Single Sideband Suppressed Carrier (SSB-SC) Modulation: Lecture Outline

- In this lecture, we consider another type of AM modulation called SSB-SC.
- We analyze this modulation technique in the time and frequency domains.
- Consider the generation and demodulation techniques.
- Study the effect of non-coherence in the phase and frequency of the locally generated carrier at the receiver, on the demodulated signal.

Normal AM Signal

Let the Fourier transform of $m(t)$ be as shown (the B.W of $m(t)$ = W Hz).

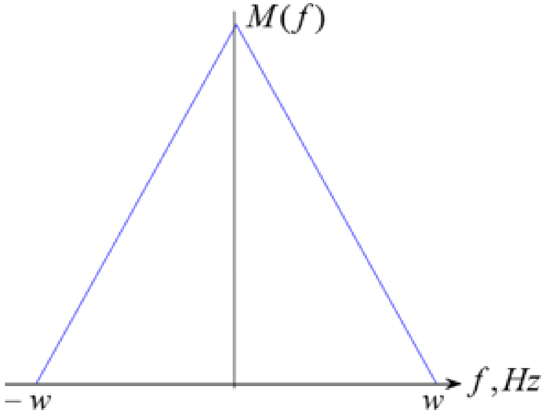
$$s(t) = A_C\left(1 + k_a m(t)\right)\cos 2\pi f_C t \qquad \text{(dc + message)*carrier}$$

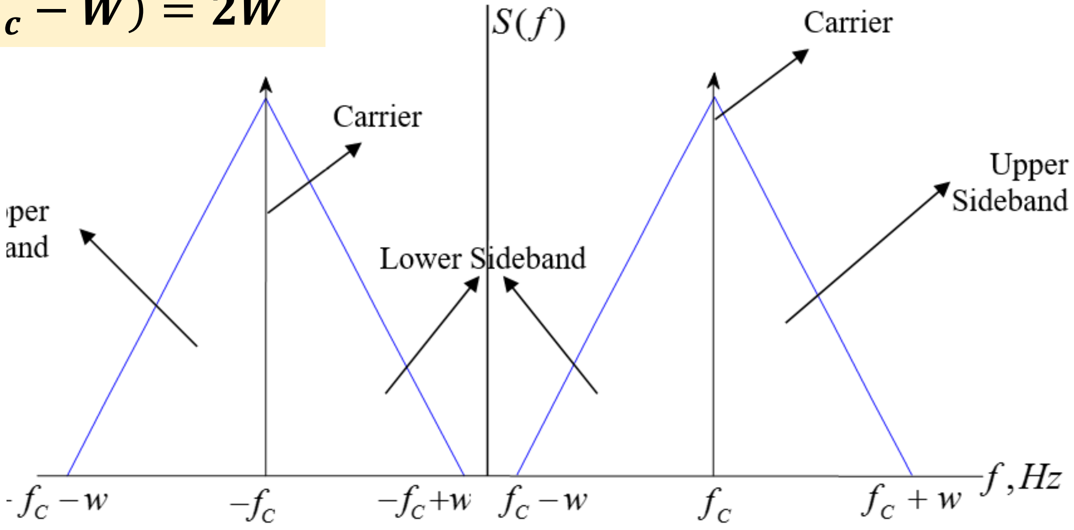
$$s(t) = A_C \cos 2\pi f_C t + A_C k_a m(t)\cos 2\pi f_C t \qquad \text{(carrier + message*carrier)}$$

Taking the Fourier transform, we get

$$S(f) = \frac{A_C}{2}\delta(f - f_C) + \frac{A_C}{2}\delta(f + f_C) + \frac{A_C k_a}{2}M(f - f_C) + \frac{A_C k_a}{2}M(f + f_C)$$

