Normal Amplitude Modulation
Time and Frequency Domain Characteristics
Lecture Outline

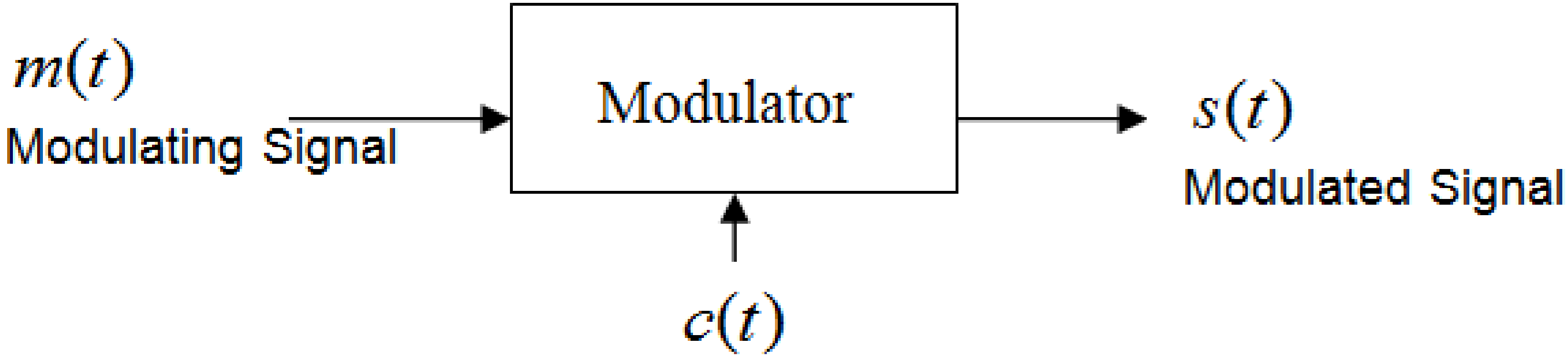
- Why do we need modulation?
- Define the normal AM signal
- The normal AM in the time and frequency domains
- Power efficiency
- Effect of the modulation index

Normal Amplitude Modulation

Modulation: is the process by which some characteristic of a high frequency signal $c(t)$, called the carrier, is varied in accordance with a message signal $m(t)$. A common form of the *carrier*, in the case of continuous wave modulation, is a sinusoidal signal

$$c(t) = A_C \cos(2\pi f_C t + \varphi)$$

The three parameters of c(t), amplitude, phase, and frequency may be varied in accordance of the message signal resulting in amplitude modulation, phase modulation, and frequency modulation.



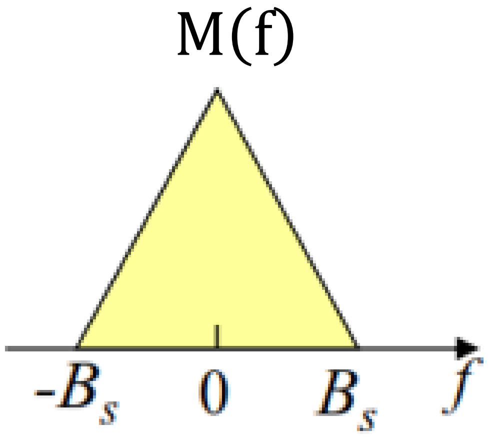
Why Modulation?

- There are several reasons why modulation is needed in a communication system.
- **Physical antenna size**: For efficient transmission of a signal, the antenna length should be about $\lambda/4$, where λ is the wavelength.
- For example, let the frequency of the message be 3KHz (**audio signal**)
- The wavelength $\lambda = \frac{c}{f} = \frac{3.0X10^8}{3.0X10^3} = 10^5 m = 100km.$
- Hence, the size of the antenna should be around (**λ/4 =25 km**), which is not at all practical.
- Now, let us find the antenna length in the GSM band (1000 MHz):
- $\lambda = \frac{c}{f} = \frac{3.0X10^8}{1000X10^6} = 0.3m$
- Hence, the size of the antenna should be around (**λ/4 = 7.5 cm**), which can easily fit into a mobile device. This is a challenging design issue in modern mobile technology.

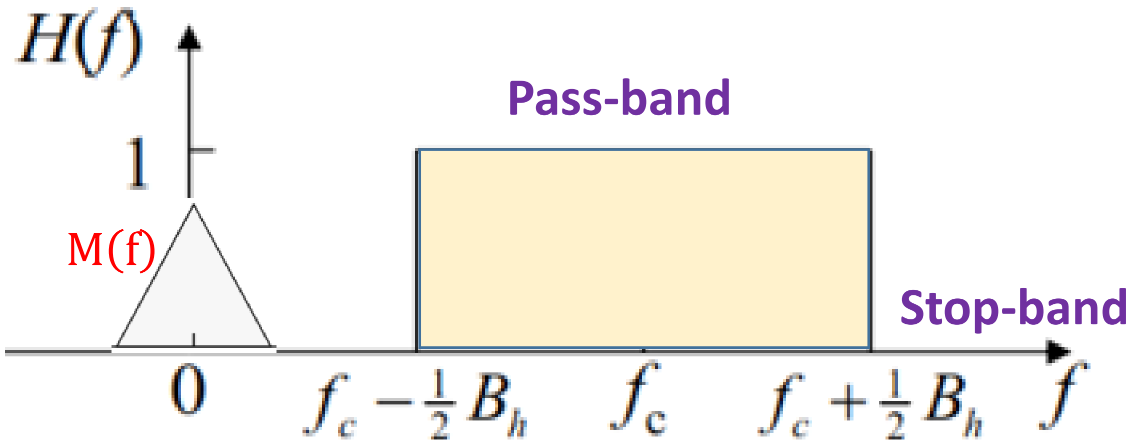
Why Modulation?

- **Band-pass channels**: Most, if not all, channels over which messages are transmitted are band-pass, while messages are low-pass signals. Hence, direct transmission of messages over band-pass channels would result in high attenuation (essentially no received signal). This necessitates shifting the message spectrum to coincide with the channel bandwidth.

$$Y(f) = M(f)H(f)$$



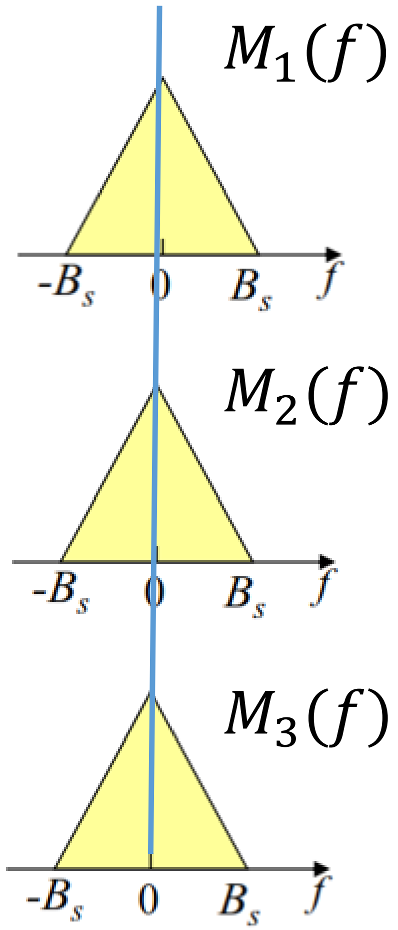
Baseband Message

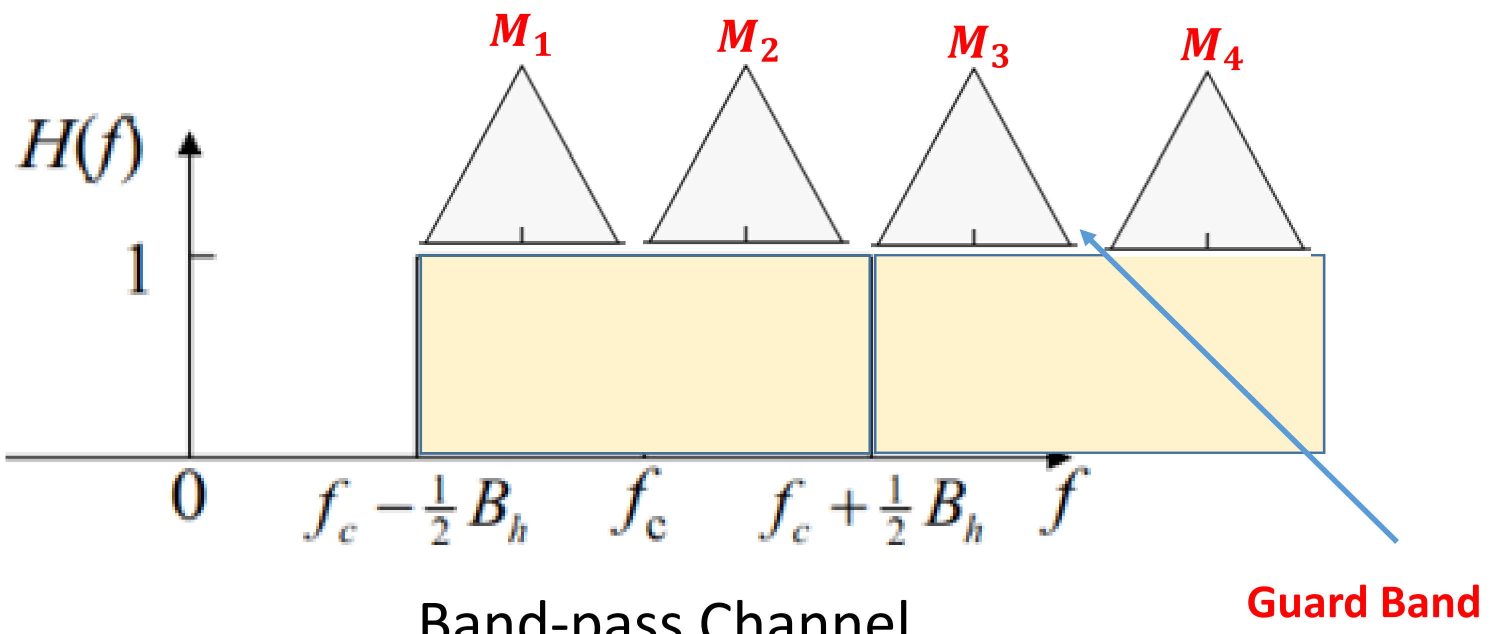


Band-pass Channel

Why Modulation?

- **Multiplexing**: Modulation allows multiple users to use the same channel by assigning each user a portion of the available bandwidth without interfering with other users.

