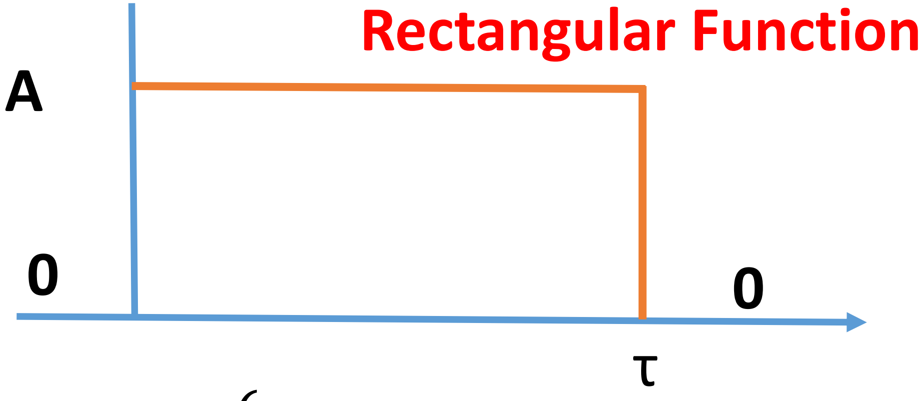
- **A periodic signal** g(t) is a function of time that satisfies the condition
$$g(t) = g(t + T_0), \forall t.$$
- The smallest value of T_0 that satisfies this condition is called the period of $g(t)$.
- **Example**: The sinusoidal signal $x(t) = \cos(2\pi(5)t)$ is periodic with period $\boldsymbol{T_0 = 1/5}$.
- The reciprocal of the period is the **fundamental frequency** $f_0 = \frac{1}{T_0}$. In this example, $\boldsymbol{f_0 = 5}$ **Hz**.
- **Example**: The saw-tooth function shown is another example of a periodic signal.
- $m(t) = \frac{A}{T_0} t, \quad 0 \le t \le T_0$
- If $T_0 = 0.001\ sec$, then the fundamental frequency $\boldsymbol{f_0 = 1000}$ **Hz**



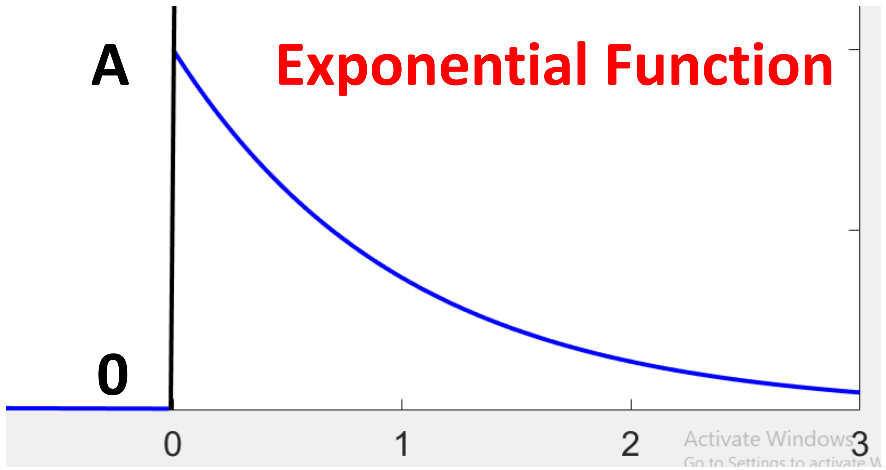


Non-periodic Signals

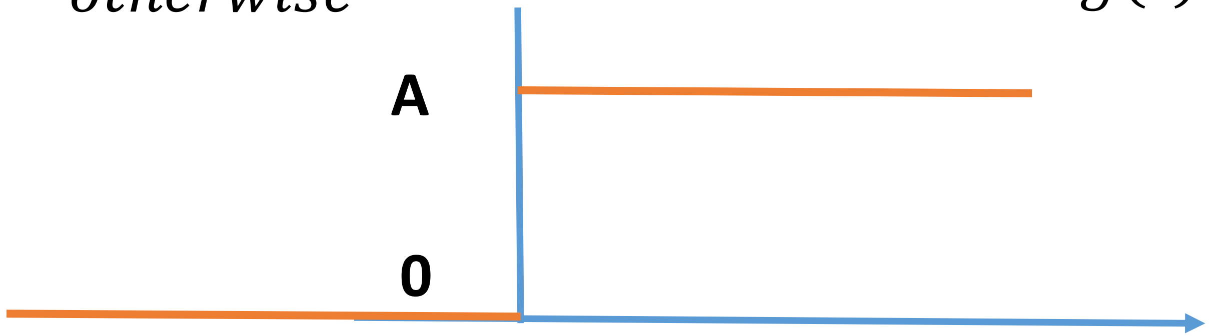
- A non-periodic signal g(t) is one for which there does not exist a T_0 for which the condition $g(t) = g(t + T_0)$ is satisfied, i.e., the signal does not repeat itself each T_0 .



