

Double Sideband Suppressed Carrier (DSB-SC) Modulation: Lecture Outline

- In this lecture, we consider a second type of AM modulation called DSB-SC.
- We analyze this modulation technique in the time and frequency domains.
- Consider the generation and demodulation techniques.
- Study the effect of non-coherence in the phase and frequency of the locally generated carrier at the receiver on the demodulated signal.

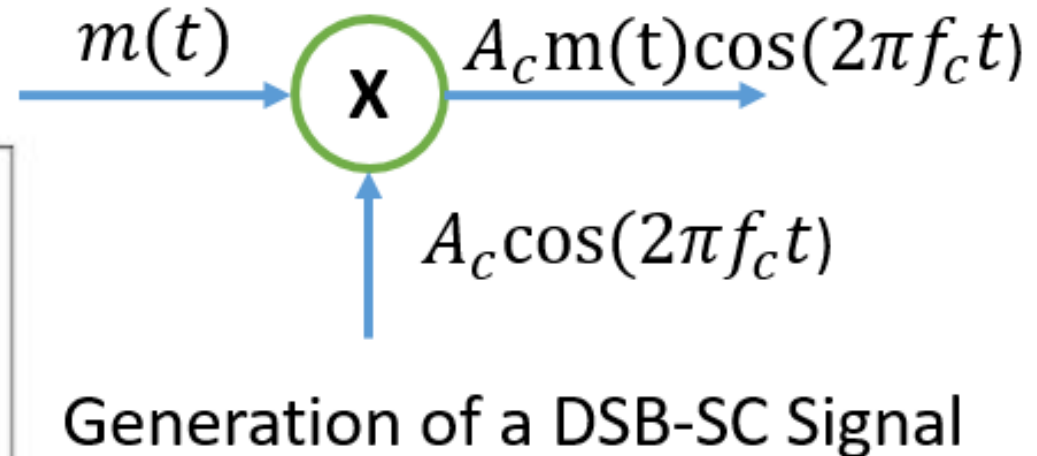
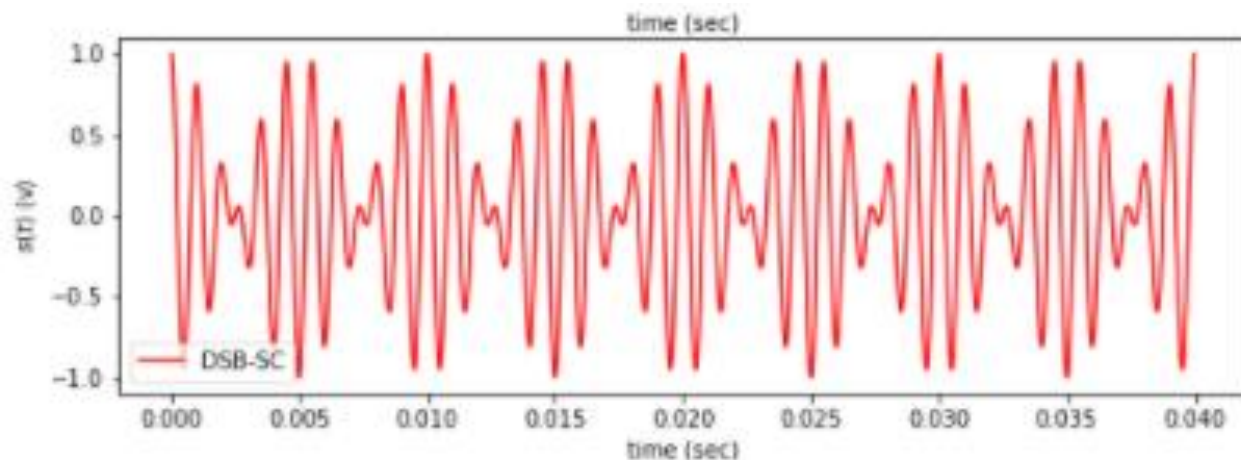
Double Sideband Suppressed Carrier (DSB-SC) Modulation

- A DSB-SC signal is an amplitude-modulated signal that has the form
- $s(t) = A_c m(t) \cos(2\pi f_c t)$, where
- $c(t) = A_c \cos(2\pi f_c t)$: is the carrier signal
- $m(t)$: is the baseband message signal
- $f_c \gg W$, W is the bandwidth of the baseband message signal $m(t)$

FIGURE: $m(t)c(t)$

$$m(t) = \cos(2\pi(100)t)$$

$$c(t) = \cos(2\pi(1000)t);$$

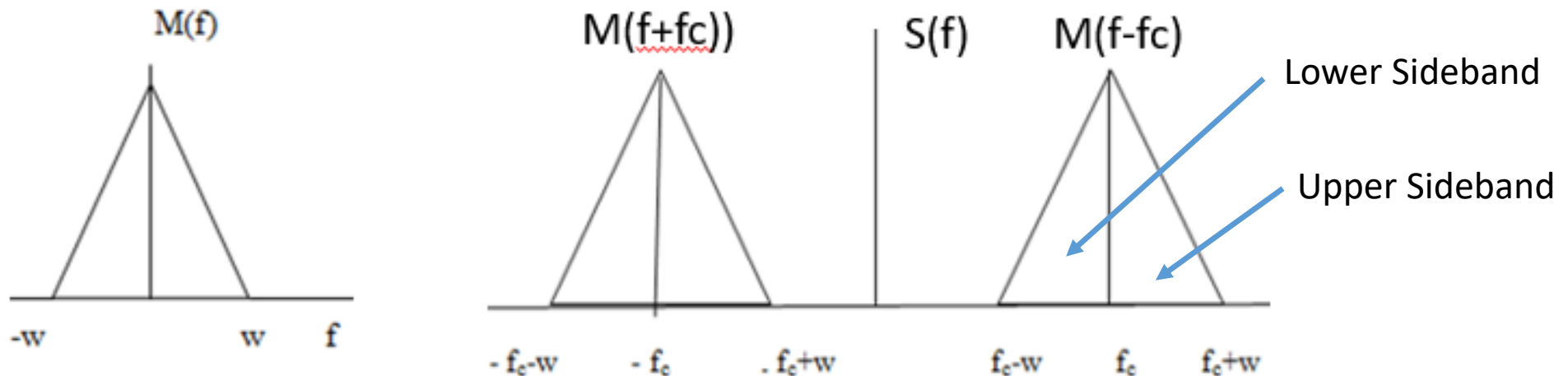


Spectrum of the Double Sideband Suppressed Carrier (DSB-SC)

- **DSB-SC:** $s(t) = A_c m(t) \cos(2\pi f_c t)$
- $S(f) = \mathfrak{T}\{A_c m(t) \cos(2\pi f_c t)\} = \frac{A_c}{2} [M(f - f_c) + M(f + f_c)]$

Remarks: Similarities and Differences with Normal AM

- 1. No impulses are present in the spectrum at $\pm f_c$, i.e., no carrier is transmitted as in the case of AM
- 2. The transmission B.W of $s(t) = 2W$; twice the message bandwidth (same as that of normal AM).
- 3. Power efficiency = $\frac{\text{power in the side bands}}{\text{total transmitted power}} = 100\%$. This is a power efficient modulation scheme.
- 4. Coherent detector is required to extract $m(t)$ from $s(t)$, as we shall demonstrate shortly.
- 5. Envelope detection cannot be used for this type of modulation.