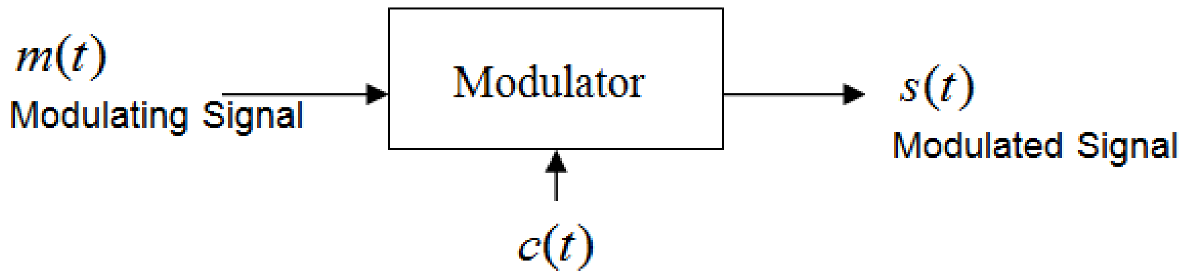




Band-pass Channel

Amplitude modulation

- **Amplitude modulation (AM)** is defined as the process in which the amplitude of the carrier $c(t)$ is varied linearly with $m(t)$.
- Three types of amplitude modulation will be considered in detail. These are
  - Normal amplitude modulation
  - Double sideband suppressed carrier modulation (DSB-SC)
  - Single sideband modulation (SSB-SC)



- The baseband (message) signal $m(t)$ is referred to as the **modulating signal** and the result of the modulation process is referred to as the **modulated signal** $s(t)$.
- **Modulation** is performed at the transmitter
- **Demodulation**, which is the process of extracting $m(t)$ from $s(t)$, is performed at the receiver.

Normal Amplitude modulation

A *normal AM* signal is defined as: $s(t) = A_c(1 + k_a m(t))\cos(2\pi f_c t)$

where, k_a is the sensitivity of the AM modulator (units in 1/volt).

$s(t)$ can be also be written in the form: $s(t) = A(t)\cos 2\pi f_C t$

The **envelope** of $s(t)$ is defined as

$$|A(t)| = A_C\,|1 + k_a m(t)|$$

Notice that the envelope of $s(t)$ has the same shape as $m(t)$ provided that:

1. $(1 + k_a m(t)) \geq 0$ Or equivalently, $|k_a m(t)| \leq 1$.
2. Over-modulation occurs when $|k_a m(t)| > 1$ resulting in envelope distortion
3. $f_C >> W$, where W is the bandwidth of $m(t)$. f_C has to be at least $10W$. This ensures the formation of an envelope, whose shape resembles the message signal.

Spectrum of the Normal AM Signal

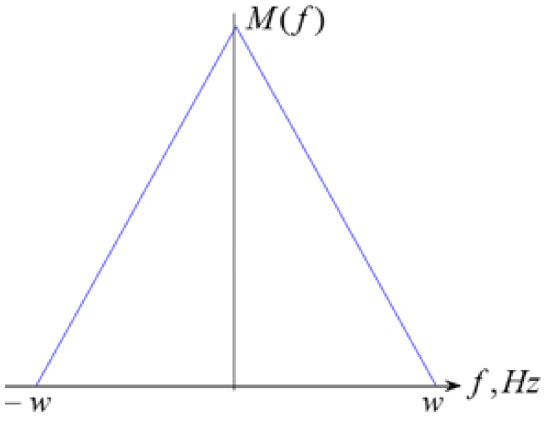
Let the Fourier transform of $m(t)$ be as shown (the B.W of $m(t) = W$ Hz).

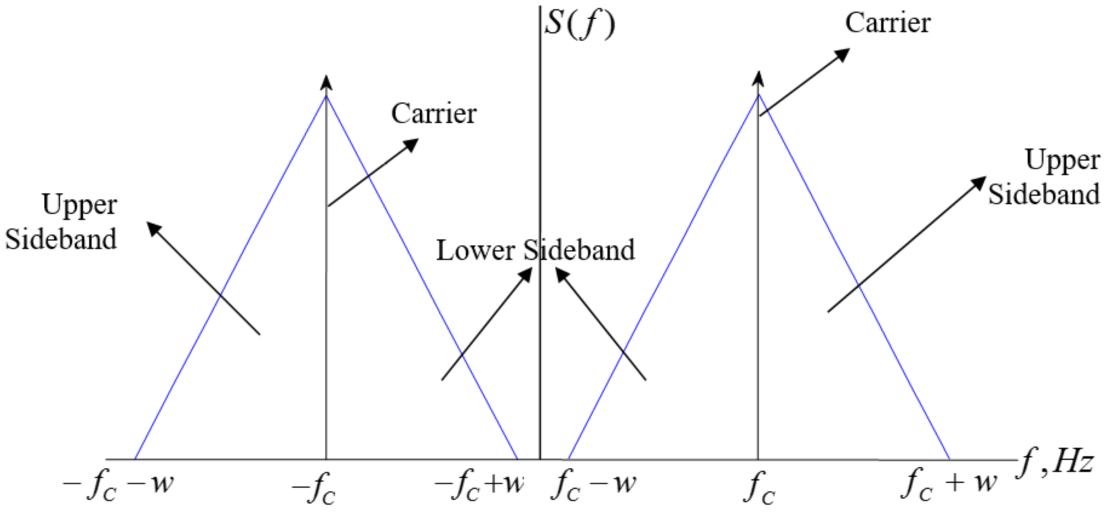
$$s(t) = A_C\big(1 + k_a m(t)\big)\cos 2\pi f_C t \qquad \text{(dc + message)*carrier}$$

$$s(t) = A_C \cos 2\pi f_C t + A_C k_a m(t)\cos 2\pi f_C t \qquad \text{(carrier + message*carrier)}$$

Taking the Fourier transform, we get

$$S(f) = \frac{A_C}{2}\delta(f - f_C) + \frac{A_C}{2}\delta(f + f_C) + \frac{A_C k_a}{2}M(f - f_C) + \frac{A_C k_a}{2}M(f + f_C)$$





Remarks

a. The baseband spectrum $M(f)$, of the message has been shifted to the bandpass region centered around the carrier frequency f_c.

b. The spectrum $S(f)$ consists of two sidebands (upper sideband and lower sideband) and a carrier.

c. The transmission bandwidth of $s(t)$ is:

$B.W. = (f_c + W) - (f_c - W) = \mathbf{2W}$ which is twice the message bandwidth.

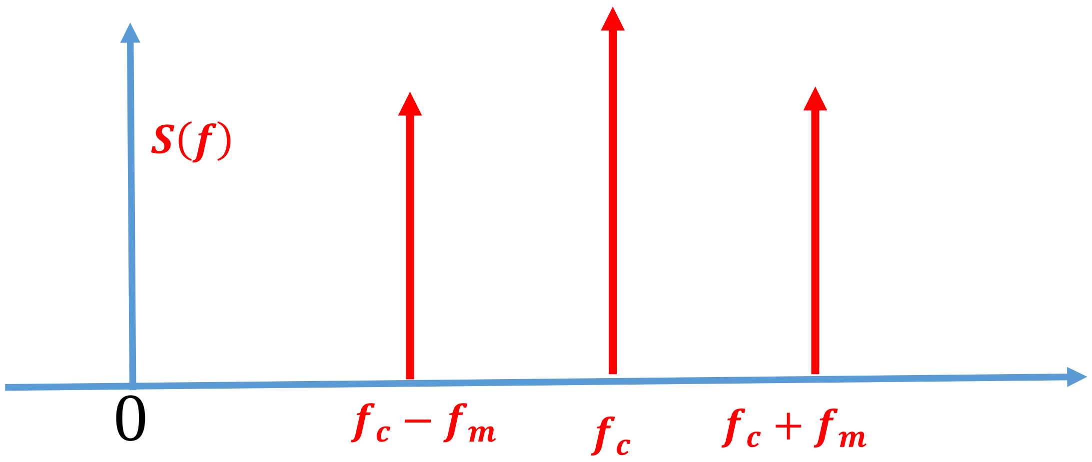
Spectrum of the Normal AM: Sinusoidal Modulation

Example: Consider the normal AM with sinusoidal modulation, where $c(t) = A_c\cos(2\pi f_c t)$; $m(t) = A_m\cos(2\pi f_m t)$; plot m(t), c(t), s(t) and find their spectrum.

Solution: $s(t) = A_c(1 + k_a m(t))\cos 2\pi(f_c)t$

- $s(t) = A_c cos(2\pi f_c t) + A_c k_a A_m\, cos(2\pi f_c t)\, cos(2\pi f_m t);$
- $s(t) = A_c cos(2\pi f_c t) + \frac{A_c A_m k_a}{2} cos(2\pi(f_c + f_m)t) + \frac{A_c A_m k_a}{2} cos(2\pi(f_c - f_m)t)$
- $S(f) = \Im\{s(t)\}$
- $M(f) = \frac{A_m}{2}\delta(f - f_m) + \frac{A_m}{2}\delta(f + f_m)$
- The next figure shows all the plots

