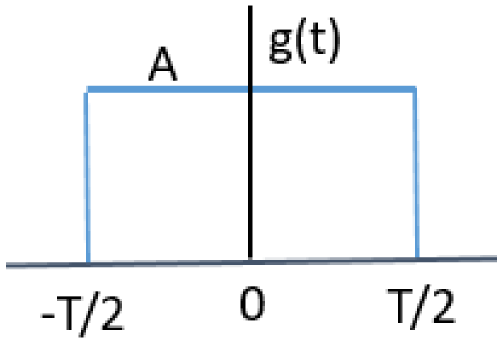
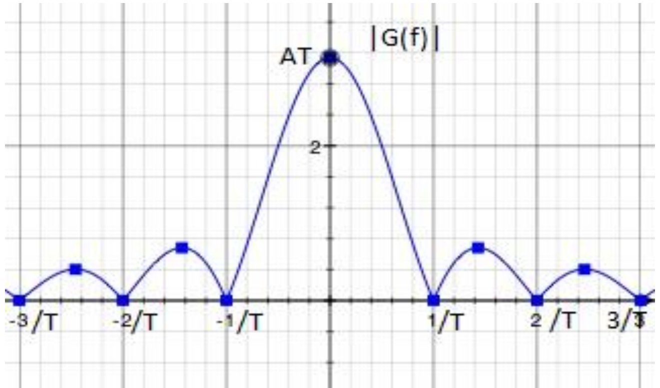
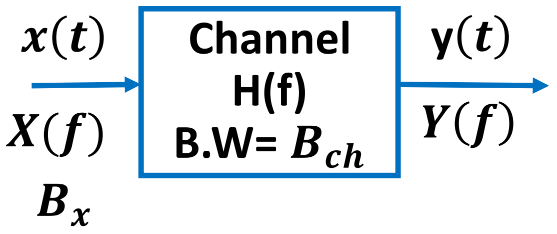
Pulse Response and Rise-time

- In this lecture, we will investigate the relationship that should exist between the pulse bandwidth and the channel bandwidth. As we know, the rectangular pulse contains significant high frequency components. When that pulse is passed through a band-limited channel, the channel will alter the shape of the input resulting in linear distortion (amplitude and phase)
- This subject is of particular importance, especially, when we study the transmission of data over band-limited channels. In the simplest form, a binary digit 1 may be represented by a pulse , $0 \leq t \leq T_b$, while binary digit 0 may be represented by the negative pulse $-A$, $0 \leq t \leq T_b$. Therefore, in order to retrieve the transmitted data, the channel bandwidth must be wide enough to accommodate the transmitted data.
- To convey this idea in a simple form, we first consider the response of a first order low pass filter to a unit step function and then to a pulse.

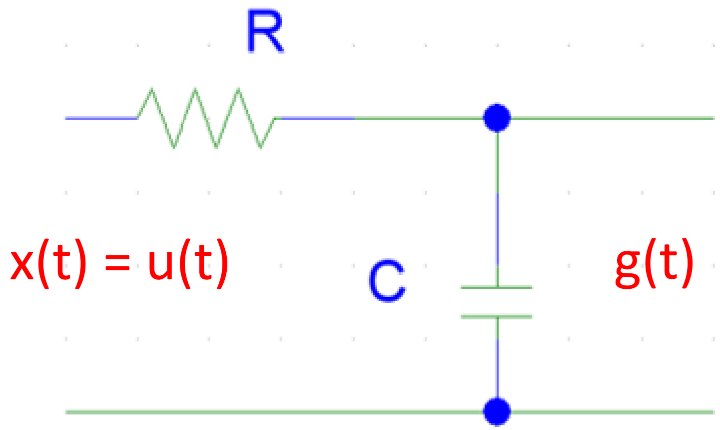






Step Response of a First Order System

- Let $x(t) = u(t)$ be applied to a first order RC circuit. This first order filter is a fair representation of a low-pass communication channel
- The system differential equation is
- $x(t) = Ri(t) + g(t) = RC\frac{dg(t)}{dt} + g(t)$
- where g(t) is the channel output.