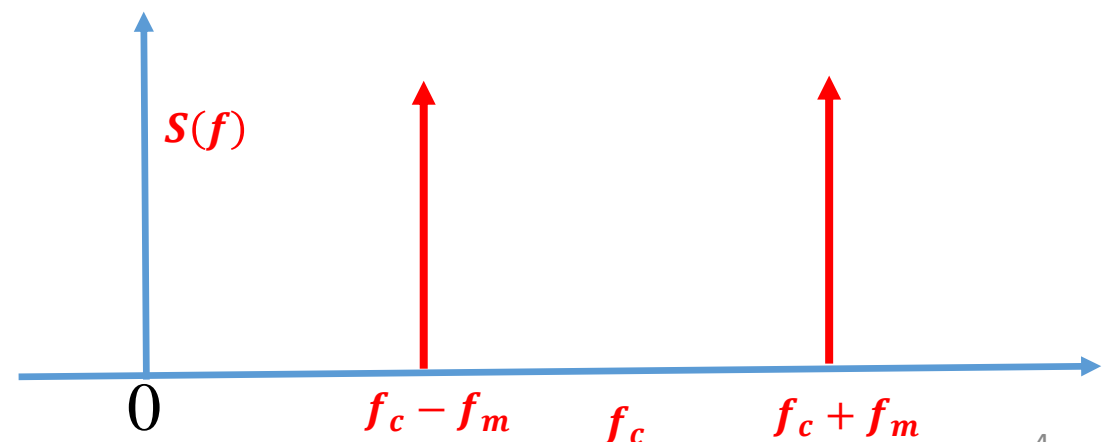


Spectrum of DSB-SC: Sinusoidal Modulation

- **Example:** Consider the sinusoidal modulation case where $c(t) = A_c \cos(2\pi f_c t)$; $m(t) = A_m \cos(2\pi f_m t)$; plot $m(t)$, $c(t)$, $s(t)$ and find their spectrum.

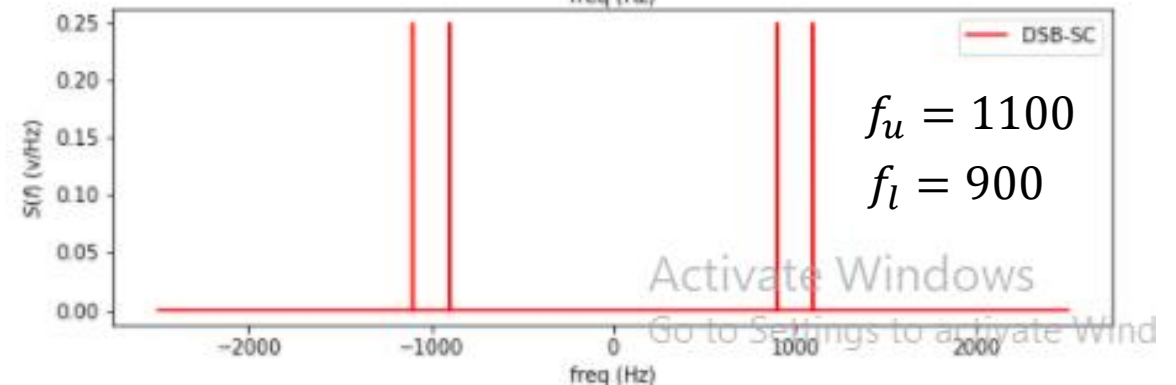
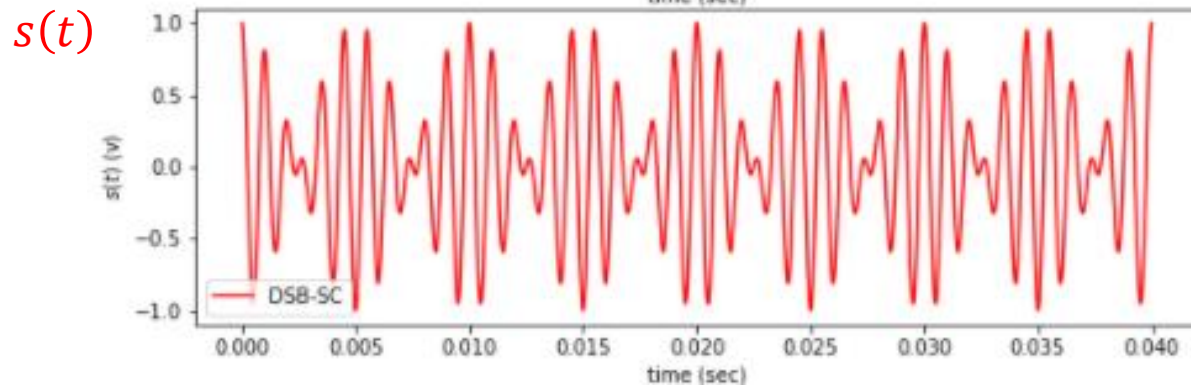
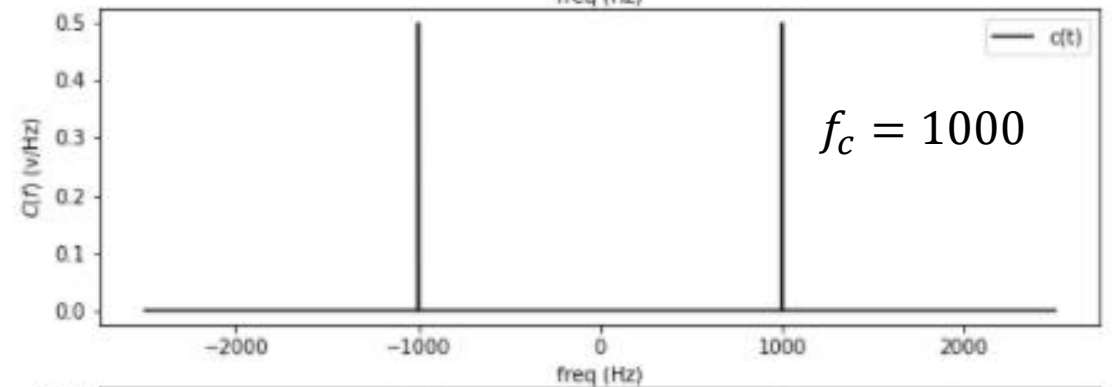
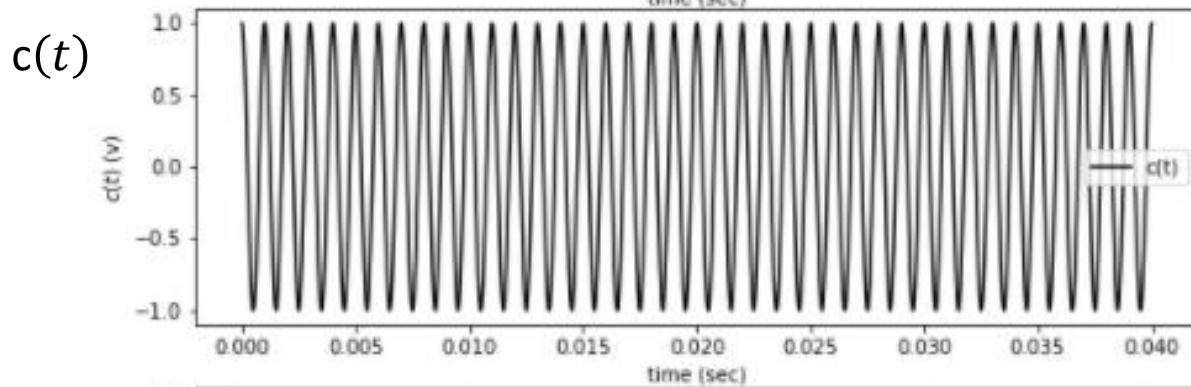
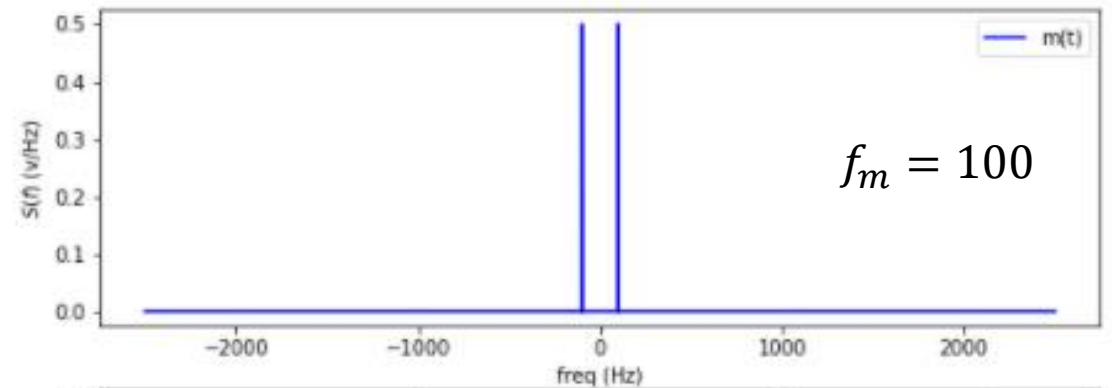
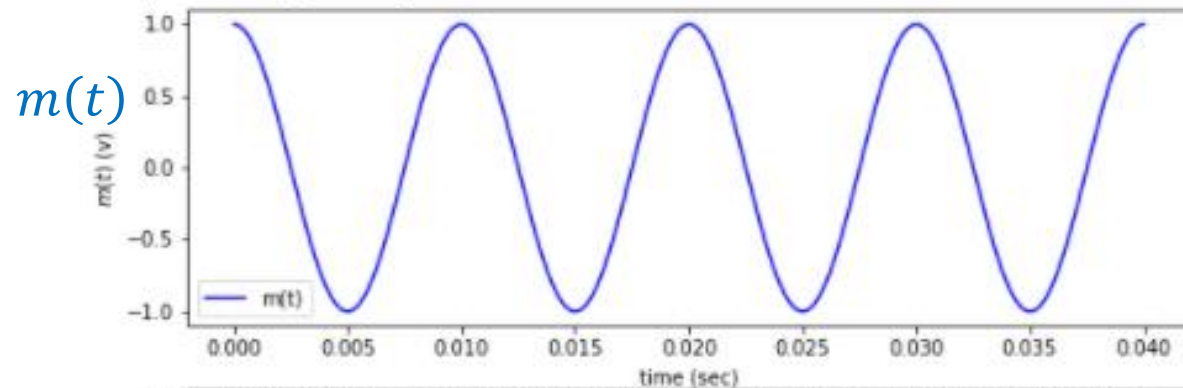
Solution:

- $s(t) = A_c m(t) \cos(2\pi f_c t) = A_c \cos(2\pi f_c t) A_m \cos(2\pi f_m t)$;
- $= \frac{A_c A_m}{2} \cos(2\pi(f_c + f_m)t) + \frac{A_c A_m}{2} \cos(2\pi(f_c - f_m)t)$
- $S(f) = \mathfrak{F}\{A_c m(t) \cos(2\pi f_c t)\} = \frac{A_c}{2} [M(f - f_c) + M(f + f_c)]$
- $M(f) = \frac{A_m}{2} \delta(f - f_m) + \frac{A_m}{2} \delta(f + f_m)$
- The next figure shows all the plots when $f_m = 100$ Hz and $f_c = 1000$ Hz



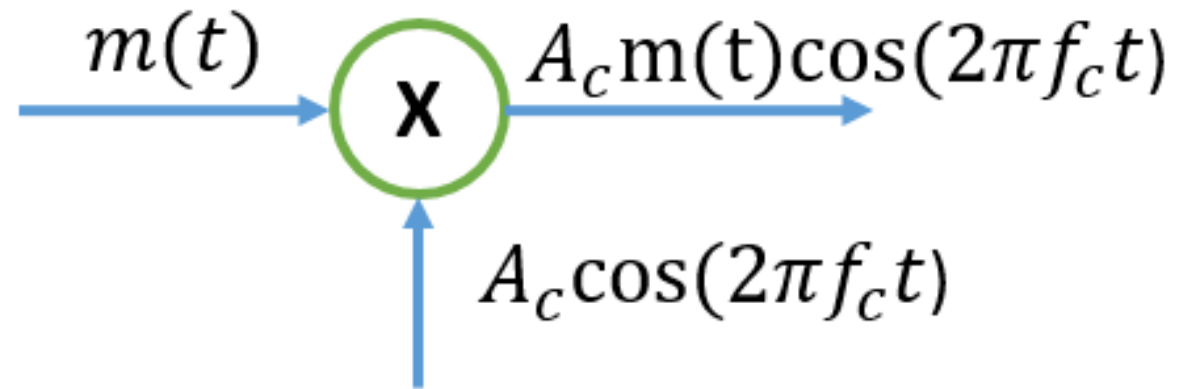
Spectrum of the DSB-SC Signal: Sinusoidal Modulation

$$m(t) = A_m \cos(2\pi(100)t); \quad c(t) = \cos(2\pi(1000)t); \quad s(t) = A_c m(t) \cos(2\pi f_c t);$$