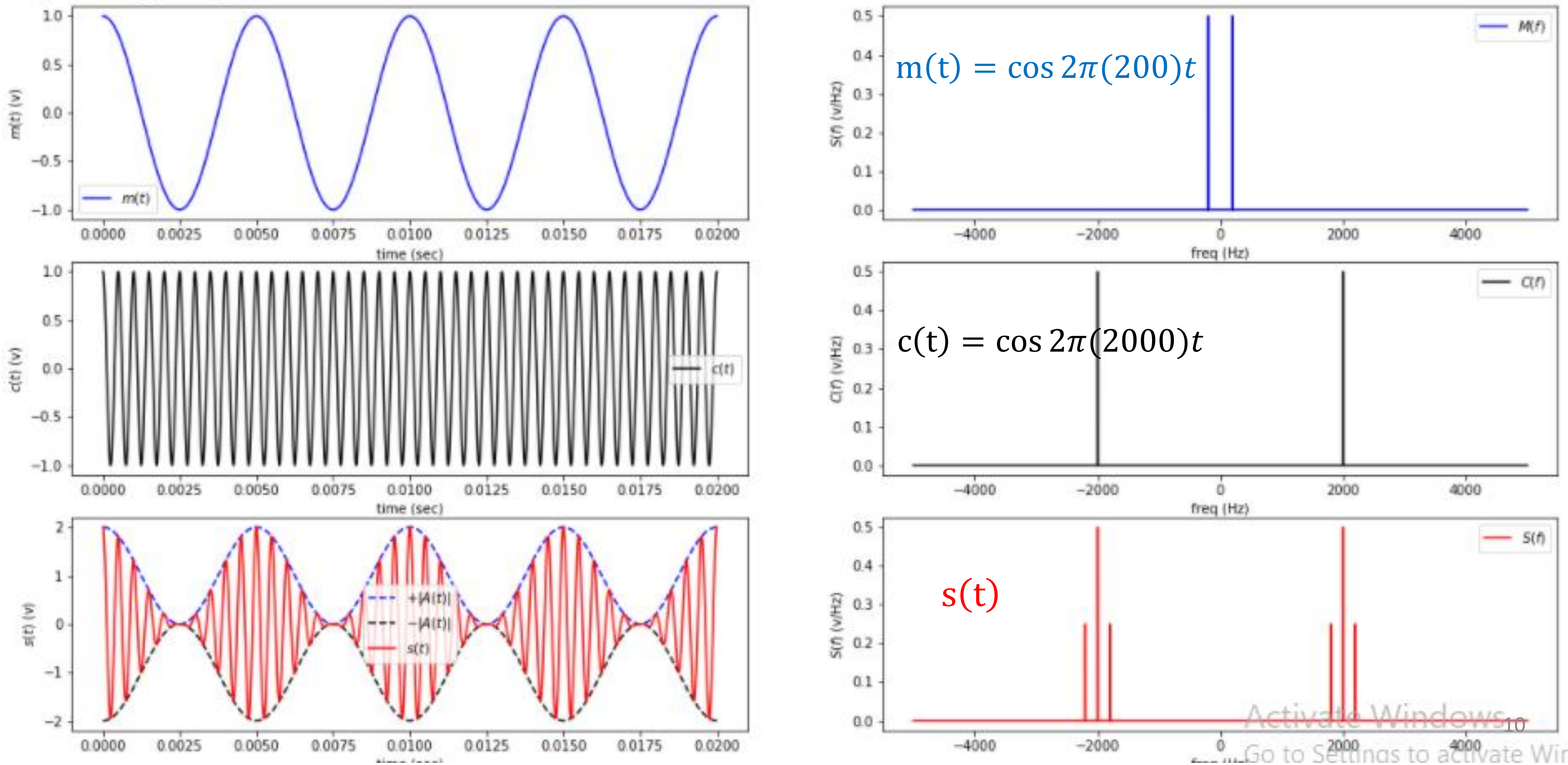
  when $f_m = 200$ Hz and $f_c = 000$ Hz



Spectrum of the Normal AM Signal

An AM signal in the time and frequency domains.

$s(t) = (1 + k_a m(t))\cos 2\pi(2000)t$ $m(t) = \cos 2\pi(200)t$ $k_a = 1.0$ $\mu = A_m k_a = 1.0$



Power Efficiency of Normal AM

The ***power efficiency*** of a normal AM signal is defined as:

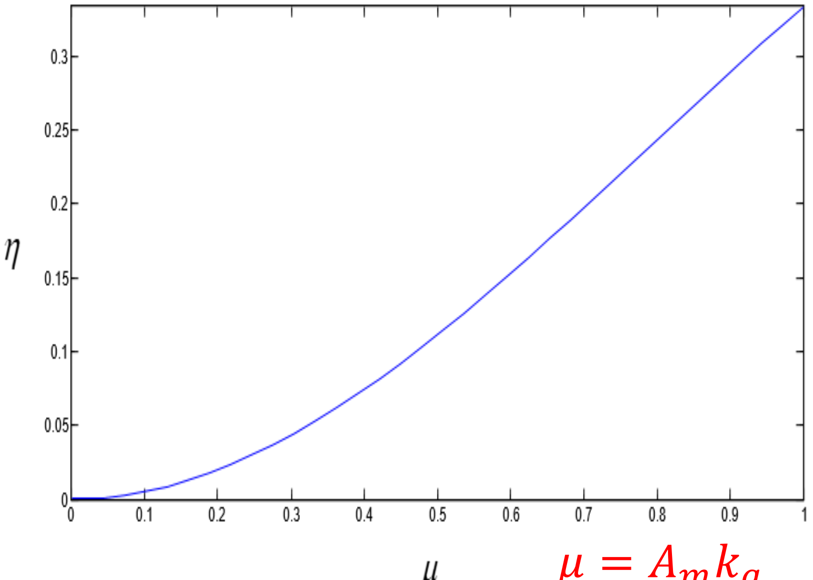
$$\eta = \frac{power \;\; in \;\; the \;\; sidebands}{power \; in \; the \; sidebands + power \; in \; the \; carrier}$$

Now, we find the power efficiency of the AM signal for the single-tone modulating signa $m(t) = A_m \cos(2\pi f_m t)$. Let $\mu = A_m k_a$, then $s(t)$ can be expressed as

$$s(t) = A_C(1 + \mu \cos 2\pi f_m t)\cos 2\pi f_C t$$

$$s(t) = A_C \cos 2\pi f_C t + A_C \mu \cos 2\pi f_C t \cos 2\pi f_m t$$

$$s(t) = A_C \cos 2\pi f_C t + \frac{A_C \mu}{2}\cos 2\pi (f_C + f_m)t + \frac{A_C \mu}{2}\cos 2\pi (f_C - f_m)t$$



$\mu = A_m k_a$

Power in carrier $= \dfrac{A_C^{\,2}}{2}$

Power in sidebands $= \dfrac{1}{2}\left(\dfrac{A_C \mu}{2}\right)^2 + \dfrac{1}{2}\left(\dfrac{A_C \mu}{2}\right)^2 = \dfrac{1}{4} A_C^{\,2} \mu^2$

Therefore, $\quad \eta = \dfrac{\frac{1}{4} A_C^{\,2} \mu^2}{\frac{A_C^{\,2}}{2} + \frac{1}{4} A_C^{\,2} \mu^2} = \dfrac{\mu^2}{2 + \mu^2} \quad ; \quad 1 \ge \mu \ge 0$

- **The maximum efficiency occurs when μ=1, i.e. for a 100% modulation index. The corresponding maximum efficiency is only η = 1/3. As a result, 2/3 of the transmitted power is wasted in the carrier**
- **Remark: Normal AM is not an efficient modulation scheme in terms of the utilization of the transmitted power.**

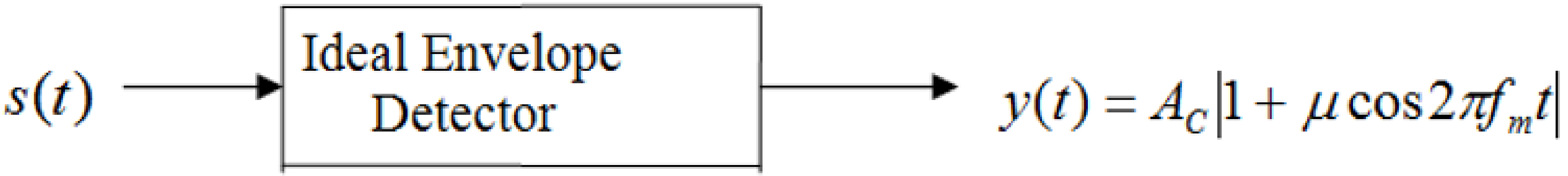
Amplitude Modulation: AM Modulation Index

Consider the AM signal: $s(t) = A_C(1 + k_a m(t)) \cos 2\pi f_C t = A(t) \cos 2\pi f_C t$

The envelope of $s(t)$ is:

$$|A(t)| = A_C |1 + k_a m(t)|$$

The following block diagram illustrate the envelope detection process for a sinusoidal message signal.



$$\mu = A_m k_a$$

To avoid distortion, the following condition must hold

$$(1 + k_a m(t)) \geq 0 \qquad \text{or} \qquad |k_a m(t)| \leq 1$$

The modulation index of an AM signal is defined as:

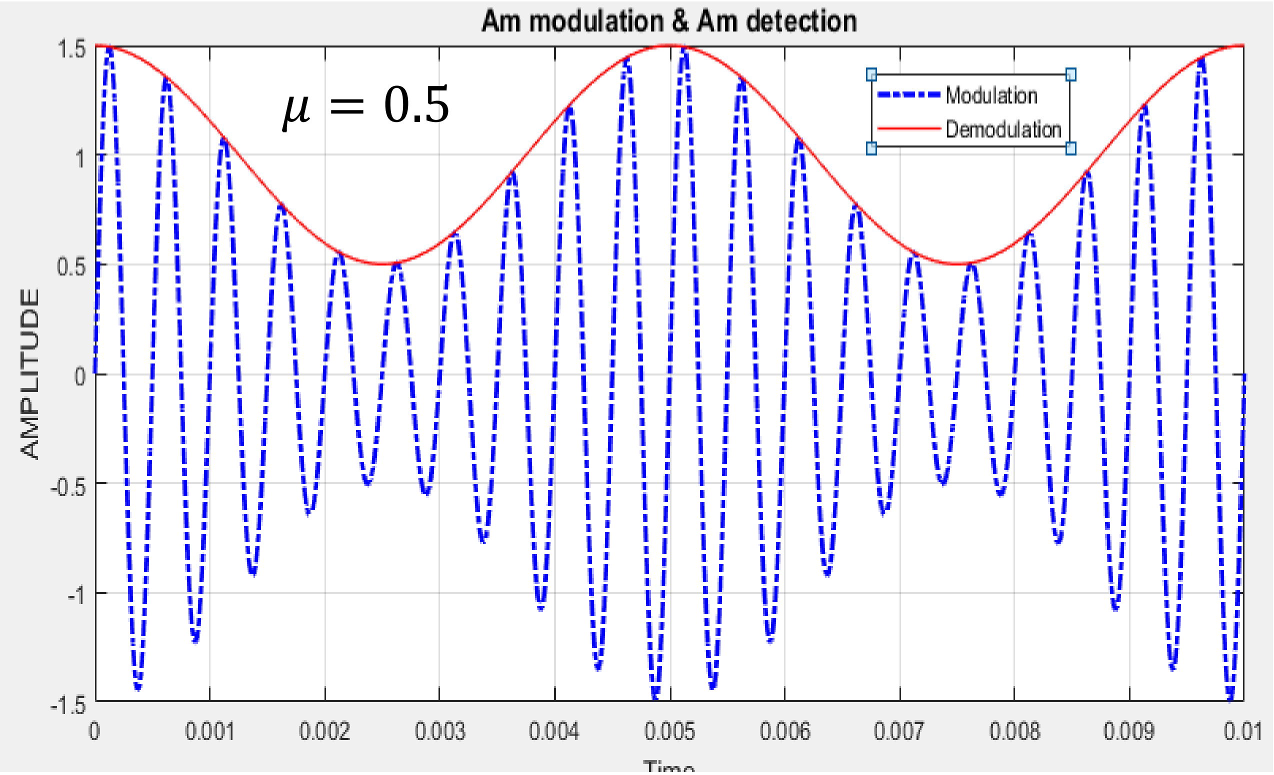
$$Modulation\ \ Index\ (M.I) = \frac{|A(t)|_{max} - |A(t)|_{min}}{|A(t)|_{max} + |A(t)|_{min}}$$