$$g(t) = \begin{cases} A, & 0 \le t \le \tau \\ 0, & otherwise \end{cases}$$



$$g(t) = \begin{cases} A\exp(-\alpha t), & 0 \le t < \infty \\ 0, & t < 0 \end{cases}$$



Step Function Au(t)

$$g(t) = \begin{cases} A, & t > 0 \\ 0 & t < 0 \end{cases}$$

Deterministic and Random Signals

- **A deterministic signal** is one about which there is no uncertainty with respect to its value at any time. It is a completely specified function of time.
- **Deterministic Signal Example:** $x(t) = Ae^{-at}u(t)$; $A = 1$ and α is a constant.
- **A random signal** is one about which there is some degree of uncertainty before it actually occurs. (It is a function of a random variable)
- **Random Signal Example:** $x(t) = Ae^{-at}u(t)$; α is a constant and A is a random variable with the following probability density function (two possible realizations shown below)

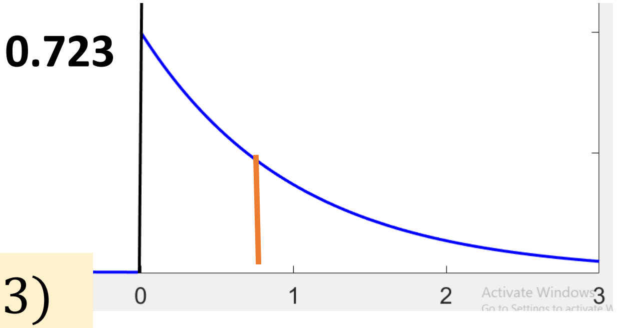
$$f_A(a) = \begin{cases} 1 & 0 \le a \le 1 \\ 0 & otherwise \end{cases}$$

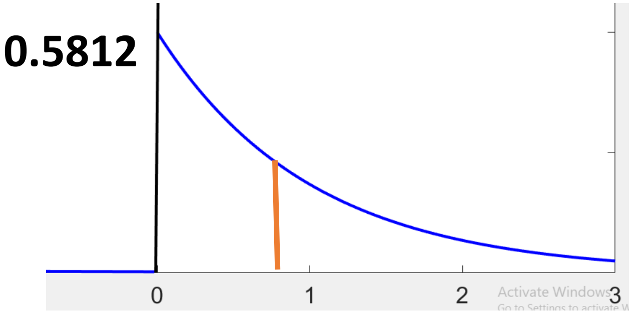
- **Random Signal Example:** $x(t) = \cos(2\pi f_c t + \Theta)$; f_c is a constant and Θ is a random variable uniformly distributed over the interval $(0, 2\pi)$ with the following probability density function (pdf).

$$f_\Theta(\theta) = \begin{cases} \frac{1}{2\pi} & 0 \le \theta \le 2\pi \\ 0 & otherwise \end{cases}$$