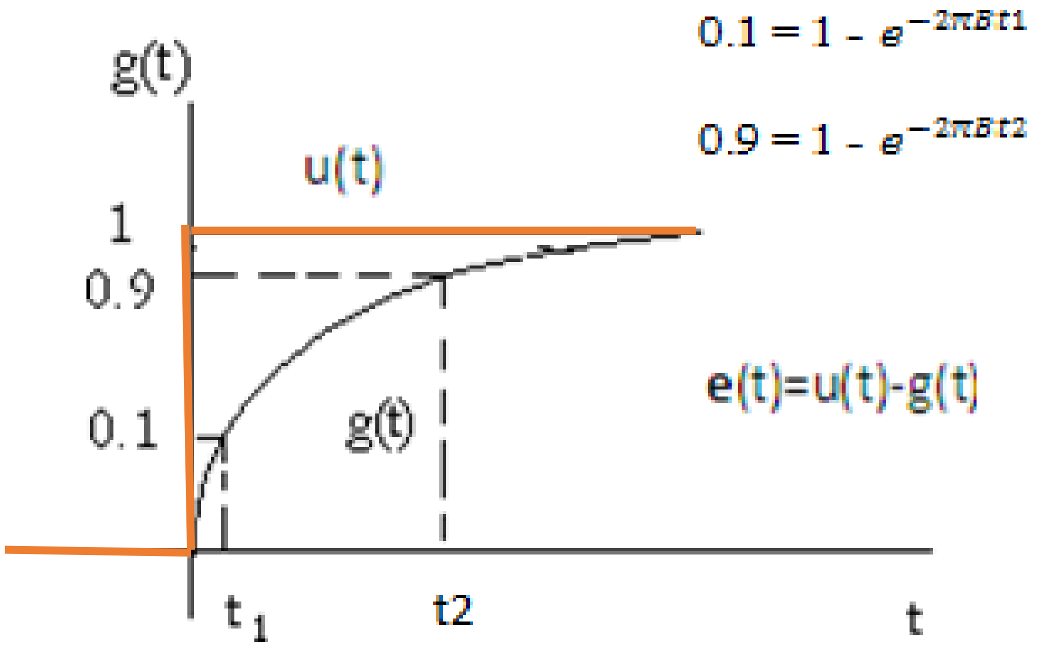


- Now let $x(t) = u(t)$. The system D.E. becomes $x(t)$
- $RC\frac{dg(t)}{dt} + g(t) = u(t)$
- The solution to this first order system is
- $g(t) = (1 - e^{-t/RC})u(t)$
- The 3- dB bandwidth of the channel (was derived in a previous example in this chapter) is $\boldsymbol{B_{ch} = \frac{1}{2\pi RC}}$
- The output $g(t)$, expressed in terms of B_{ch} becomes

$$\boldsymbol{g(t) = (1 - e^{-2\pi B_{ch} t})u(t)}$$

Step Response and Rise-time of a First Order System

- Define the difference between the input and the output as
- $\boldsymbol{e(t) = u(t) - g(t) = e^{-2\pi B_{ch} t}}$
- Note that $e(t)$ decreases as B_{ch} increases. This means that as the channel bandwidth increases, the output becomes closer and closer to the input.
- In the ideal case, when the channel bandwidth becomes infinity, the output becomes a step function.
- In essence, to reproduce a step function (or a rectangular pulse), **a channel with infinite bandwidth is needed**.