$$\boldsymbol{B.W. = (f_c + W) - (f_c - W) = 2W}$$





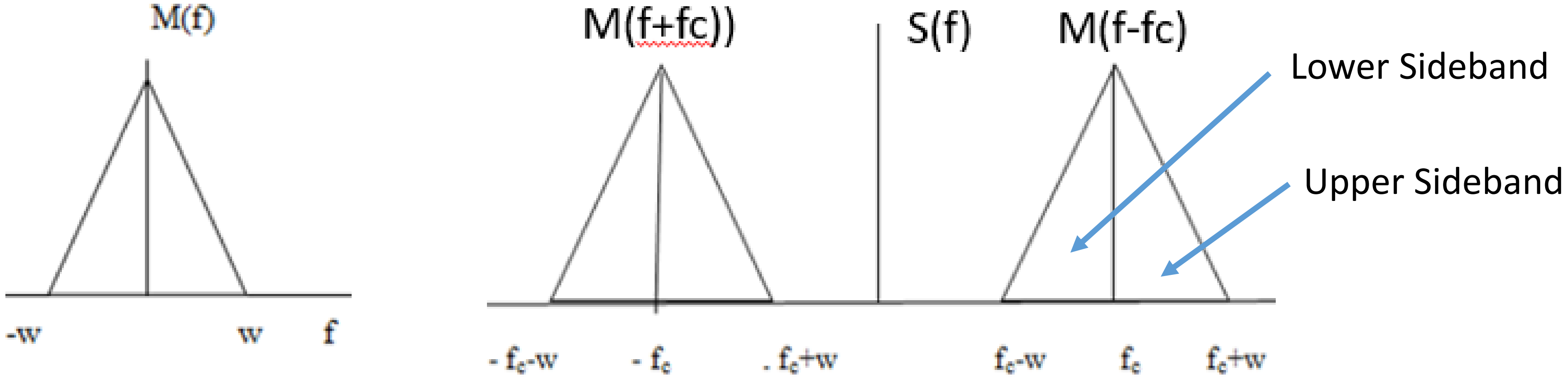
- **Two impulses are present in the spectrum at $\pm$ f_c ,**
- **2. The transmission B.W of s(t) = 2W; twice the message bandwidth**
- **Poor power efficiency.**
- **Envelope detection is used for this type of modulation.**

Double Sideband Suppressed Carrier (DSB-SC)

- **DSB-SC:** $s(t) = A_c m(t)\cos(2\pi f_c t)$
- $S(f) = \Im\{A_c m(t)\cos(2\pi f_c t)\} = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]$

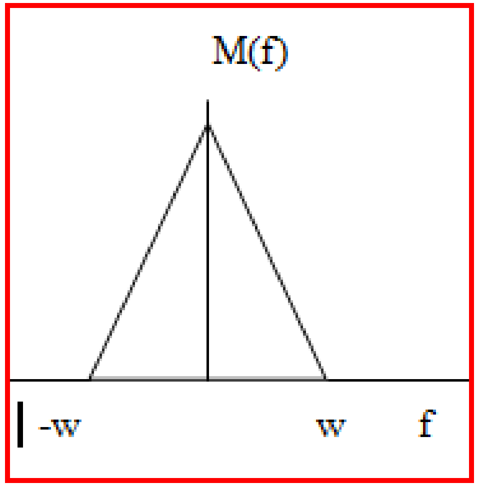
Remarks:

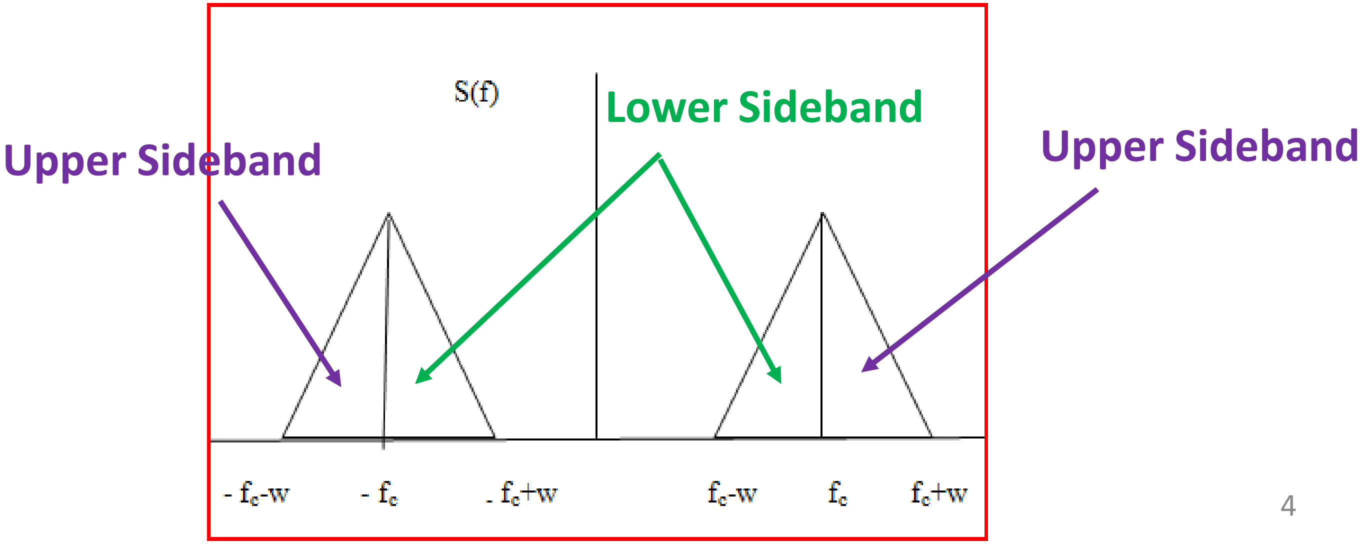
- 1. No impulses are present in the spectrum at $\pm f_c$, i.e., no carrier is transmitted as in the case of AM
- 2. The transmission B.W of s(t) = 2W; twice the message bandwidth (same as that of normal AM).
- 3. Power efficiency = $\frac{power\ in\ the\ side\ bands}{total\ transmitted\ power}$ = 100%. This is a power efficient modulation scheme.
- 4. Coherent detector is required to extract m(t) from s(t)
- 5. Envelope detection cannot be used for this type of modulation.