$x(t) = \cos(2\pi f_c t + 30)$

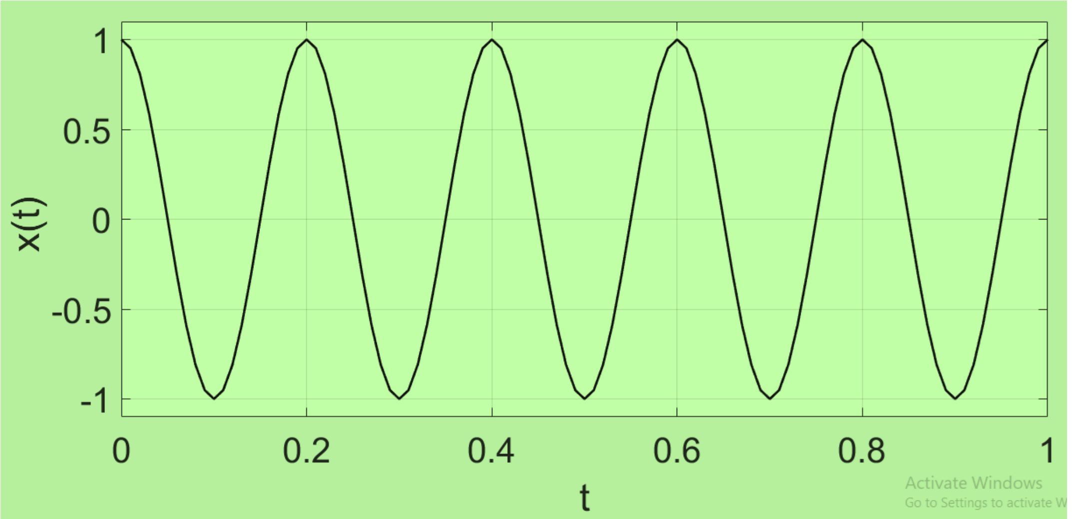
$x(t) = \cos(2\pi f_c t + 63)$

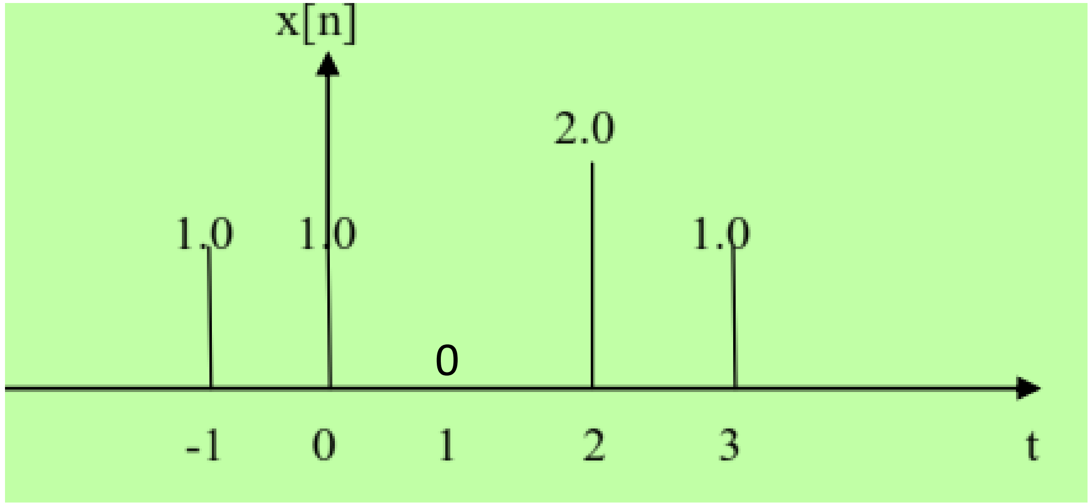




Analog and Digital Signals

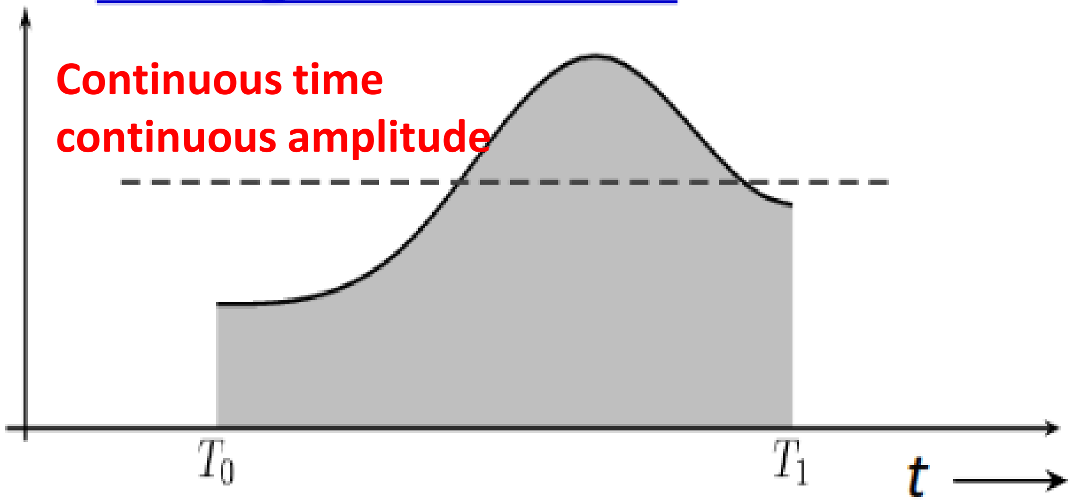
- In an **analog signal** the amplitude takes on any value within a defined range of continuous values.
- **Example**: The sinusoidal signal $x(t) = A\cos 2\pi f_0 t$, $-\infty < t < \infty$, is an example of an analog signal.
- ***A digital signal*** : The values assumed by the signal belong to a finite and countable set.
- **Example**: The sequence x[n] shown below is an examples of a digital signal. The amplitudes are drawn from the finite set {1, 0, 2}.