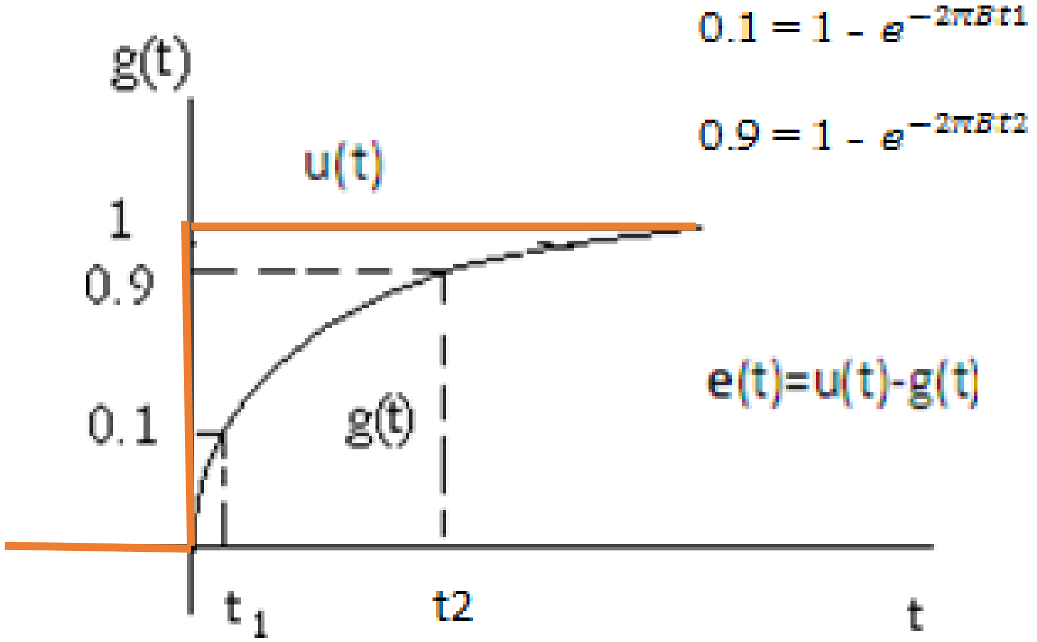
Step Response and Rise-time of a First Order System

- The rise-time is a measure of the speed of rise of the output of a system due to step function applied at its input.
- One common measure is the 10-90 % rise-time, defined as the time it takes for the output to rise between 10% to 90% of the final steady state value when a unit step function is applied to the system input.
- The 10% - 90% rise-time for the first order RC circuit considered above is

$$\boldsymbol{T_r = t_2 - t_1 = \frac{0.35}{B_{ch}}}$$

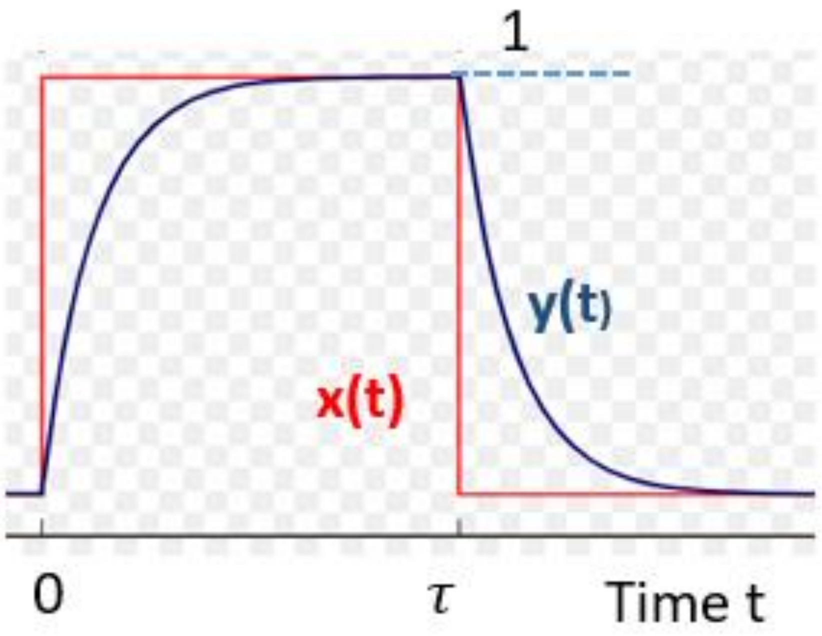
- **<u>Exercise:</u>** For the system above, verify that the rise-time is given as $T_r = \frac{0.35}{B_{ch}}$.

- From this result, we conclude that **increasing the bandwidth of the channel will decrease the rise-time**, implying a faster response.