Single Sideband Modulation

- **Rationale:** The information representing the modulating waveform is contained in both the upper and the lower sidebands of the DSB signal. $\Rightarrow$ Redundant Transmission.
- Therefore, it is not essential to transmit both side-bands. The transmission of one sideband will suffice in reconstructing the message signal at the receiver.
- In SSB-SC the carrier is suppressed and one of the two sideband is transmitted.
- Hence, power saving and bandwidth saving are achieved
- Sometimes, an attenuated part of the carrier is transmitted that will ease the process of demodulation called residual carrier SSB signal, but this will not be addressed in this lecture.



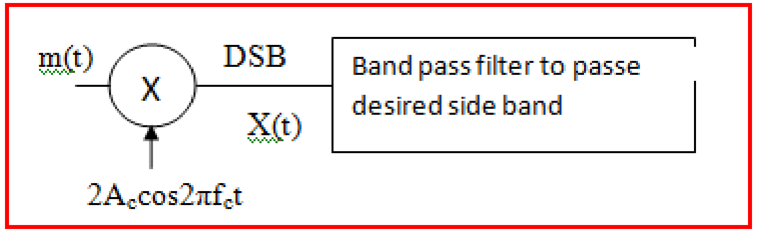


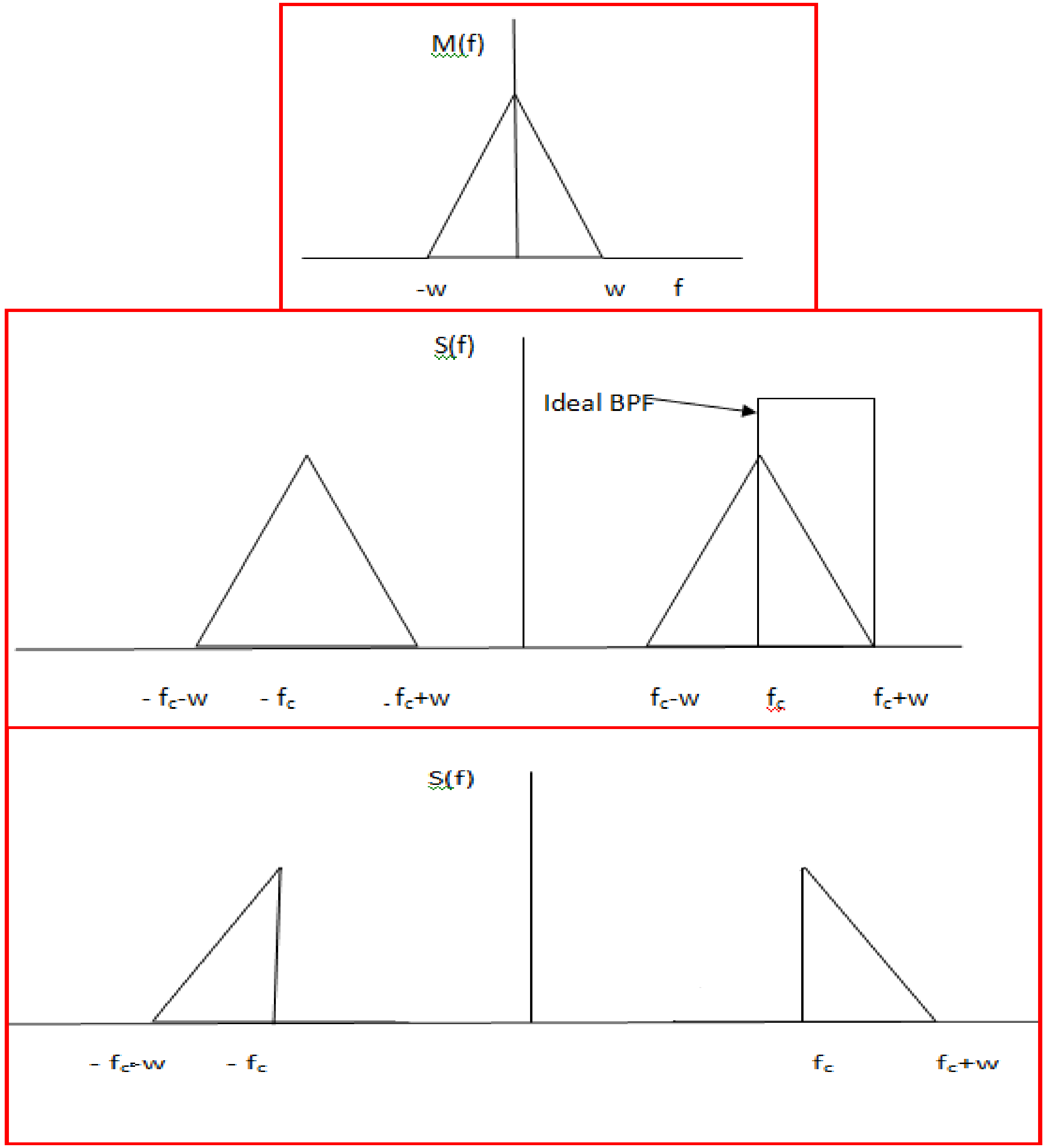
Single Sideband Suppressed Carrier (DSB-SC) Modulation

- In this type of modulation, only one of the two sidebands of a DSB-SC is retained while the other sideband is suppressed. This means that the B.W of the SSB signal is one half that of DSB-SC. The saving in the bandwidth comes at the expense of increasing modulation/demodulation complexity.
- The time-domain representation of a SSB signal is