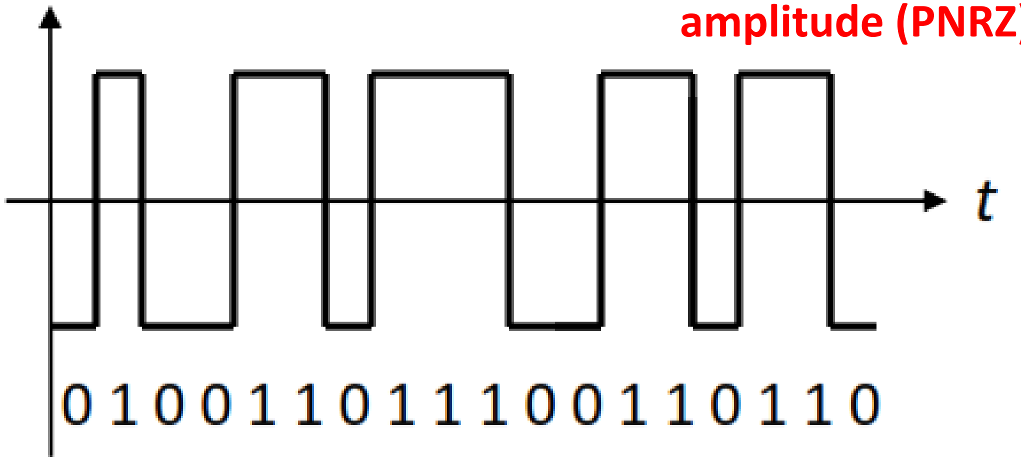
Analog and Digital Signals: Continuous Valued and Discrete Valued

Analog & continuous

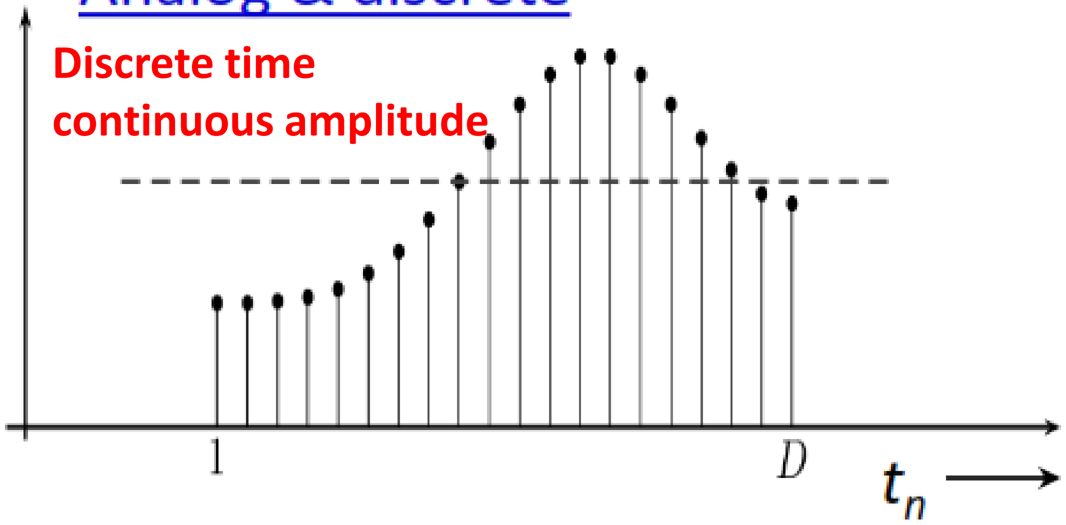


Digital & continuous

Continuous time discrete amplitude (PNRZ)



