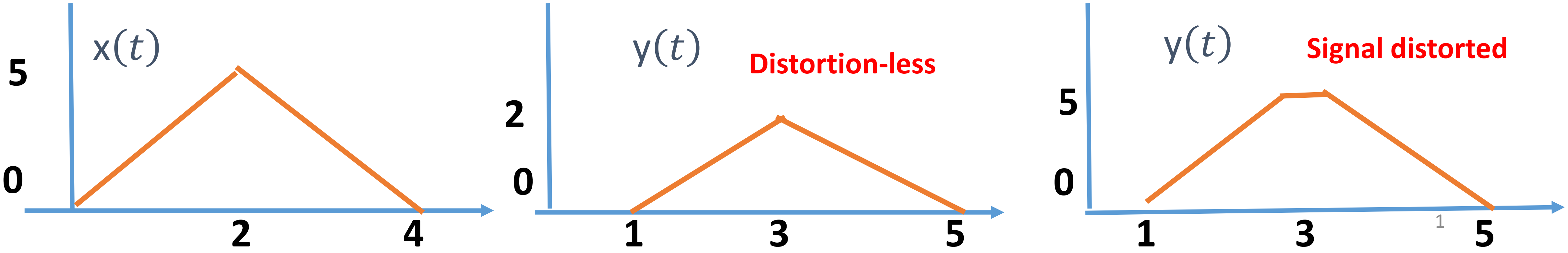
Signal Distortion in Transmission

- The objective of a communication system is to deliver to the receiver almost an exact copy of what the source generates.
- However, communication channels are not perfect in the sense that impairments on the channel will cause the received signal to differ from the transmitted one. During the course of transmission, the signal undergoes **attenuation**, **phase delay**, **interference** from other transmissions, **Doppler shift** in the carrier frequency, **AWGN**, and many other effects.
- **In this lecture, we consider the conditions for a distortion-less transmission over a channel. In addition, we consider linear and non-linear distortion**
- **Distortion-less Transmission**: A signal transmission is said to be *distortion-less* if the output signal y(t) is an exact replica of the input signal x(t) , i.e., y(t) has the same shape as the input, except for a constant amplification (or attenuation) and a constant time delay.