- $\boldsymbol{s(t) = A_C m(t) cos\omega_c t \pm A_C \hat{m}(t) sin\omega_c t}$
- $m(t)$: is the baseband message signal with bandwidth W.
- $\hat{m}(t)$: Hilbert transform of $m(t)$ obtained by passing $m(t)$ through a 90-degrees phase shifter.
- \- sign: upper sideband is retained.
- \+ sign: lower sideband is retained.
- $c(t) = A_c \cos(2\pi f_c t)$: is the high frequency carrier signal; $f_c >> W$.

Generation of SSB: Filtering Method

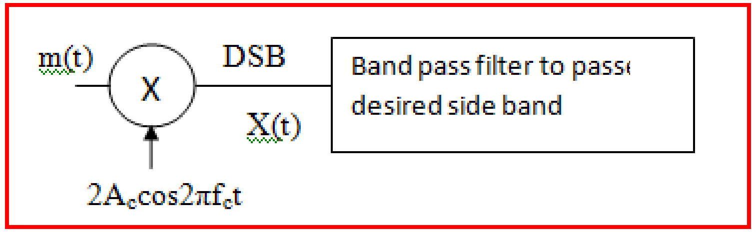
- A DSB-SC signal $x(t) = 2A_C m(t) cos\omega_c t$ is generated first. A band pass filter with appropriate B.W and center frequency is used to pass the desired side band only and suppress the other sideband.
- The pass band of the filter must occupy the same frequency range as the desired sideband.
- **Remark**: Ideal filter do not exist in practice meaning that a complete elimination of the undesired side band is not possible. The consequence of this is that either part of the undesired side band is passed or the desired one will be highly attenuated. SSB modulation is suitable for signals with low frequency components that are not rich in terms of their power content, as we shall see next.