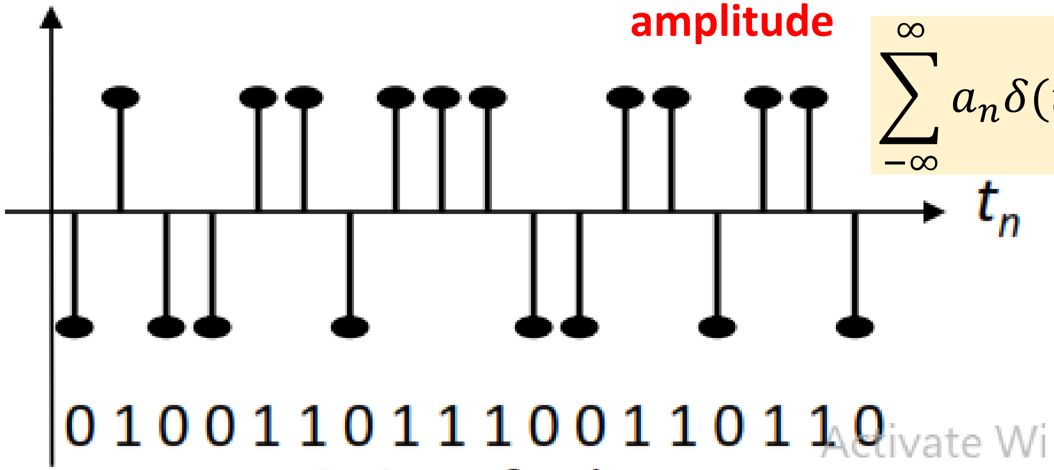
Analog & discrete



Digital & discrete

discrete time discrete amplitude

$$\sum_{-\infty}^{\infty} a_n \delta(t - kT_s)$$



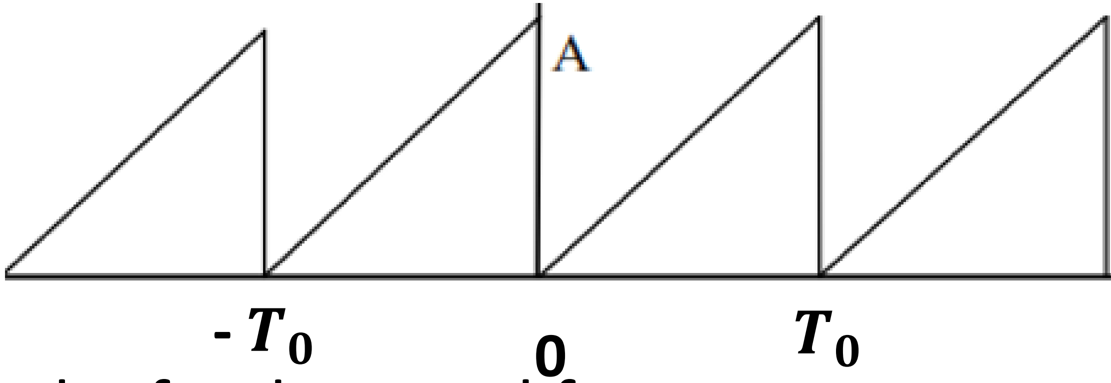
String of values

Average Value of a Signal

- ***The average value of a signal*** $g(t)$ over an observation interval of 2T centered at the origin is:

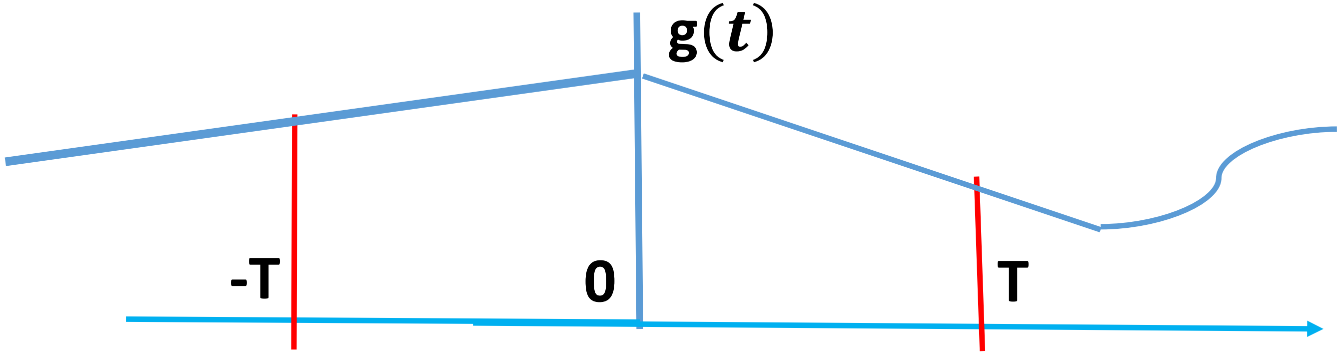
$$g_{av} = \frac{1}{2T}\int_{-T}^{T} g(t)\, dt$$

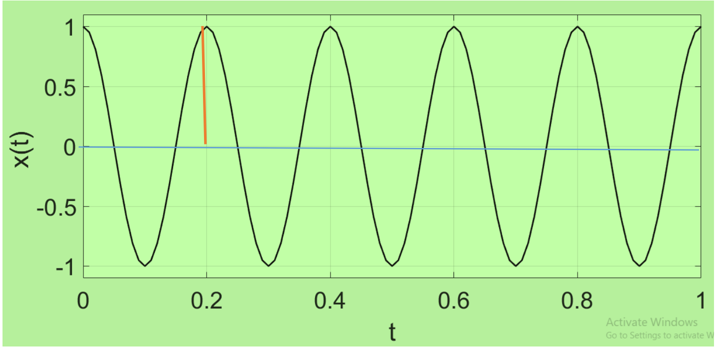
- The average value of a periodic signal $g(t)$ is



$$\boldsymbol{g_{av} = \frac{1}{T_0}\int_{0}^{T_0} g(t)\, dt}$$; T_0 is the period; f_0 is the fundamental frequency.

- **Example**: Find the average value of the sinusoidal signal
- $x(t) = A\cos 2\pi f_0 t\;, -\infty < t < \infty$
- **Solution**: $x_{av} = \frac{1}{T_0}\int_{0}^{T_0} A\cos 2\pi f_0 t dt = -\frac{A\sin 2\pi f_0 t}{2\pi f_0}\Big|_0^{T_0} = 0$





Energy and Power Signals

- ***The instantaneous power*** in a signal g(t) is defined as that power dissipated in a 1-Ω resistor, i.e.,

$$p(t) = |g(t)|^2$$

- ***The average power*** over an observation interval of 2T centered at the origin is:

$$P_{av} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |g(t)|^2 \, dt$$

- The total energy of a signal $g(t)$ is

$$E = \lim_{T\to\infty} \int_{-T}^{T} |g(t)|^2 \, dt$$

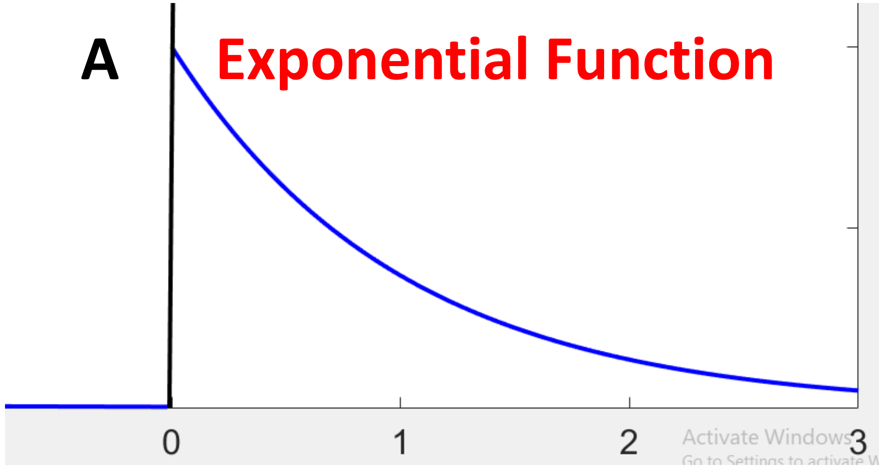
- A signal g(t) is classified as **energy signal** if it has a finite energy, i.e,$0 < E < \infty$.
- A signal g(t) is classified as **power signal** if it has a finite power, i.e, $0 < P_{av} < \infty$.
- The average power in a periodic signal $g(t)$ is

$$\boldsymbol{P_{av} = \frac{1}{T_0} \int_{0}^{T_0} |g(t)|^2 \, dt}$$; T_0 is the period; f_0 is the fundamental frequency.