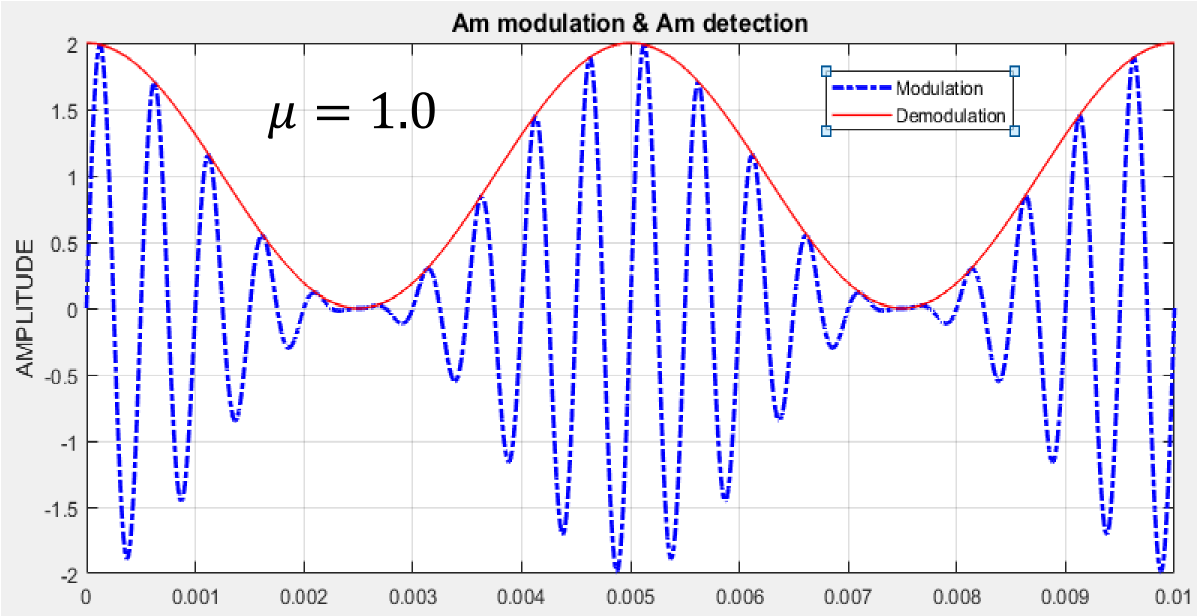
The modulation index μ (modulation depth) of an amplitude modulated signal is defined as the measure or extent of amplitude variation about an un-modulated carrier. In other words the amplitude modulation index describes the amount by which the modulated carrier envelope varies about the static level.

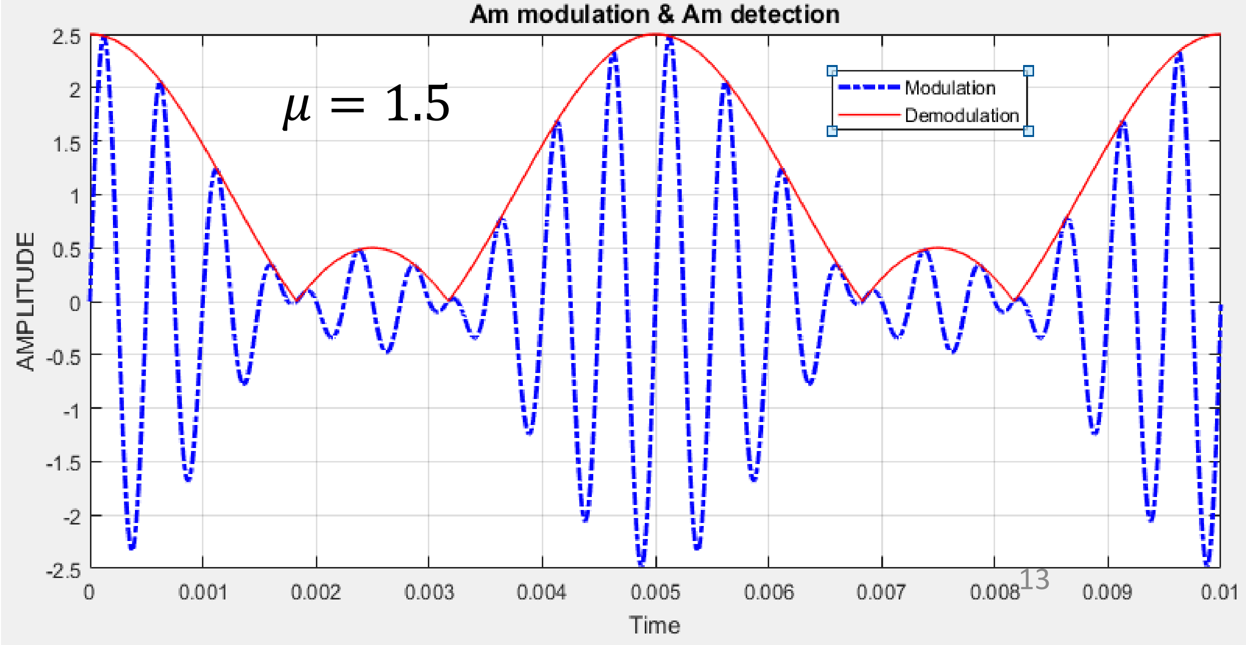
Amplitude Modulation: Effect of the Modulation Index

$$s(t) = (1 + \mu m(t))\cos 2\pi(2000)t$$

$$m(t) = \cos 2\pi(200)t \qquad \mu = k_a|m(t)|$$







Amplitude Modulation: Multi-tone Modulation

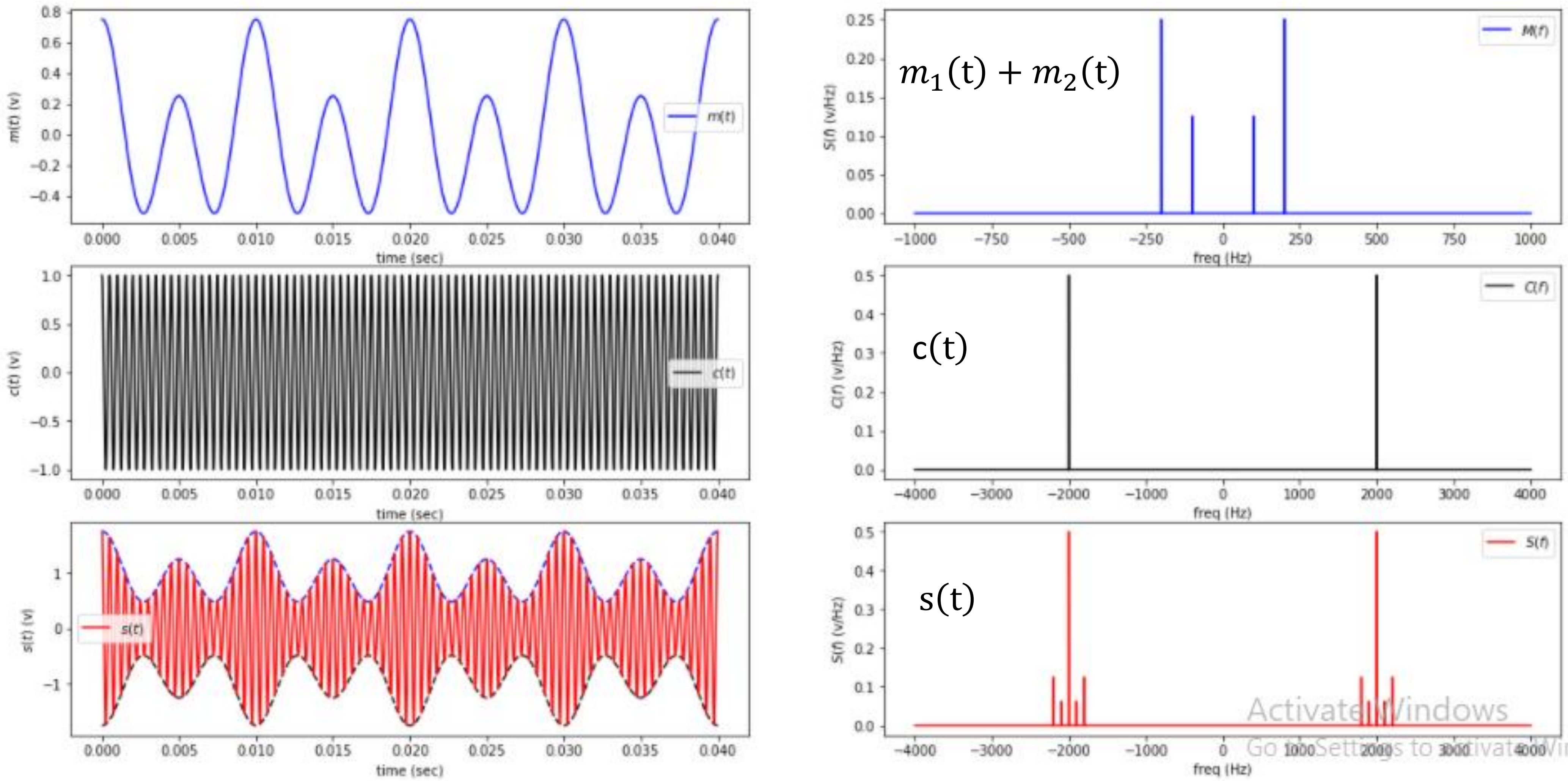
$s(\text{t}) = \ (1 + m_1(\text{t}) + m_2(\text{t}))\cos 2\pi(2000)t$