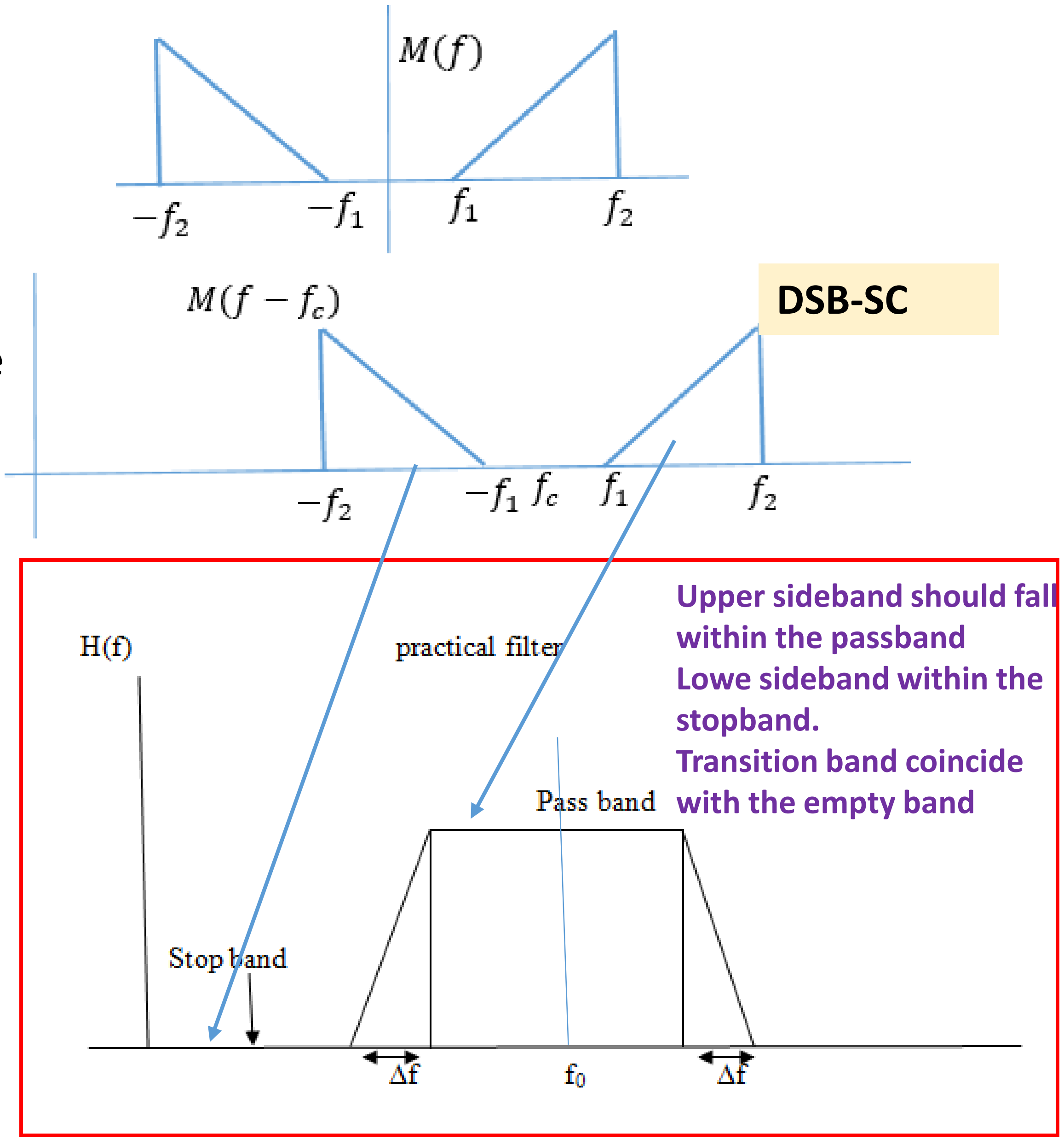




Generation of SSB: Practical Consideration on the Filtering Method

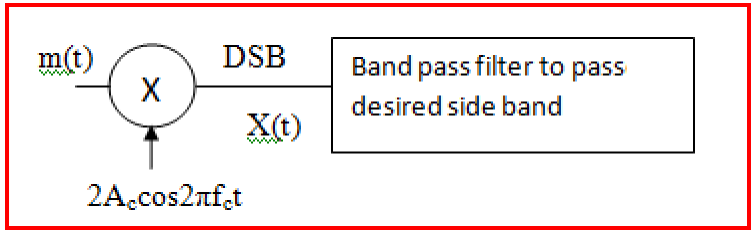
- The following practical considerations must be taken into account:
  - The pass band of the filter must occupy the same frequency band as the desired sideband.
  - The width of the transition band of the filter separating the pass band and the stop band must be at least 1% of the center frequency of the filter. i.e., **$0.01f_0 \leq \Delta f$**. This is sort of a rule of thumb for realizable filters on the relationship between the transition band and the center frequency.
  - The width of the transition band of the filter should be at most twice the lowest frequency components of the message signal so that a reasonable