Generation of DSB-SC: The Product Modulator

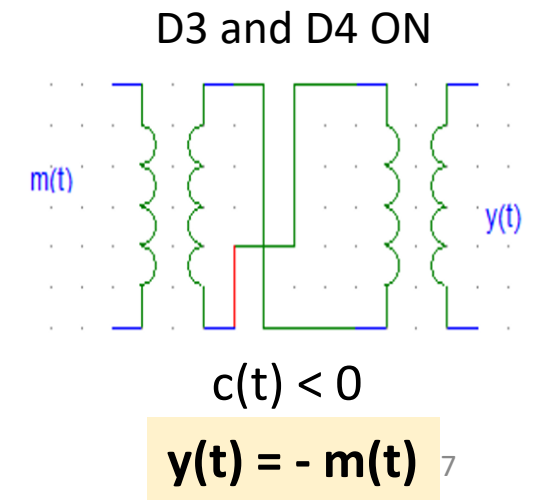
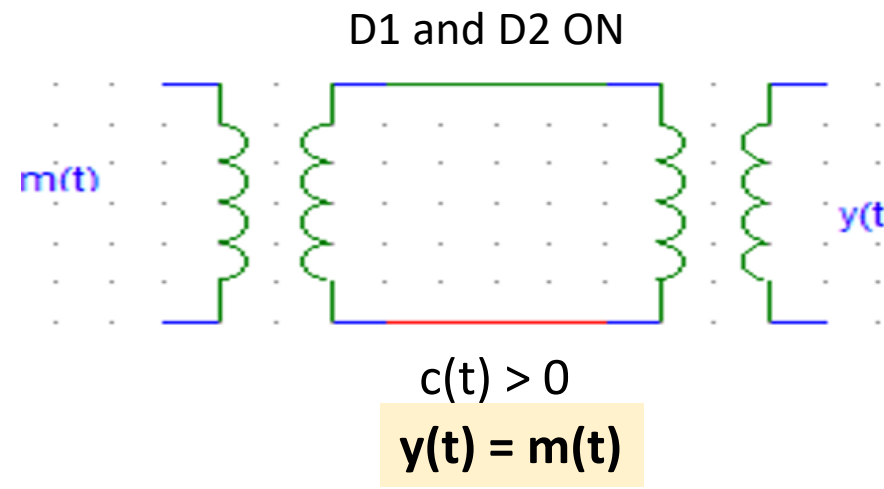
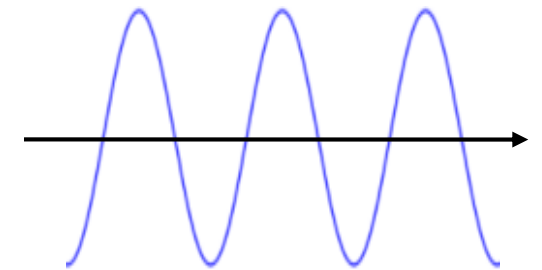
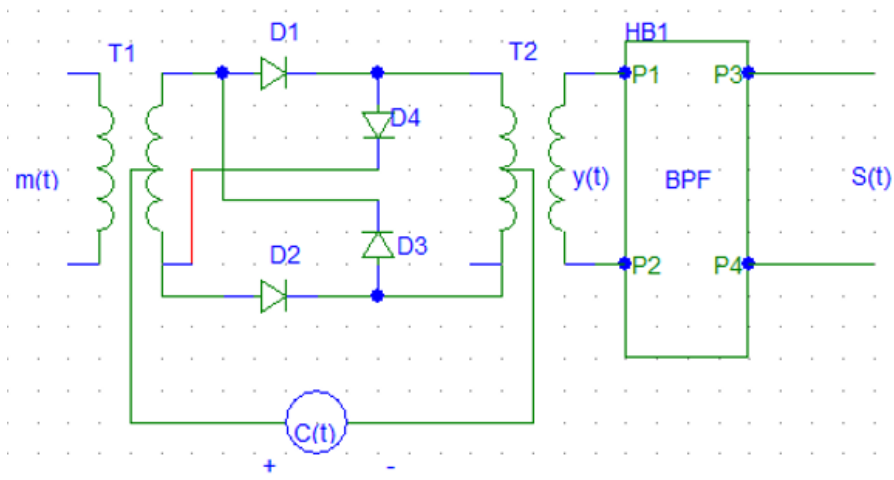
- **Product Modulator:** It multiplies the message signal $m(t)$ with the carrier $c(t)$. This technique is usually applicable when low power levels are possible and over a limited carrier frequency range.



Generation of a DSB-SC Signal

Generation of DSB-SC: The Ring Modulator

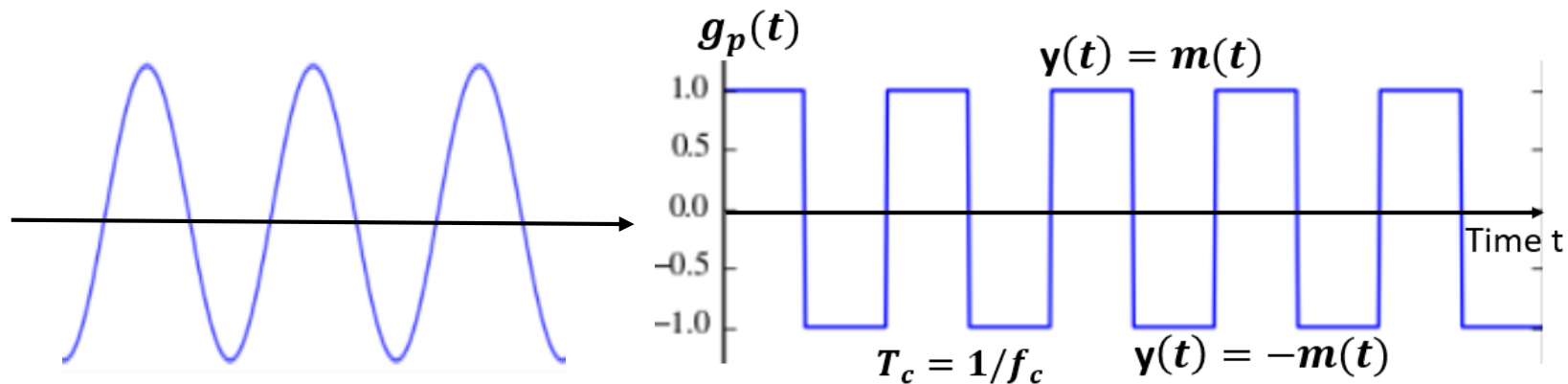
- Consider the scheme shown in the figure.
- Let $c(t) \gg m(t)$. Here the carrier $c(t)$ controls the behavior of the diodes .
 - During the positive half cycle of $c(t)$, $c(t) > 0$, and D1 and D2 are ON while D3 and D4 are OFF. Here, $y(t) = m(t)$.
 - During the negative half cycle of $c(t)$, $c(t) < 0$ and D3 and D4 are ON while and D1 and D2 are OFF. Here, $y(t) = -m(t)$.
 - So $m(t)$ is multiplied by +1 during the +ve half cycle of $c(t)$ and $m(t)$ is multiplied by -1 during the -ve half cycle of $c(t)$