Pulse Response of a First Order System

- It is the response of the circuit to a pulse of duration τ. For the same RC circuit, considered above, let us apply the pulse
- $x(t) = u(t) - u(t-\tau)$
- Using the linearity and time invariance properties, the output due to the pulse can be obtained from the step response g(t) as



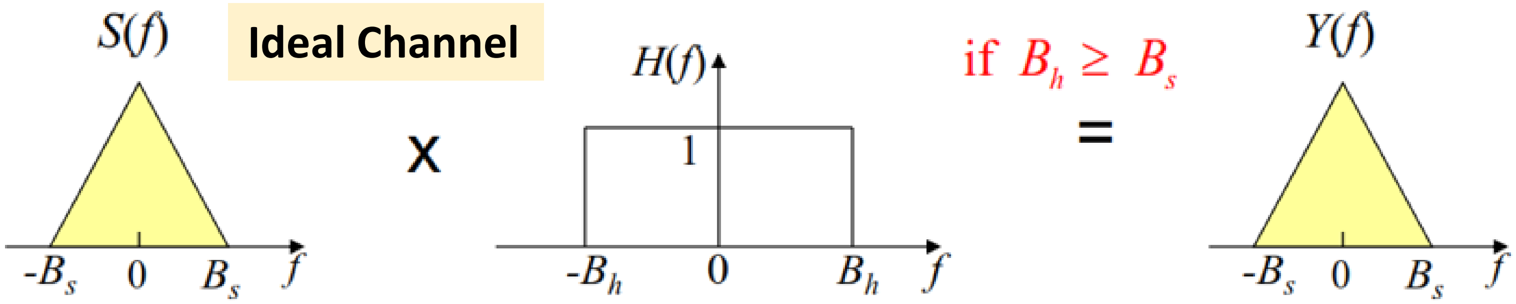
- $y(t) = g(t) - g(t-\tau)$
- $y(t) = \begin{cases} 0 & t < 0 \\ 1 - e^{-2\pi B_{ch} t} & 0 < t < \tau \\ (1 - e^{-2\pi B_{ch}\tau}) e^{-2\pi B_{ch}(t-\tau)} & t > \tau \end{cases}$
- This response is sketched in the figure below.
- From the equation above, we observe that the output $y(t)$ approximates the input $x(t)$ provided that ($y(\tau) > 0.99$)

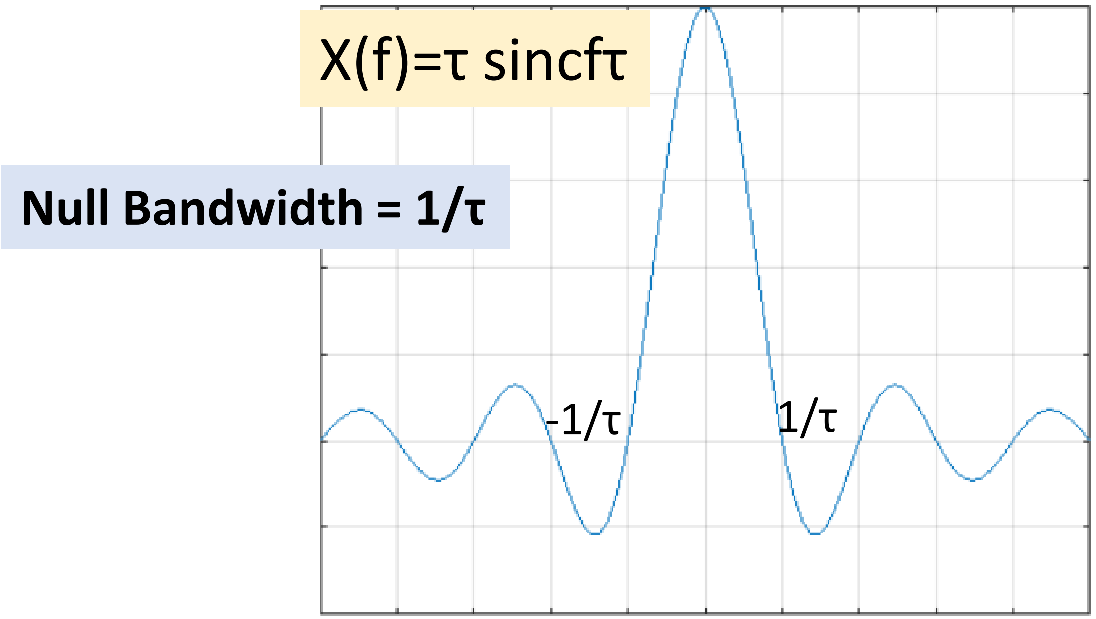
$$\boldsymbol{B_{ch}\tau \geq 1} \quad \textbf{or} \quad \boldsymbol{B_{ch} \geq \frac{1}{\tau}}$$