Energy and Power Signals

- **Example: Consider the Exponential Pulse**

$g(t) = Ae^{-\alpha t}u(t)$. **Is it an energy or a power signal?**



Solution: Let us first find the energy in the signal

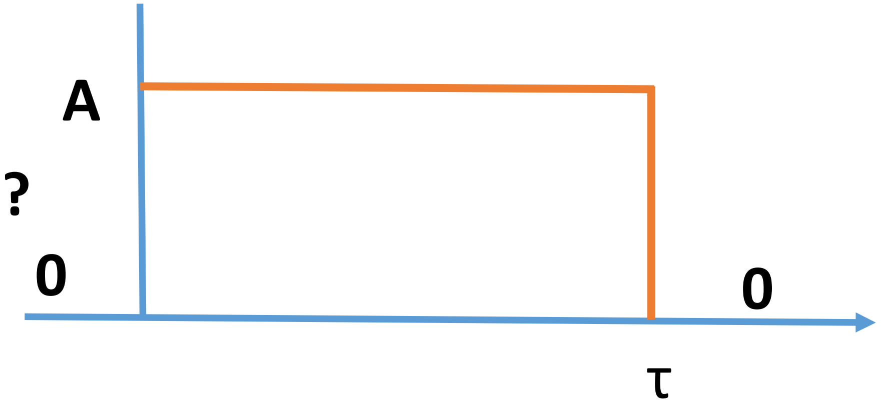
$$E = \int_0^{\infty} A^2\, e^{-2\alpha t} dt = A^2 \frac{-e^{-2\alpha t}}{2\alpha} \Big|_0^{\infty} = \frac{A^2}{2\alpha}.$$

Since E is finite, then $g(t)$ is an energy signal.

- **Example: Consider the Rectangular Pulse**

$$g(t) = \begin{cases} A, & 0 < t < \tau \\ 0, & o.w \end{cases}$$

Is it an energy or a power signal?



- **Solution**: Let us first find the energy in the signal

$E = \int_0^{\tau} A^2\, dt = A^2\tau$. This is an energy signal since E is finite.

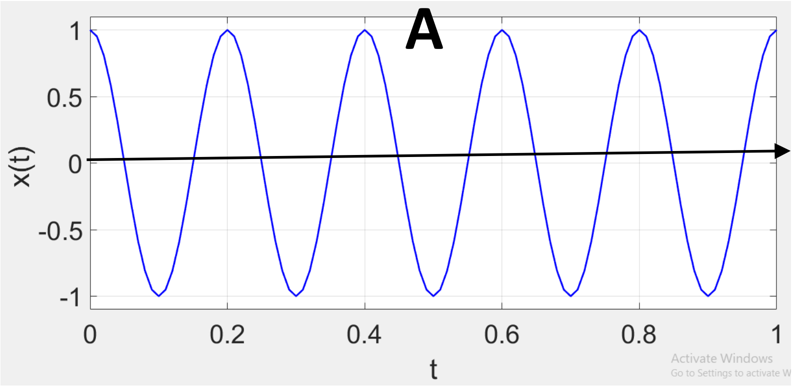
Energy and Power Signals

Example: Consider the Periodic Sinusoidal Signal

- $g(t) = A\cos(2\pi f_0 t),\ -\infty < t < \infty$; **Is it an energy or a power signal**
- Since $g(t)$ is periodic, then