$k_a = 1; A_c = 1$

$m_1(\text{t}) = 0.5\cos 2\pi(200)t$

$m_2(\text{t}) = 0.25\cos 2\pi(100)t$

A non-envelope distortion case for multitoned transmission

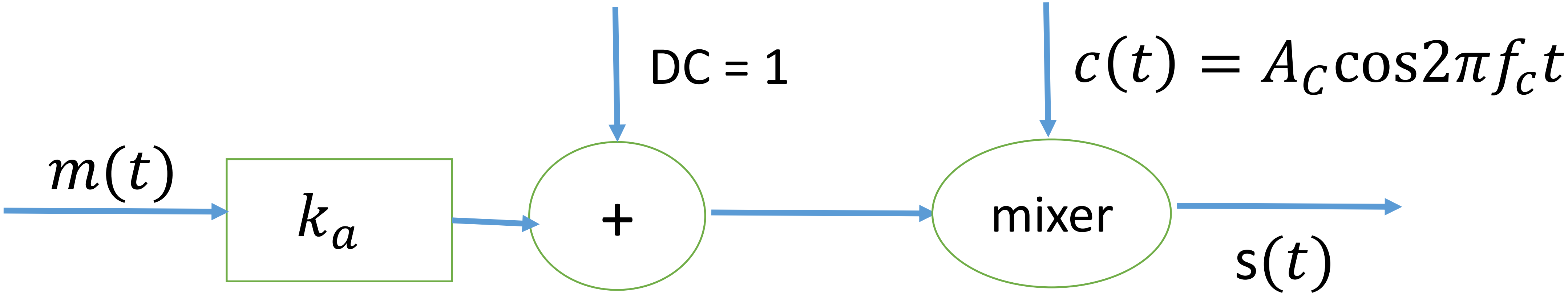


Normal Amplitude Modulation Generation and Demodulation Lecture Outline

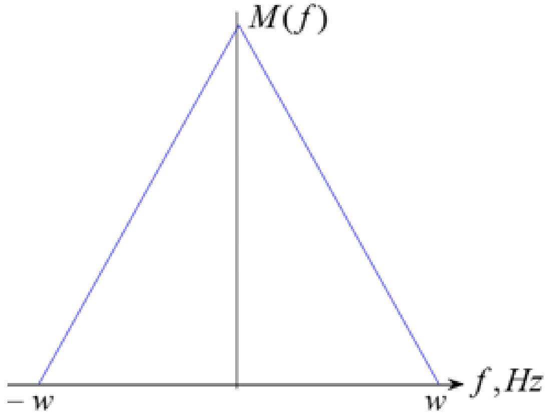
- Last Lecture:
  - Why do we need modulation?
  - Define the normal AM signal
  - The normal AM in the time and frequency domains
  - Power efficiency
  - Effect of the modulation index
- This Lecture:
  - AM generation techniques: the switching modulator
  - The envelope detector

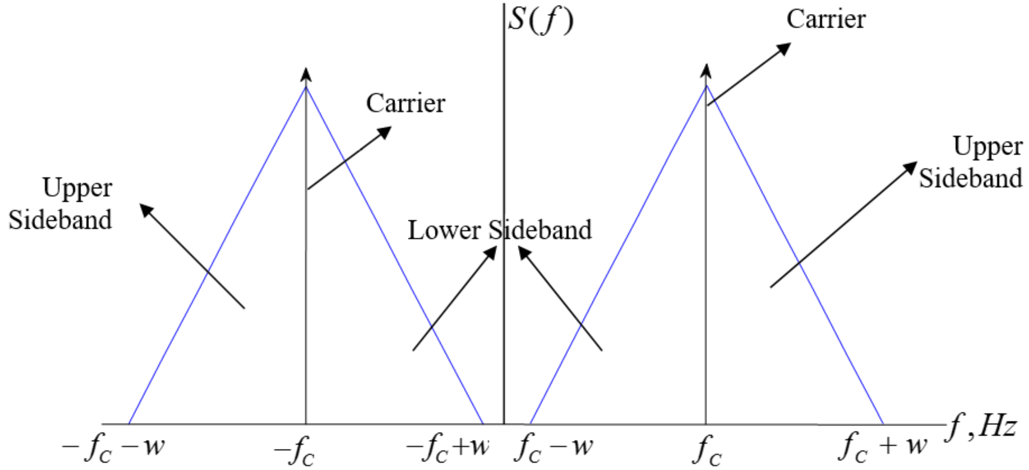
Normal Amplitude Modulation: Standard Form

$$s(t) = A_c(1 + k_a m(t))\cos 2\pi f_c t$$



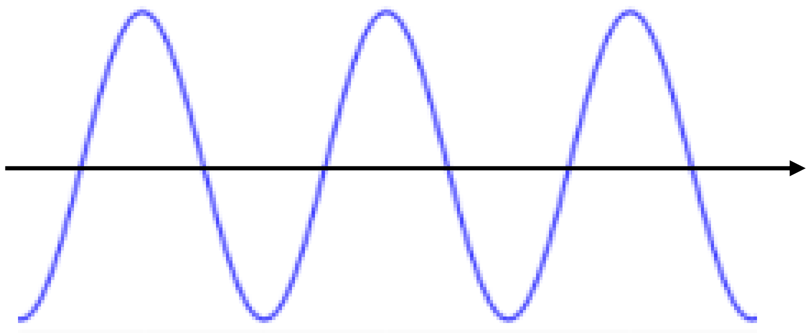
$$S(f) = \frac{A_c}{2}\delta(f - f_c) + \frac{A_c}{2}\delta(f + f_c) + \frac{A_c k_a}{2}M(f - f_c) + \frac{A_c k_a}{2}M(f + f_c)$$





Generation of a Normal Amplitude Modulation: the Switching Modulator

Assume that the carrier $c(t)$ is large in amplitude so that the diode –shown in the figure below- acts like an ideal switch.

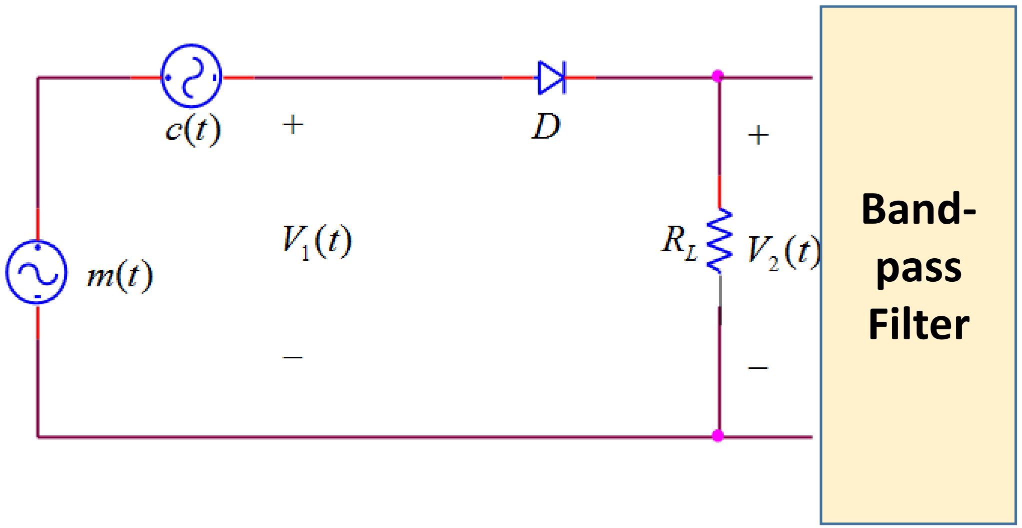


$c(t) = A_C cos2\pi f_c t$

When $m(t)$ is small compared to $|c(t)|$,

$$V_2(t) = \begin{cases} m(t) + A_C \cos\omega_C t & ; \quad c(t) > 0 \\ 0 & ; \quad c(t) < 0 \end{cases}$$

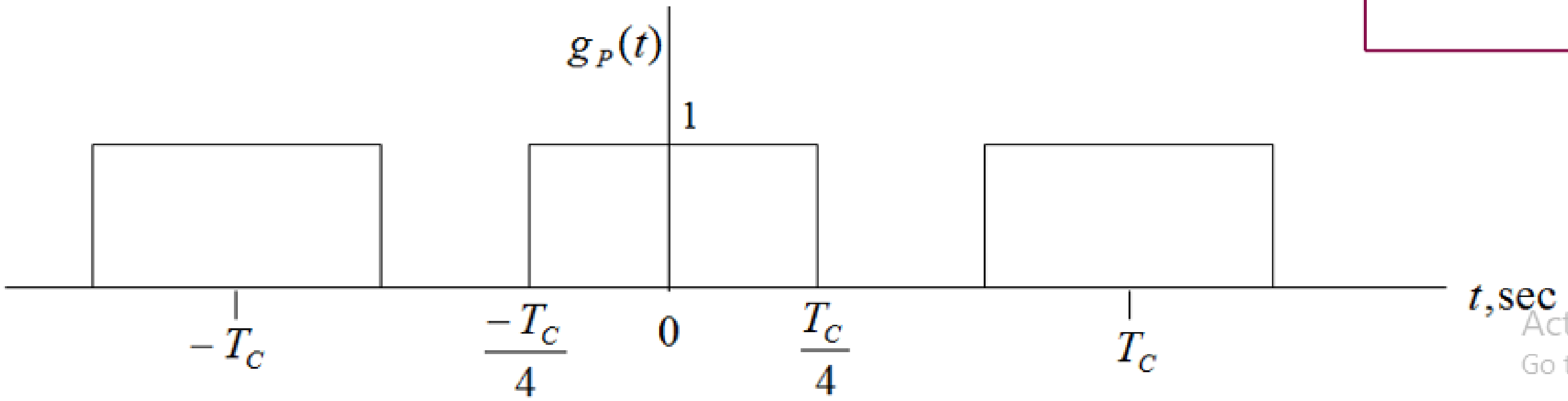
Here, the diode opens and closes at a rate equals to the carrier frequency f_C. This switching mechanism can be modeled as:



$$V_2(t) = [A_C \cos\omega_C t + m(t)]g_P(t)$$

where $g_P(t)$ is the periodic square function, expanded in a Fourier series as

$$g_P(t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_C t - \frac{1}{3}\cos 3\omega_C t + \frac{1}{5}\cos 5\omega_C t + \ldots\right)$$