Generation of DSB-SC: The Ring Modulator

- So $m(t)$ is multiplied by +1 during the +ve half cycle of $c(t)$ and $m(t)$ is multiplied by -1 during the -ve half cycle.
- Mathematically, $y(t)$ behaves as if $m(t)$ is multiplied by the switching function $g_p(t)$ where $g_p(t)$ is the square periodic function with period $T_c = \frac{1}{f_c}$; T_c the period of $c(t)$. By expanding $g_p(t)$ in a Fourier series, we get
- $y(t) = m(t) g_p(t) = m(t) \left[\frac{4}{\pi} \cos 2\pi f_c t - \frac{4}{3\pi} \cos 3(2\pi f_c t) + \frac{4}{5\pi} \cos 5(2\pi f_c t) \right]$
- $= m(t) \frac{4}{\pi} \cos 2\pi f_c t - m(t) \frac{4}{3\pi} \cos 3(2\pi f_c t) + m(t) \frac{4}{5\pi} \cos 5(2\pi f_c t)$
- When $y(t)$ passes through the BPF with center frequency f_c , and bandwidth = $2W$, the only component that appears at the output is the desired DSB-SC signal, which is

$$s(t) = \frac{4}{\pi} m(t) \cos 2\pi f_c t$$

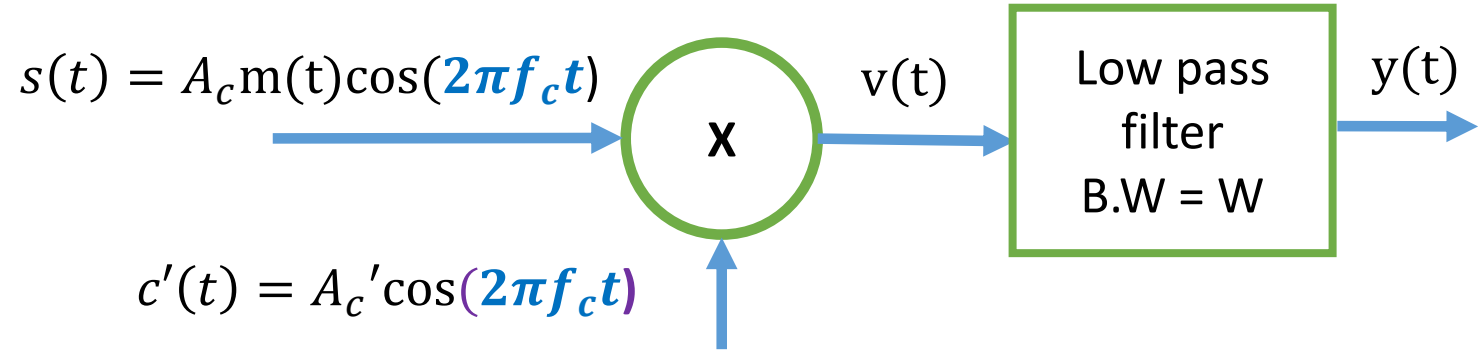


Demodulation of DSB-SC

- A DSB-SC signal is demodulated using what is known as **coherent demodulation**. This means that the modulated signal $s(t)$ is multiplied by a locally generated signal at the receiver which has the same frequency and phase as that of the carrier $c(t)$ at the transmitting side

Perfect Coherent Demodulation

- Let $c(t) = A_c \cos(2\pi f_c t)$
- $c'(t) = A_c' \cos(2\pi f_c t)$



- Mixing the received signal with the version of the carrier at the receiving side, we get
- $v(t) = s(t)A_c' \cos(2\pi f_c t) = A_c A_c' m(t) \cos^2 2\pi f_c t$
- $= \frac{A_c A_c'}{2} m(t) [1 + \cos 2(2\pi f_c t)] = \frac{A_c A_c'}{2} m(t) + \frac{A_c A_c'}{2} m(t) \cos 2(2\pi f_c t)$
- The first term on the RHS is proportional to $m(t)$, while the second term is a DSB signal modulated on a carrier with frequency $2f_c$. The high frequency component can be eliminated using a LPF with B.W = W. The output is **$y(t) = \frac{A_c A_c'}{2} m(t)$**
- Therefore, $m(t)$ has been recovered from $s(t)$ without distortion, i.e., the whole modulation-demodulation process is distortion-less.

Effect of Carrier Non-Coherence on Demodulated Signal: Constant Phase Shift

A constant phase difference between $c(t)$ and $c'(t)$

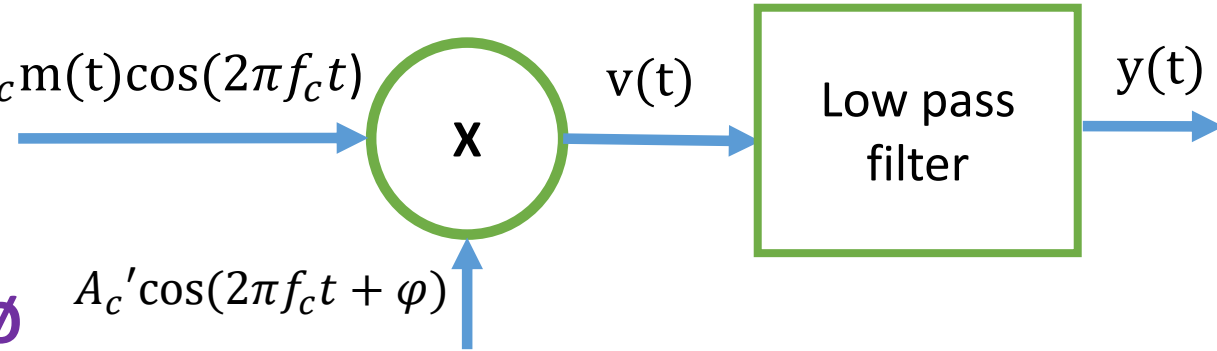
- Let $c(t) = A_c \cos 2\pi f_c t$, $c'(t) = A_c' \cos(2\pi f_c t + \varphi)$

- We use the same demodulator

- $v(t) = A_c m(t) \cos 2\pi f_c t \cdot A_c' \cos(2\pi f_c t + \varphi)$ $s(t) = A_c m(t) \cos(2\pi f_c t)$

- $= \frac{A_c A_c'}{2} m(t) [\cos(4\pi f_c t + \varphi) + \cos \varphi]$

- $= \frac{A_c A_c'}{2} m(t) \cos(4\pi f_c t + \varphi) + \frac{A_c A_c'}{2} m(t) \cos \varphi$ $A_c' \cos(2\pi f_c t + \varphi)$



- The low pass filter suppresses the first high frequency term and admits only the second low frequency term. The output is $y(t) = \frac{A_c A_c'}{2} m(t) \cos \varphi$

- For $0 < \varphi < \frac{\pi}{2}$, $0 < \cos \varphi < 1$, $y(t)$ suffers from an attenuation due to φ .

- However, **for $\varphi = \frac{\pi}{2}$, $\cos \varphi = 0$ and $y(t) = 0$, i.e., receiver loses the signal.**

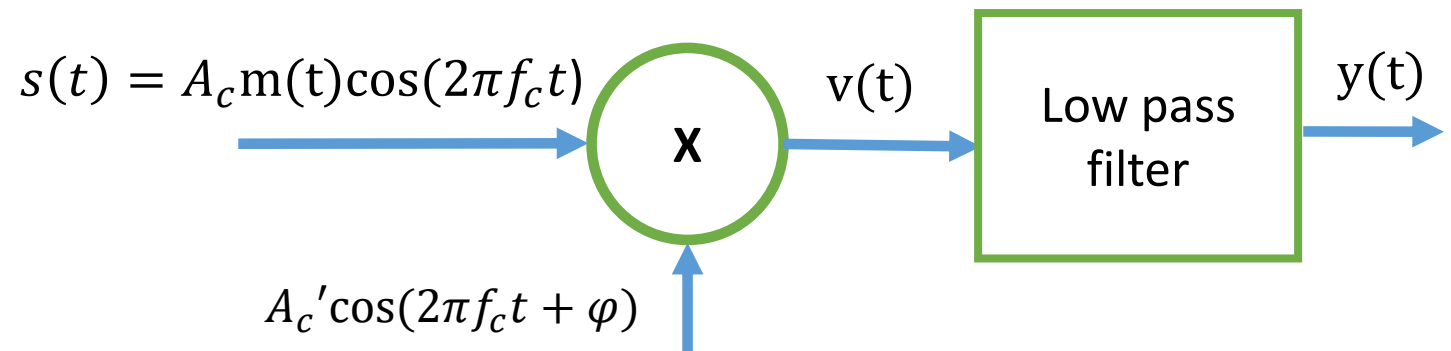
- The disappearance of a message component at the demodulator output is called **quadrature null effect**. This highlights the importance of maintaining synchronism between the transmitting and receiving carrier signals $c'(t)$ and $c(t)$.