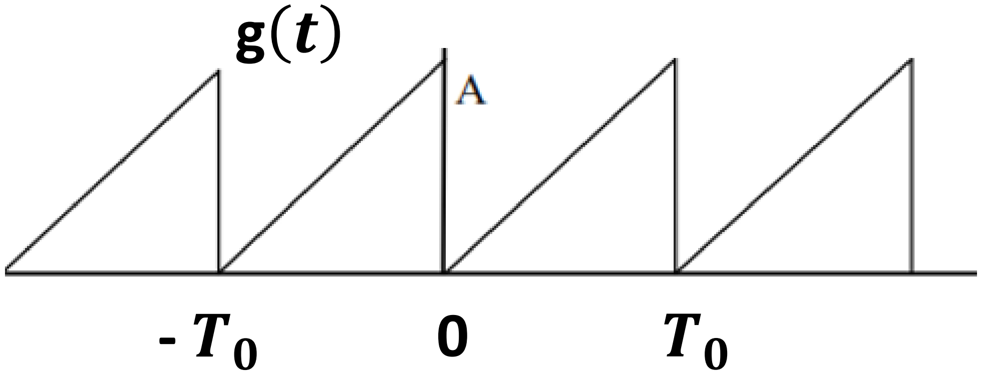
$$P_{av} = \frac{1}{T_0}\int_0^{T_0} A^2 \cos^2 \omega t\, dt = \frac{A^2}{T_0}\int_0^{T_0}\left(\frac{1+\cos 2\omega t}{2}\right) = \left(\frac{A^2}{T_0}\right).\left(\frac{T_0}{2}\right) \Rightarrow \boldsymbol{P_{av} = \frac{A^2}{2}}.$$

- Here, P_{av} is finite. Therefore, $g(t)$ is a power signal.



Example: Consider the Periodic Saw-tooth Signal

- $g(t) = \frac{A}{T_0}t,\ 0 \le t \le T_0$. **Is it an energy or a power signal?**
- Let us evaluate the average power in $g(t)$
- $P_{av} = \frac{1}{T_0}\int_0^{T_0} \frac{A^2}{T_0^2} t^2 dt = \frac{1}{T_0}\frac{A^2}{T_0^2}\frac{t^3}{3}\Big|_0^{T_0} = \frac{A^2 T_0^3}{3\, T_0^3} = \frac{A^2}{3}$.
- Here, P_{av} is finite. Therefore, $g(t)$ is a power signal.



Energy and Power Signals

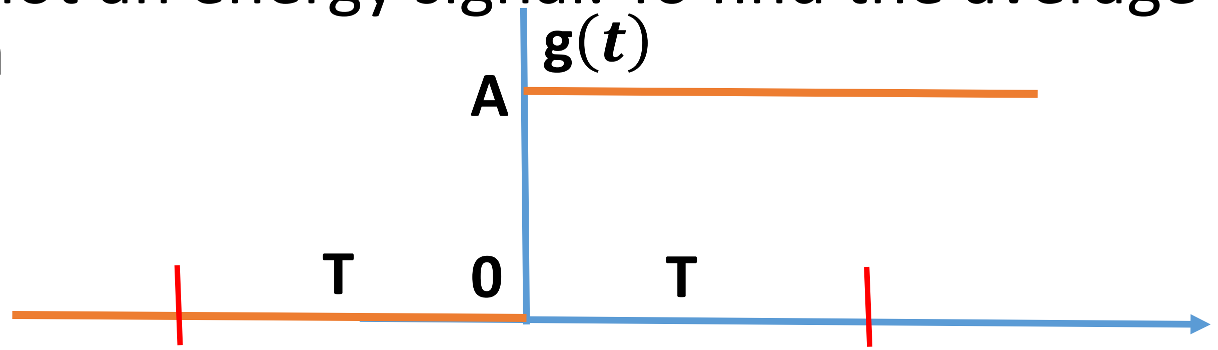
Example: The Unit Step Function $g(t) = Au(t)$

- This is a non-periodic signal. Let us first try to find its energy

$$E = \int_0^\infty A^2 \, dt \to \infty$$

- Sine E is not finite, then $g(t)$ is not an energy signal. To find the average power, we employ the definition

$$P_{av} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |g(t)|^2 \, dt,$$



- where 2T is chosen to be a symmetrical interval about the origin.
- $$P_{av} = \lim_{T\to\infty} \frac{1}{2T} \int_0^T A^2 \, dt = \lim_{T\to\infty} \frac{A^2 \, T}{2T} = \frac{A^2}{2}.$$
- So, even-though $g(t)$ is non-periodic, it turns out that it is a power signal.
- **Remark**: This is an example where the general rule (periodic signals are power signals and non-periodic signals are energy signals) fails to hold.