Generation of a Normal Amplitude Modulation: the Switching Modulator

$$V_2(t) = [A_C \cos\omega_C t + m(t)]\left(\frac{1}{2}\right) + \left(\frac{2}{\pi}\cos\omega_C t\right)(A_C \cos\omega_C t + m(t)) - \left(\frac{2}{3\pi}\cos 3\omega_C t\right)$$
$$(m(t) + A_C \cos\omega_C t) + \ldots$$

$$V_2(t) = \frac{m(t)}{2} + \frac{A_C}{2}\cos\omega_C t + \frac{2}{\pi}m(t)\cos\omega_C t + \frac{A_C}{\pi} + \frac{A_C}{\pi}\cos 2\omega_C t + \frac{2}{3\pi}m(t)\cos 3\omega_C t$$
$$+ \frac{2}{3\pi}A_C \cos 2\omega_C t + \ldots$$

$$\boldsymbol{V_2(t) = [A_C\cos\omega_C t + m(t)]g_P(t)}$$

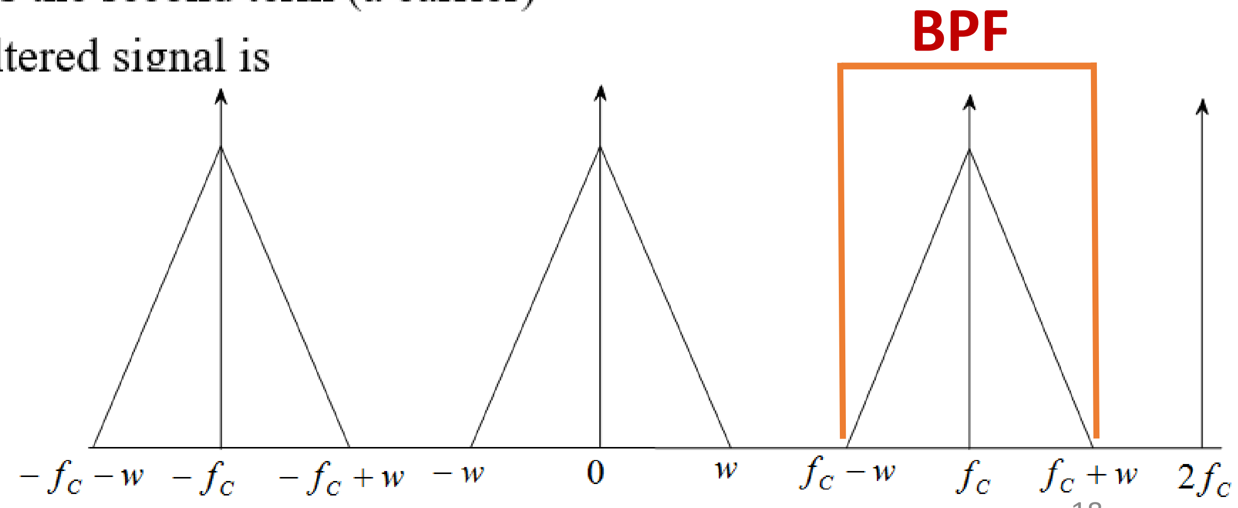
$$\boldsymbol{g_P(t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_C t - \frac{1}{3}\cos 3\omega_C t + \frac{1}{5}\cos 5\omega_C t + \ldots\right)}$$

A band-pass filter with a bandwidth $2w$, centered at f_C, passes the second term (a carrier) and the third term (a carrier multiplied by the message). The filtered signal is

$$s(t) = \frac{A_C}{2}\cos\omega_C t + \frac{2}{\pi}m(t)\cos\omega_C t$$

$$s(t) = \frac{A_C}{2}\left(1 + \frac{4}{\pi A_C}m(t)\right)\cos\omega_C t \quad \text{; Desired AM signal.}$$

Modulation Index $= M.I = \frac{4}{\pi A_C}|m(t)|_{\max}$



Demodulation of a Normal Amplitude Modulation: Envelope Detection

The Ideal Envelope Detector: The ideal envelope detector responds to the envelope of the signal, but is insensitive to phase variation. If

$$s(t) = A_C\left(1 + k_a m(t)\right)\cos 2\pi f_C t$$

then, the output of the ideal envelope detector is $y(t) = A_C\left|1 + k_a m(t)\right|$

To avoid envelope distortion,
$|1 + k_a \mathrm{m(t)}|$ should equal $(1 + k_a m(t))$
That is, $(1 + k_a m(t)) \geq 0$ for all time

Example: single tone modulation (under-modulation)

Example: Let $s(t) = A_C(1+\mu\cos 2\pi f_m t)\cos 2\pi f_C t$ be applied to an ideal envelope detector, sketch the demodulated signal for $\mu = 0.25, 1.0, \textit{and } 1.25$.

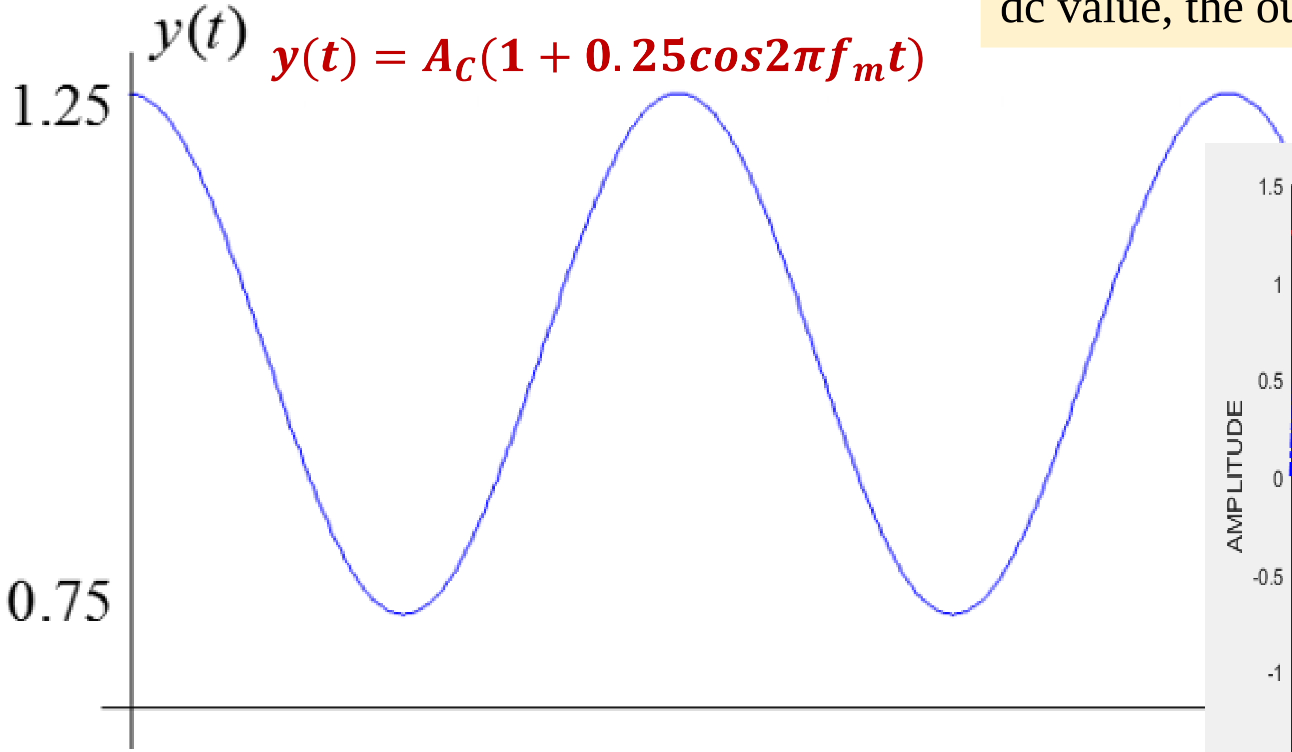
As was mentioned before, the output of the envelope detector is $y(t) = A_C\left|1+\mu\cos 2\pi f_m t\right|$

Case1 : ($\mu = 0.25$)

$$y(t) = A_C\left|1+0.25\cos 2\pi f_m t\right|$$

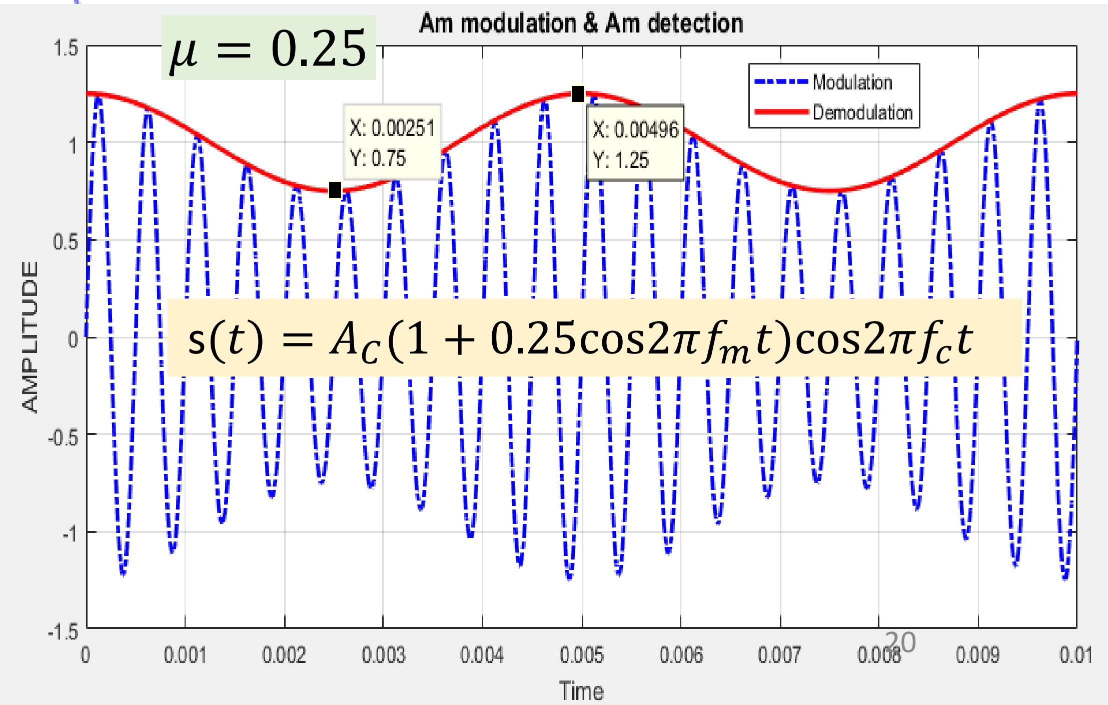
Here, m(t) can be extracted without distortion. $(1 + k_a m(t)) \geq 0$ for all time. $|1 + k_a m(t)| = (1 + k_a m(t))$. By removing the dc value, the output will be proportional to the message