Effect of Carrier Non-Coherence on Demodulated Signal: Constant Phase Shift

Example: Let $m_1(t) = \cos 2\pi(1000)t$; $m_2(t) = \cos 2\pi(2000)t$; $m(t) = m_1(t) + m_2(t)$
 $c(t) = \cos 2\pi(10000)t$ and let $\phi = 50$ degrees.

Solution: From the analysis above,

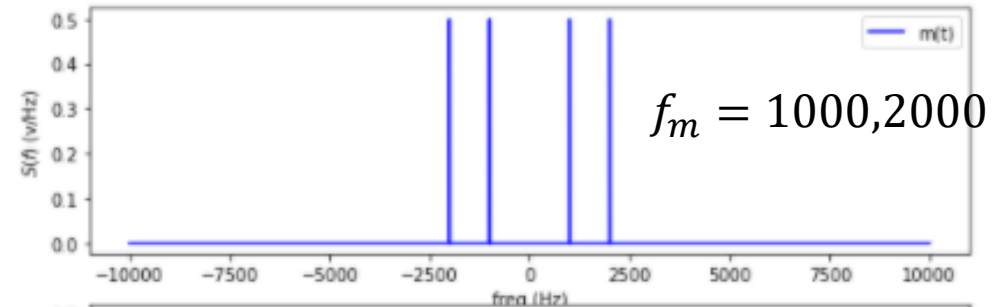
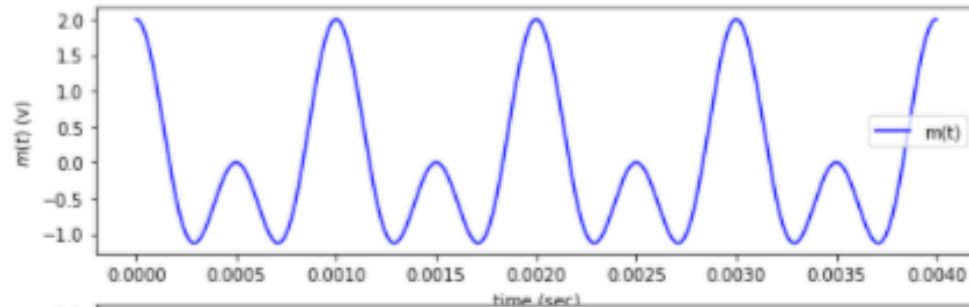
- $y(t) = \frac{A_c A'_c}{2} m(t) \cos \phi$
- The next figure shows the input message, carrier, modulated, and demodulated signals in the time and frequency domains.



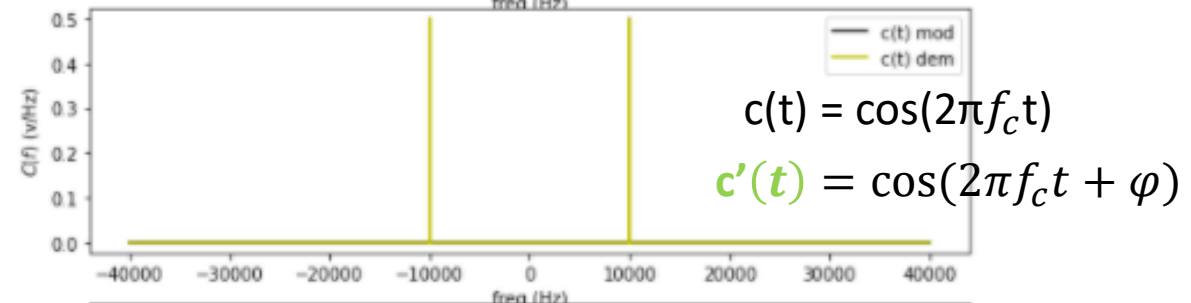
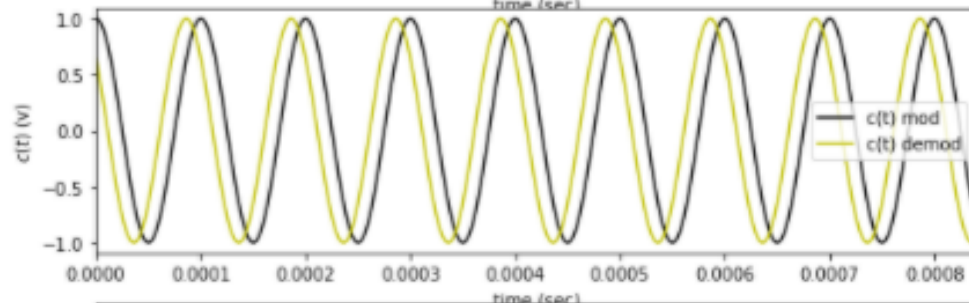
Effect of Carrier Non-Coherence on Demodulated Signal: Constant Phase Shift

$m_1(t) = \cos 2\pi(1000)t$; $m_2(t) = \cos 2\pi(2000)t$; $m(t) = m_1(t) + m_2(t)$ $c(t) = \cos 2\pi(10000)t$ and let $\phi = 50$ degrees.

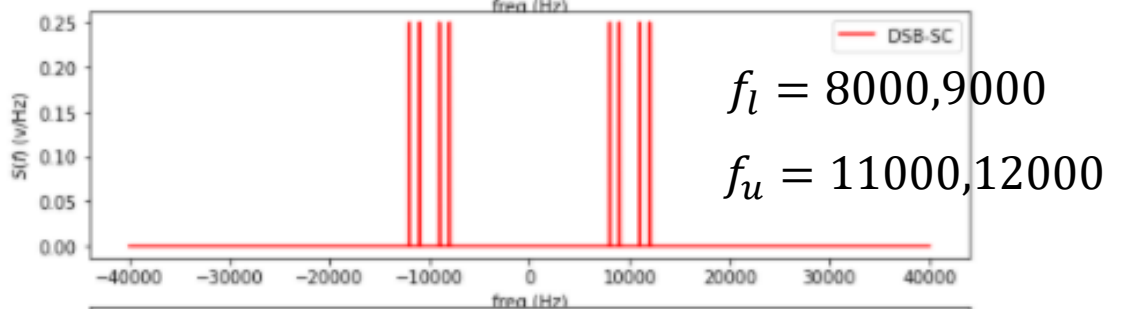
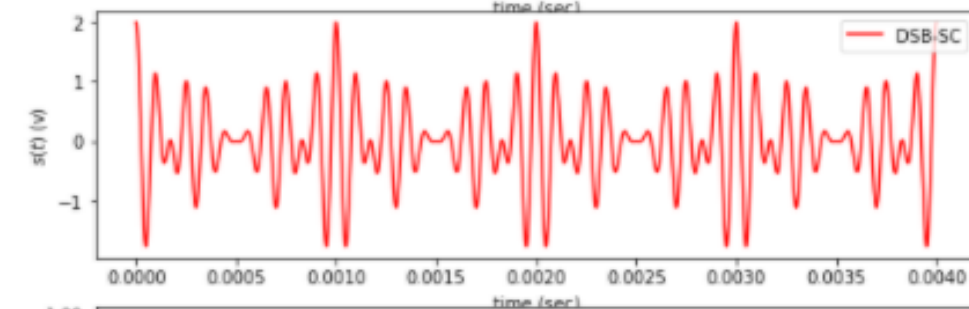
$m(t)$