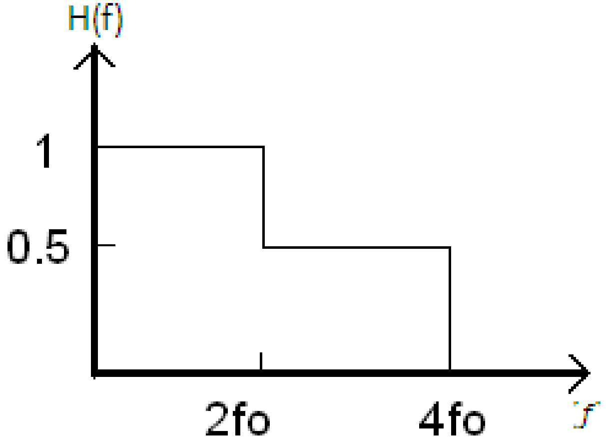


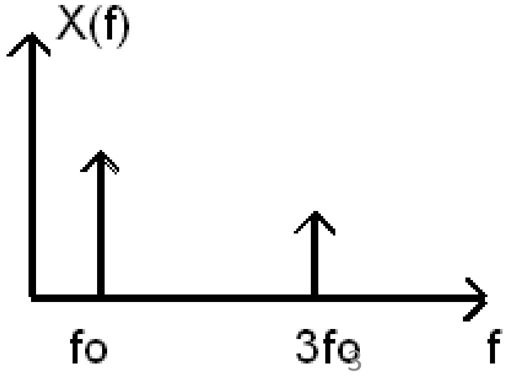
Signal Distortion in Transmission

- Condition for **distortion-less transmission in the time-domain:**
- $\boldsymbol{y(t) = kx(t - t_d)}$; where k is a constant amplitude scaling, t_d is a constant time delay.
- **In the frequency domain**, the condition for a distortion-less transmission becomes
- $Y(f) = kX(f)e^{-j2\pi f t_d}$ or $\boldsymbol{H(f) = \frac{Y(f)}{X(f)} = ke^{-j2\pi f t_d} = ke^{-j\theta(f)}}$
- That is, for a distortion-less transmission, the transfer function should satisfy two conditions:
- **|H(f)| = k** ; The magnitude of the transfer function is constant (gain or attenuation) over the frequency range of interest.
- $\boldsymbol{\theta(f) = -2\pi f t_d = -(2\pi t_d) f}$; The phase function is linear in frequency with a negative slope that passes through the origin (or multiples of π).
- When |H(f)| is not constant for all frequencies of interest, *amplitude distortion* results.
- When $\theta(f) \neq -2\pi f t_d \pm 180^0$, then we have *phase distortion* (or delay distortion).
- The following examples demonstrate the two types of distortion mentioned above.

Example: amplitude distortion

- Consider the signal $x(t) = \ cos\ w_0 t\ - \frac{1}{3}\ cos\ 3w_0 t$. If this signal passes through a channel with zero time delay (i.e., $t_d = 0$) and amplitude spectrum as shown in the figure
- Find y(t)
- Is this a distortion-less transmission?
- **Solution:** x(t) consists of two frequency components, f_0 and $3f_0$. Upon passing through the channel, each component will be scaled by a different factor.
- $y(t) = (\mathbf{1}) cos\ w_0 t\ - (\mathbf{\frac{1}{2}})\ .\ \frac{1}{3}\ cos\ 3w_0 t$
- Since $y(t) = \left(cos\ w_0 t - \frac{1}{2}\ .\ \frac{1}{3}\ cos\ 3w_0 t \right) \neq k \left(cos\ w_0 t\ - \frac{1}{3}\ cos\ 3w_0 t \right)$
- then this is not a distortion-less transmission.