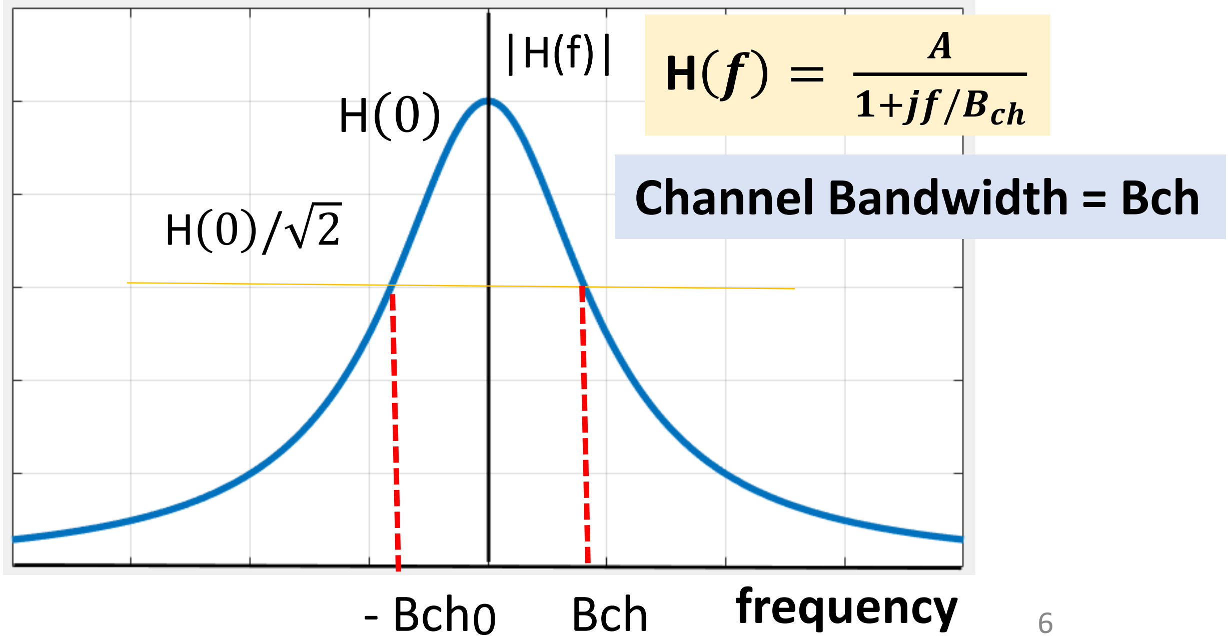
Pulse Response of a First Order System

- The figure below shows the Fourier transform of the input and the channel.
- To reproduce the input, the channel bandwidth should be wider than the message bandwidth
- Y (f) = X(f) H(f)
- $Y(f) \simeq X(f)$

- If the channel bandwidth is much wider than the message bandwidth, then







Relationship to data transmission

- In digital communication systems, data are transmitted at a rate of R_b bits/sec. The time allocated for each bit is $\tau = \frac{1}{R_b}$. To enable the receiver to recognize the transmitted bit within its allocated slot and to prevent cross talk between neighboring time slots, we require that

$$\boldsymbol{B_{ch} \geq \frac{1}{\tau} = R_b}$$

- **Result**: the channel bandwidth in binary digital communication systems should be larger than the rate of the data sent over the channel.