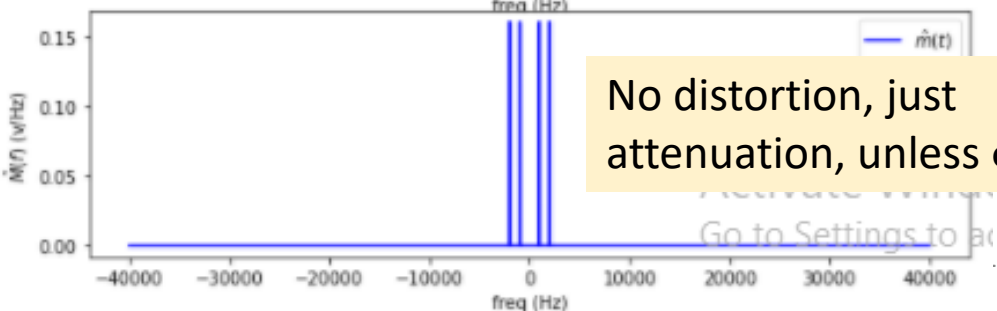
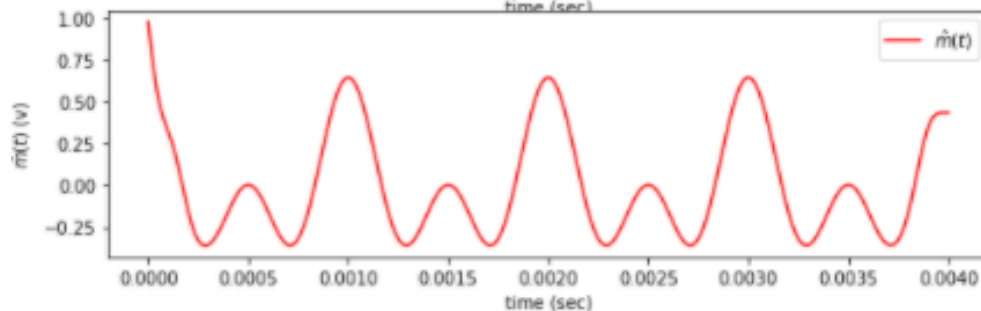
$c(t)$
 $c'(t)$



$s(t)$



$y(t)$

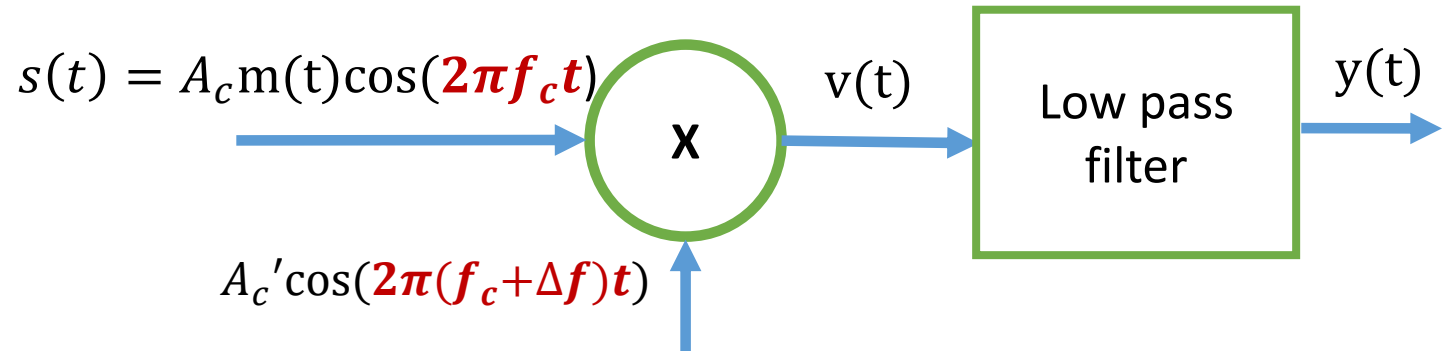


Effect of Carrier Non-Coherence on Demodulated Signal: Constant Frequency Difference

Constant Frequency Difference between $c(t)$ and $c'(t)$

- Let $c(t) = A_c \cos 2\pi f_c t$, $c'(t) = A_c' \cos(2\pi(f_c + \Delta f)t)$
- Again, we use the same receiver structure as before.
- $v(t) = A_c m(t) \cos(2\pi f_c t) \cdot A_c' \cos(2\pi(f_c + \Delta f)t)$
- $= \frac{A_c A_c'}{2} m(t) [\cos(4\pi f_c t + 2\pi \Delta f t) + \cos 2\pi \Delta f t]$
- After low-pass filtering,

$$y(t) = \frac{A_c A_c'}{2} m(t) \cos(2\pi \Delta f t)$$



- As you can see, $y(t) \neq km(t)$, but rather $m(t)$ is multiplied by a time function. Hence, the system is not distortion-less.
- In addition, $y(t)$ appears as a **double side band modulated signal with a carrier with magnitude Δf** . The next example illustrates this case more.

Effect of Carrier Non-Coherence on Demodulated Signal: Constant Frequency Difference

Example: Let $m(t) = \cos 2\pi(1000)t$; $c(t) = \cos 2\pi(10000)t$ and let $\Delta f = 500$ Hz

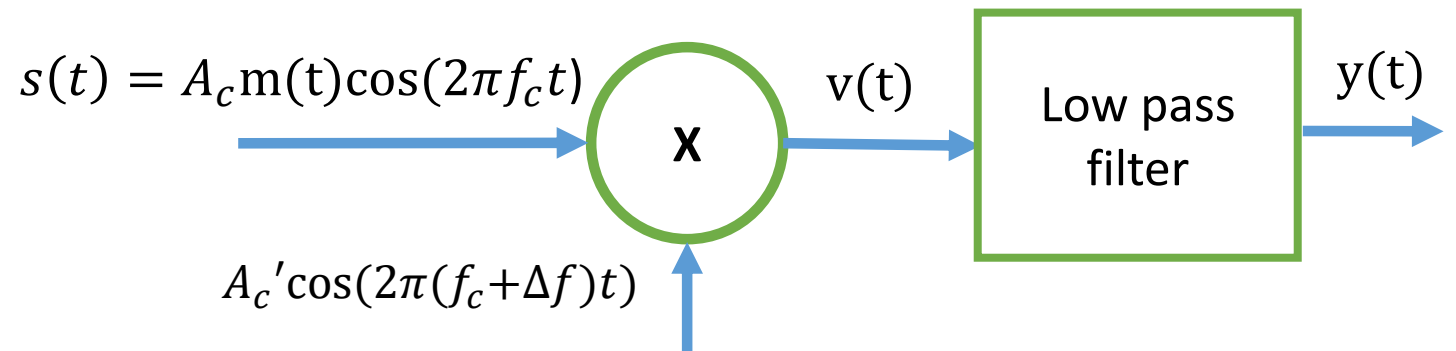
Solution: From the analysis in case 2 above,