separation of the two side band is possible. If the message significant frequency components extends between (f_1, f_2), then $\boldsymbol{2f_1 \geq \Delta f}$.

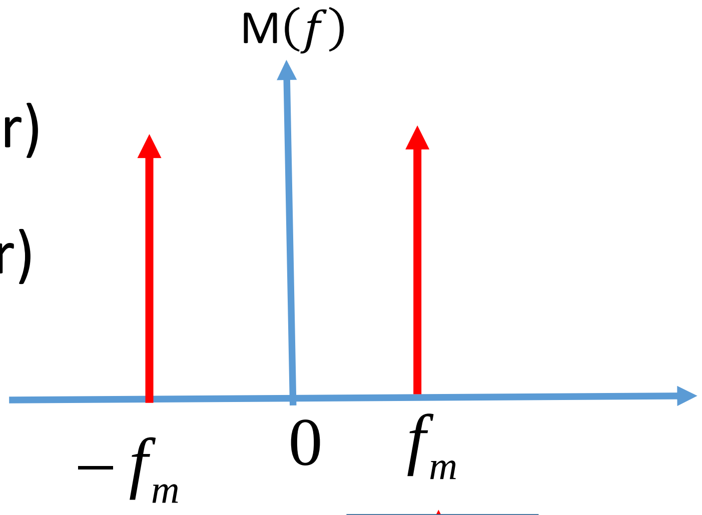




Single Tone Modulation: Filtering Requirements

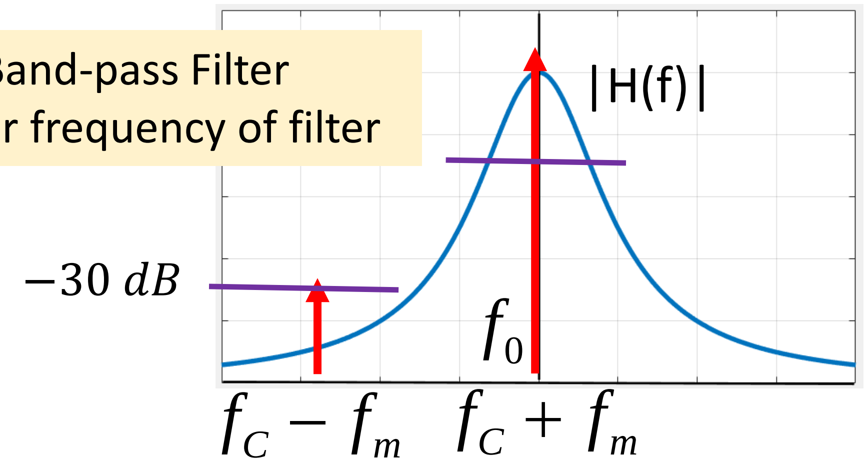
- Let $m(t) = A_m \cos(2\pi f_m t)$; message signal
- Let $\mathrm{c}(t) = A_c \cos(2\pi f_c t)$; carrier signal
- $s_{DSB}(t) = A_c A_m \cos(2\pi f_c t)\cos(2\pi f_m t)$; DSB-SC modulation
- $s_{DSB}(t) = \frac{A_c A_m}{2}\cos(2\pi(f_c + f_m)t) + \frac{A_c A_m}{2}\cos(2\pi(f_c - f_m)t)$;
- $\boldsymbol{s_{SSB}(t) = \frac{A_c A_m}{2}\cos(2\pi(f_c + f_m)t)}$; upper SSB (using ideal filter)
- $\boldsymbol{s_{SSB}(t) = \frac{A_c A_m}{2}\cos(2\pi(f_c - f_m)t)}$; lower SSB (using ideal filter)