$y(t) = A_C(1 + 0.25\cos 2\pi f_m t)$



$\mu = 0.25$

$s(t) = A_C(1 + 0.25\cos 2\pi f_m t)\cos 2\pi f_c t$

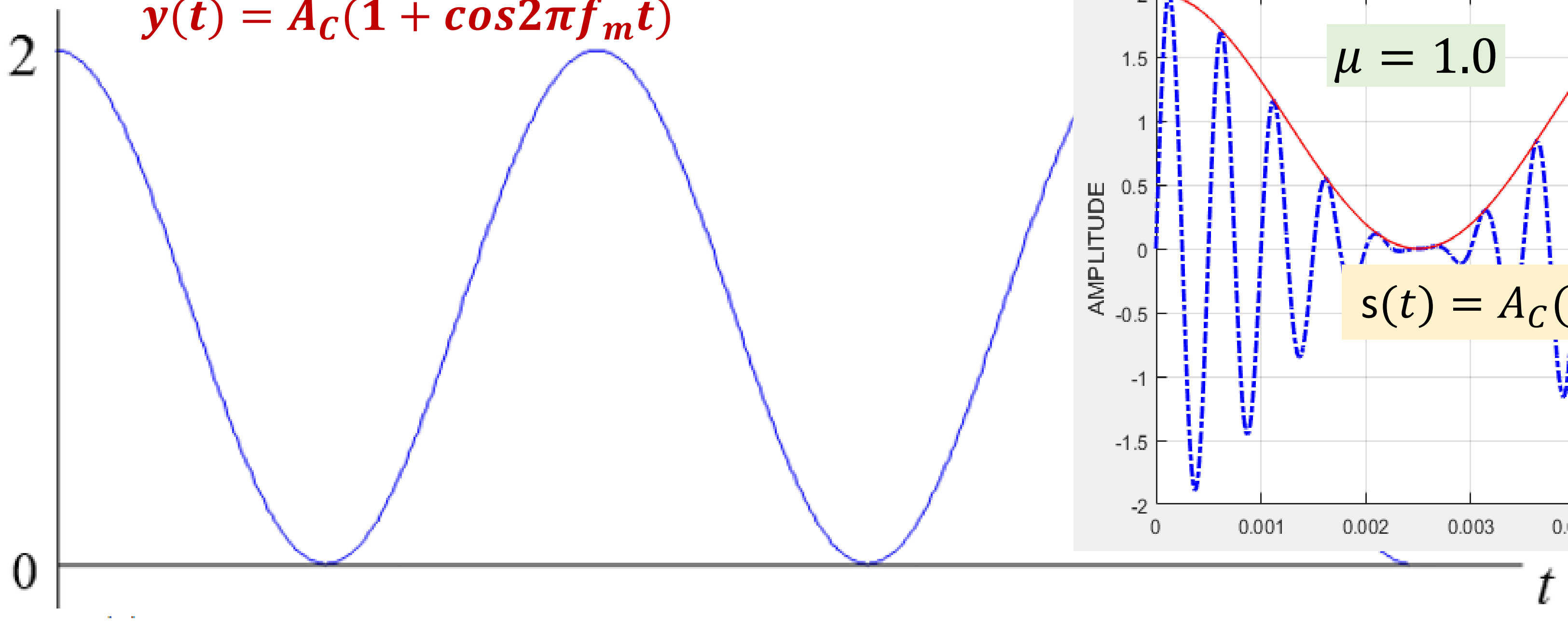


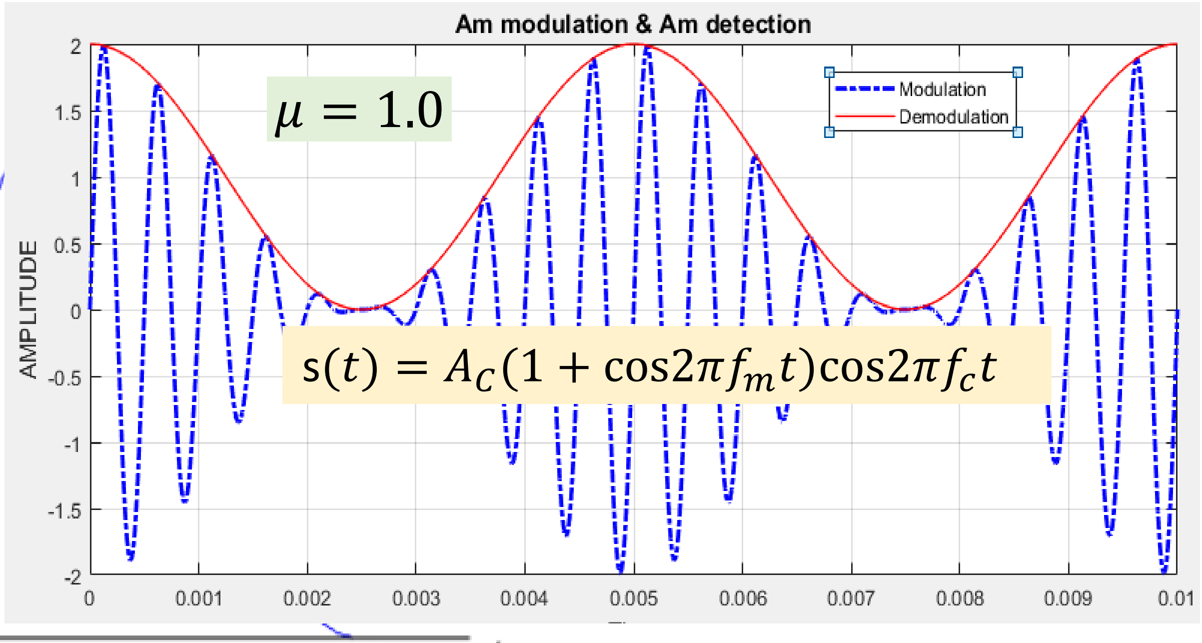
Example: single tone modulation (100% - modulation)

Case2: ($\mu = 1.0$)

$$y(t) = A_C |1 + \cos 2\pi f_m t|$$

$$y(t)$$



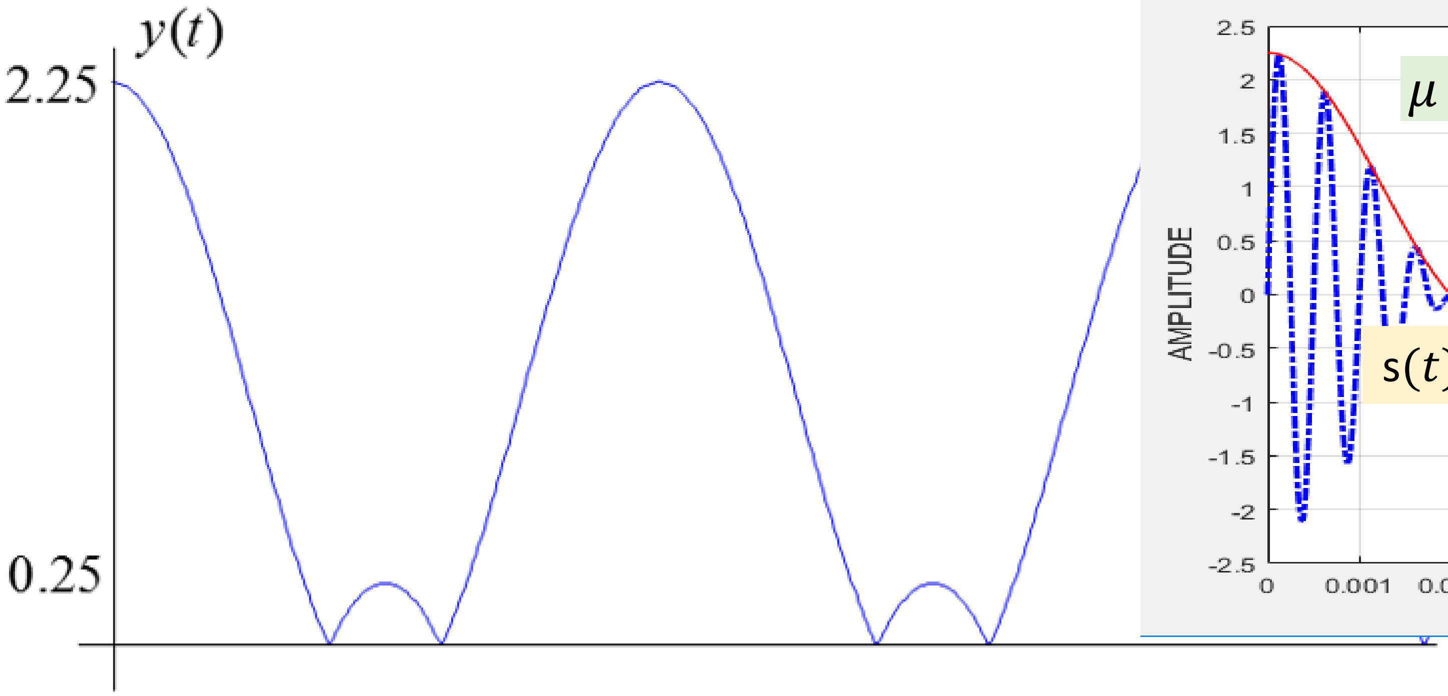


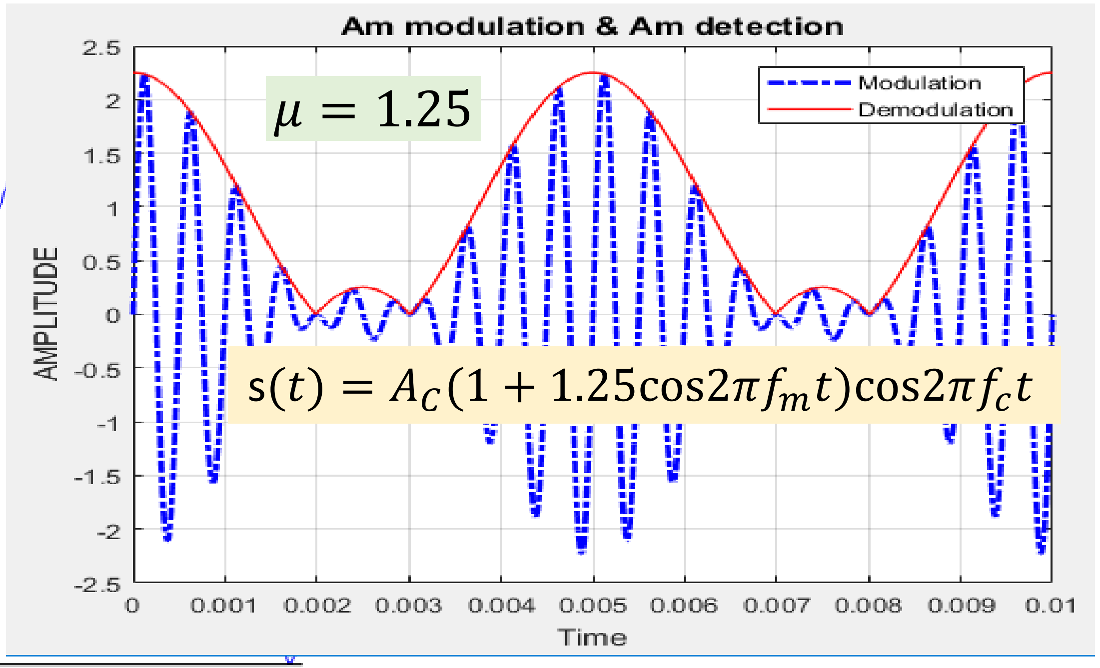
Here, m(t) can be extracted without distortion. $(1 + k_a m(t)) \geq 0$ for all time. That is, $|1 + k_a \mathrm{m(t)}| = (1 + k_a m(t))$. By removing the dc value, the output will be proportional to the message