$$y(t) = \frac{A_c A_c'}{2} m(t) \cos(2\pi \Delta f t)$$

$$y(t) = \frac{A_c A_c'}{2} \cos 2\pi(1000)t \cos 2\pi(500)t$$

$$= \frac{A_c A_c'}{4} [\cos 2\pi(1500)t + \cos 2\pi(500)t]$$

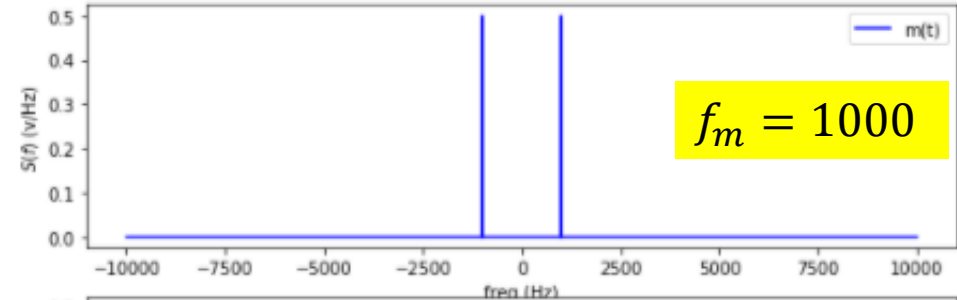
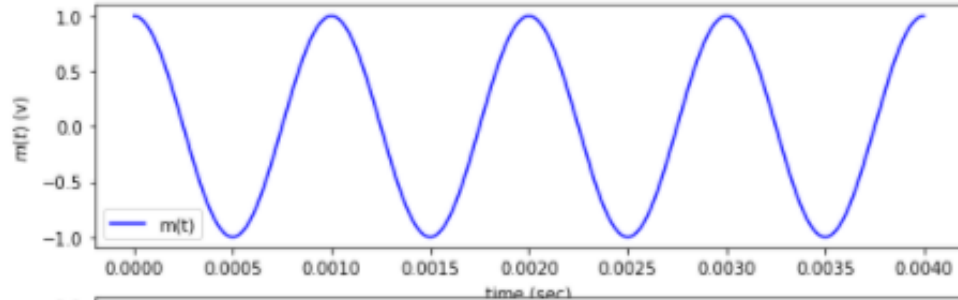
- The original message is a signal with a single frequency of 1000 Hz, while the output consists of a signal with two frequencies at $f_1 = 1500$ Hz and $f_2 = 500$ Hz
- \Rightarrow ***Distortion***



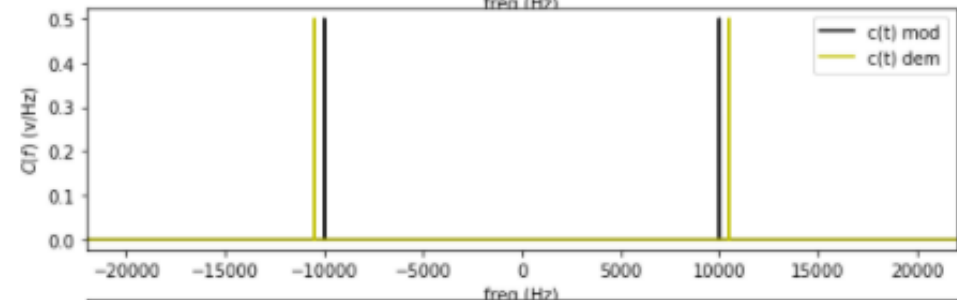
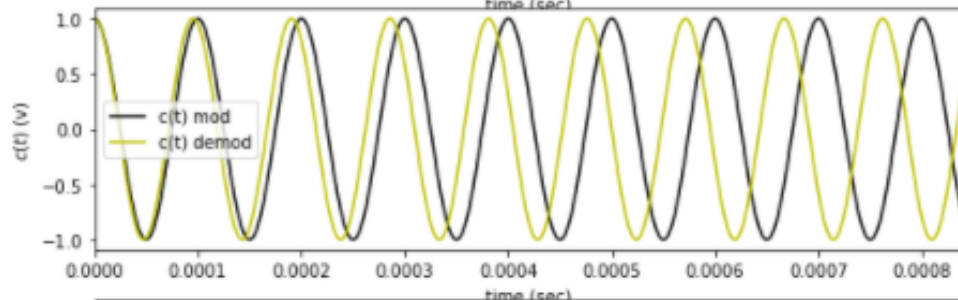
Effect of Carrier Non-Coherence on Demodulated Signal: Constant Frequency Difference

$$m(t) = A_c \cos(2\pi(1000)t); c(t) = \cos(2\pi(10000)t); c'(t) = \cos(2\pi(10500)t); \Delta f = 500$$

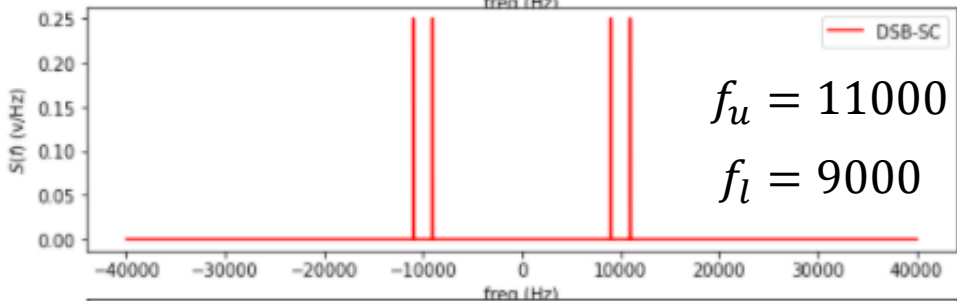
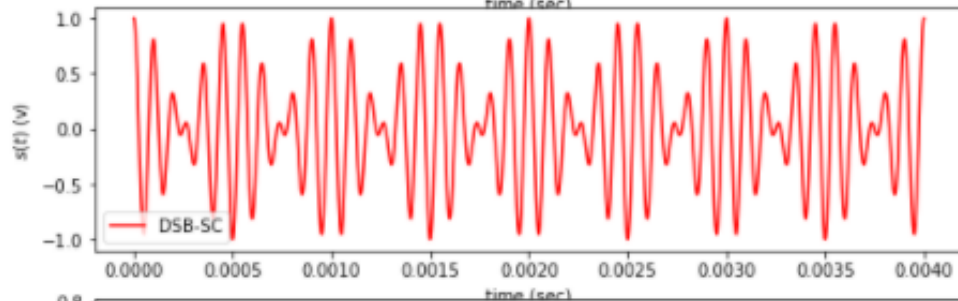
$m(t)$



$c(t)$
 $c'(t)$



$s(t)$



$y(t)$