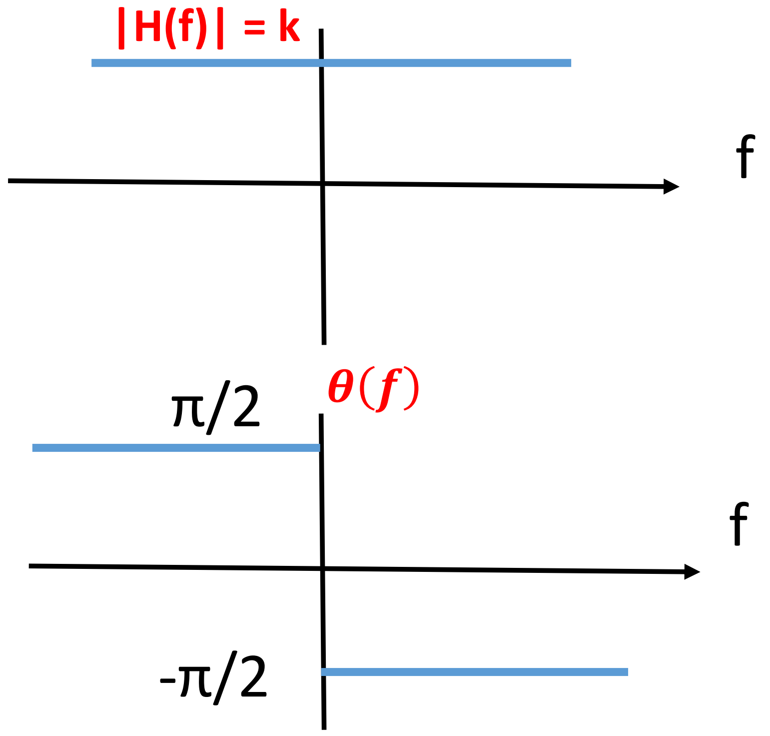




In this figure, only the positive part of the spectrum is shown

Example: phase distortion

- Consider the signal $x(t) = \ cos\ w_0 t\ - \frac{1}{3}\ cos\ 3w_0 t$. If x(t) passes through a channel whose amplitude spectrum is a constant k. Each component in x(t) suffers a $-\frac{\pi}{2}$ phase shift.
- Find y(t).
- Is this a distortion-less transmission?
- **Solution:**
- $x(t) = \cos w_o t - \frac{1}{3} \cos 3w_o t$
- $y(t) = \text{k}\ \cos(w_o t - \frac{\pi}{2})\ -\ \frac{1}{3}\ k\ \cos\left(3w_o t - \frac{\pi}{2}\right)$
- $y(t) = \text{k}\ \cos w_o(t - \frac{\pi}{2w_o})\ -\ \frac{1}{3}\ k\ \cos\left(3w_o(t - \frac{\pi}{2x3w_o})\right)$
- $y(t) = \text{k}\ \cos w_o(t - t_{d1})\ -\ \frac{1}{3}\ k\ \cos(3w_o(t - t_{d2}))$
- Since $t_{d1} \neq t_{d2}$, we cannot write $y(t) = kx(t - t_d)$. Here, each component in $x(t)$ suffers from a different time delay. Hence, this transmission introduces phase (delay) distortion.



Nonlinear distortion

- When a system contains nonlinear elements, it is **not** described by a transfer function H(f), but rather by a transfer characteristic of the form
- $y(t) = a_1 x(t) + a_2 x^2(t) + a_3 x^3(t) + \dots$ (time domain)
- In the frequency domain,
- $Y(f) = a_1 X(f) + a_2 X(f)*X(f) + a_3 X(f)*X(f)*X(f) + \dots$
- Here, the output contains new frequencies not originally present in the original signal. The nonlinearity produces undesirable frequency component for $|f| \leq W$, in which W is the signal bandwidth.

Harmonic distortion in nonlinear systems

- Let the input to a nonlinear system be the single tone signal $x(t) = \cos(2\pi f_0 t)$.
- This signal is applied to a channel with characteristic $y(t) = a_1x(t) + a_2x(t)^2 + a_3x(t)^3$;
- $y(t) = a_1 \cos(2\pi f_0 t) + a_2(\cos(2\pi f_0 t))^2 + a_3(\cos(2\pi f_0 t))^3$;
- upon substituting x(t) and arranging terms, we get
- $y(t) = \frac{1}{2}a_2 + \left(a_1 + \frac{3}{4}a_3\right)\cos 2\pi f_0 t + \frac{1}{2}\, a_2 \cos 4\pi f_0 t + \frac{1}{4} a_3 cos 6\pi\, f_0 t$
- Note that the output contains a component proportional to x(t), which is
- $\left(a_1 + \frac{3}{4}a_3\right)\cos 2\pi f_0 t$, in addition to a second and a third harmonic terms (terms at twice and three times the frequency of the input).
- These new terms are the result of the nonlinear characteristic and are, therefore, considered as harmonic distortion. The DC term does not constitute a distortion, for it can be removed using a blocking capacitor.
- Note: Use was made of the inequalities $Cos^2x = \frac{1}{2}\{1 + cos2x\}$; $Cos^3x = \frac{1}{4}\{3cosx + cos3x\}$.

Harmonic distortion in nonlinear systems

- Let the input to a nonlinear system be the single tone signal
- $y(t) = a_1x(t) + a_2x(t)^2 + a_3x(t)^3$; $\quad x(t) = \cos(2\pi f_0 t)$;
- $y(t) = \frac{1}{2}a_2 + \left(a_1 + \frac{3}{4}a_3\right)\cos 2\pi f_0 t + \frac{1}{2}a_2 \cos 2(2\pi f_0 t) + \frac{1}{4}a_3 \cos 3(2\pi f_0 t)$

- Define the second harmonic distortion
- $D_2 = \frac{|amplitude\ of\ second\ harmonic\ |}{|amplitude\ of\ fundamental\ term|}$; $\quad D_2 = \frac{|\frac{1}{2}a_2\ |}{|\left(a_1+\frac{3}{4}a_3\right)|}\ x\ 100$
- In a similar way, we can define the third harmonic distortion as:
- $D_3 = \frac{|amplitude\ of\ third\ harmonic\ |}{|amplitude\ of\ fundamental\ term|}$; $\quad D_3 = \frac{|\frac{1}{4}a_3\ |}{|\left(a_1+\frac{3}{4}a_3\right)|}\ x\ 100\%.$