Example: single tone modulation (over-modulation)

Case3: (μ = 1.25)

$$y(t) = A_C \left| 1 + 1.25\cos 2\pi f_m t \right|$$

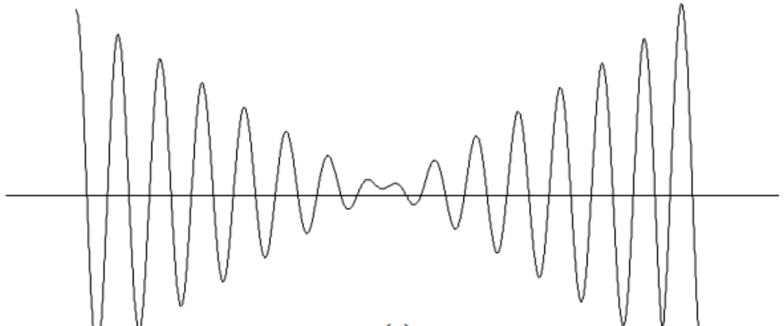




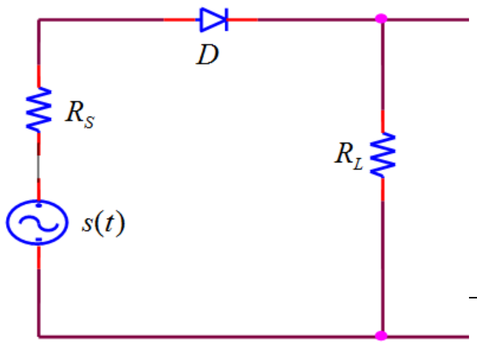
Here, m(t) cannot be extracted without distortion. The shape of the envelope is not the same as the shape of the message. $(1 + k_a m(t))$ fails to remain positive for all time. $|1 + k_a \mathrm{m(t)}| \neq (1 + k_a m(t))$

A Simple Practical Envelope Detector

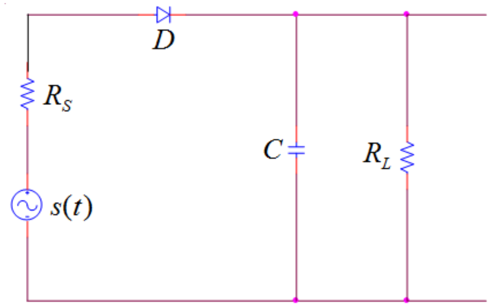
- A practical envelope detector consists of a diode followed by an RC circuit that forms a low pass filter.
- During the positive half cycle of the input, the diode is forward biased and C charges rapidly to the peak value of the input.
- When s(t) falls below the maximum value, the diode becomes reverse biased and C discharges slowly through R_L.
- To follow the envelope of s(t), the circuit time constant should be chosen such that : $\frac{1}{f_C} << R_L C << \frac{1}{W}$ where W is the message B.W and f_C is the carrier frequency.
- When a capacitor C is added to a half wave rectifier circuit, the output follows the envelope of s(t). The circuit output (with C connected) follows a curve that connects the tips of the positive half cycles, which is the envelope of the AM signal.

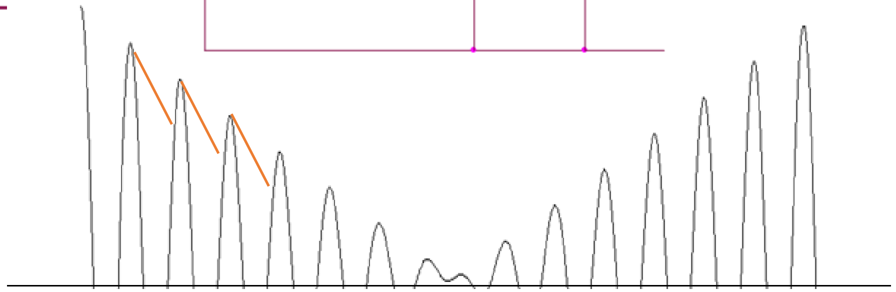


$s(t)$



Half – Wave Rectifier

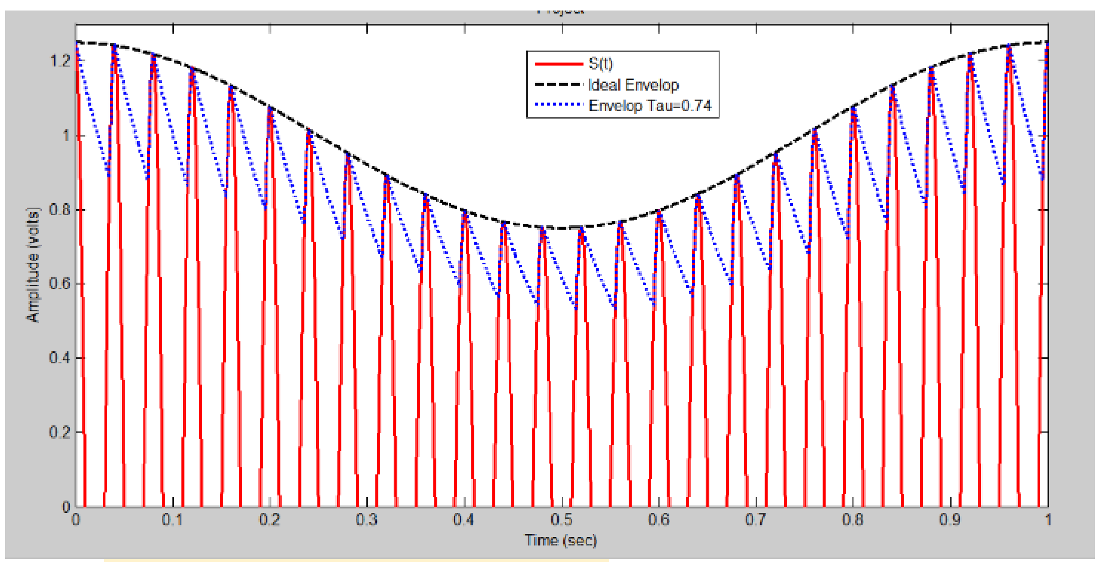




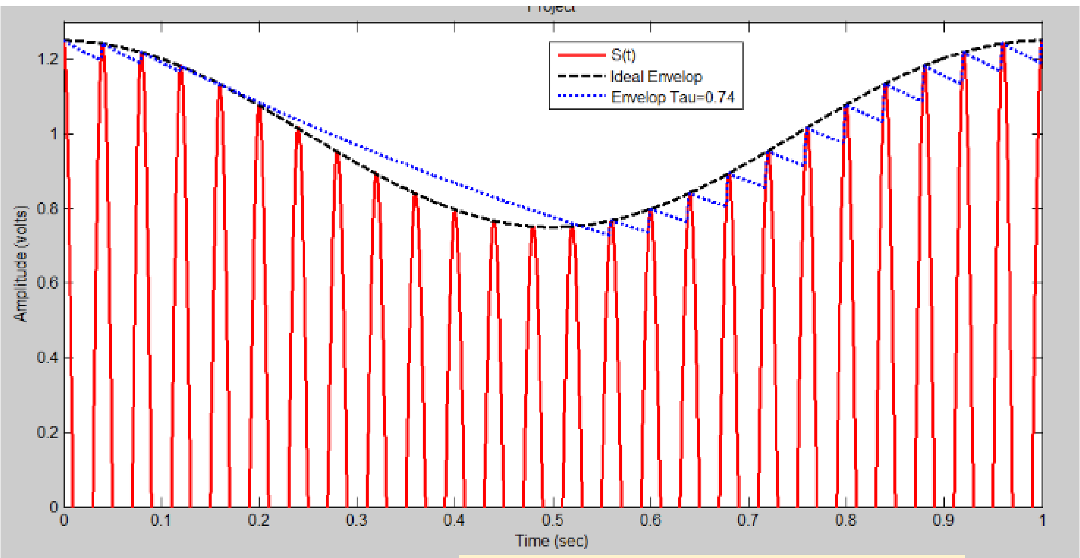
Output of half wave rectifier (without C)

A Simple Practical Envelope Detector: Effect of the Time Constant

Consider the AM signal $s(t) = A_c[1 + \mu \cos(2\pi f_m t)]\cos(2\pi f_c t)$ that is demodulated using the envelope detector. Assume $R_s = 0$, $\mu = 0.25$, $A_c = 1$, $f_m = 1\text{Hz}$, $f_c = 25\text{Hz}$. We show the effect of the time constant $\tau = R_L C$ on the detected signal

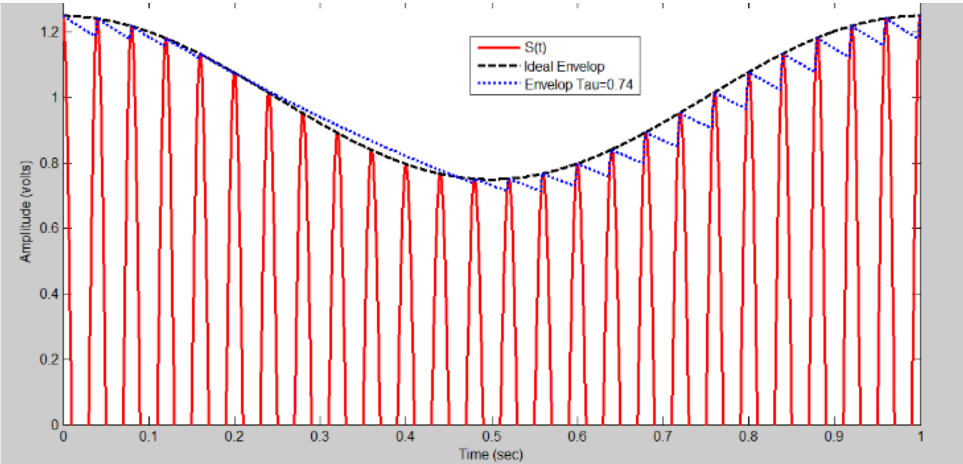


RC output when tau 0.1



RC output when tau 0.9

$T_C << R_L C << T_m$



RC optimum tau 0.74