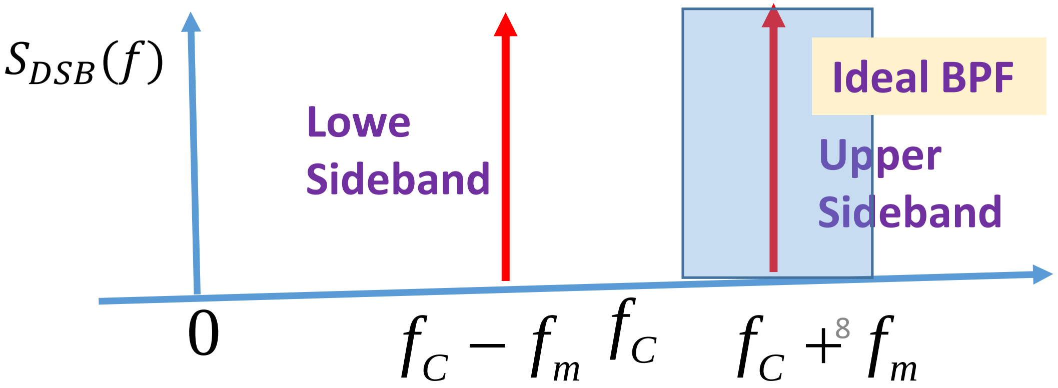




Practical Band-pass Filter

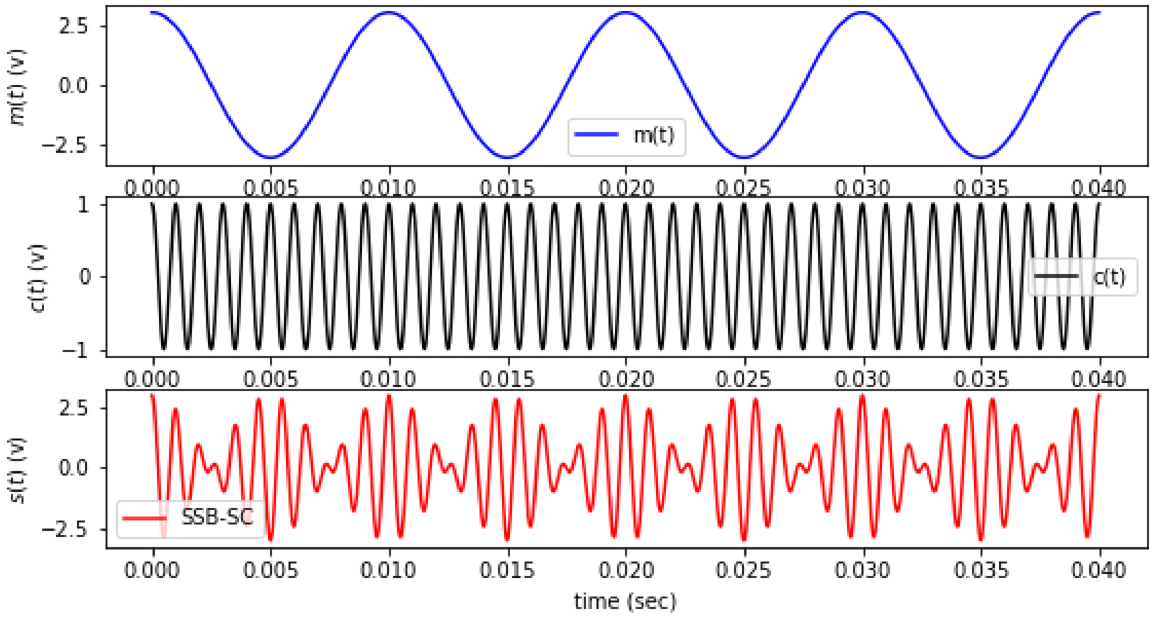
f_0 : Center frequency of filter

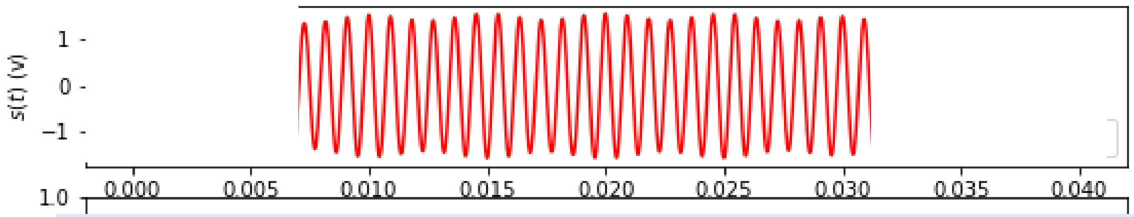




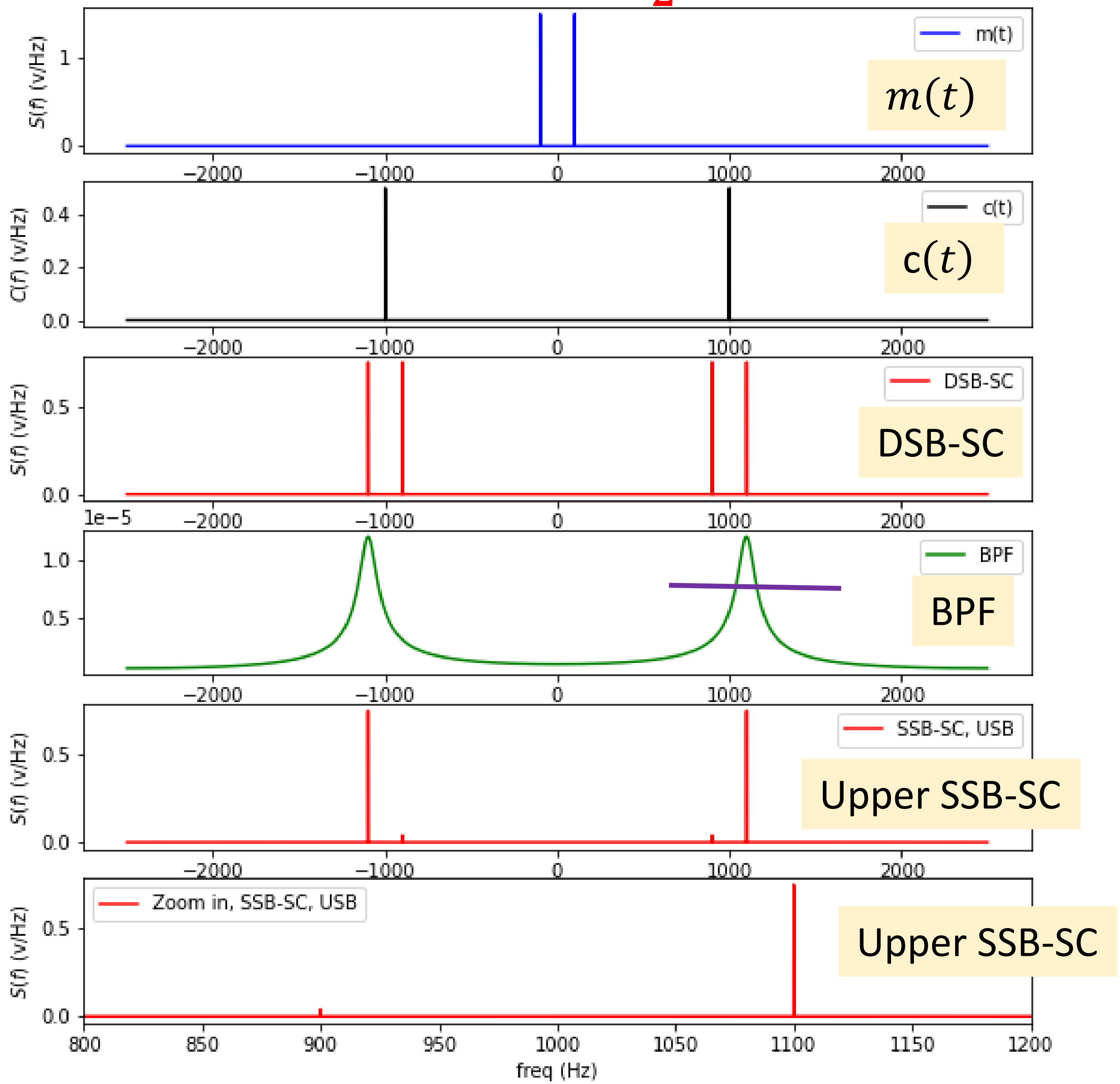
Upper SSB Signal in the Time and Frequency Domains: Single Tone Message

- Example: $m(t) = 3\cos(2\pi 100 t)$; Let $c(t) = \cos(2\pi 1000 t)$; $\mathbf{s(t) = \frac{A_c A_m}{2} cos(2\pi 1100 t)}$

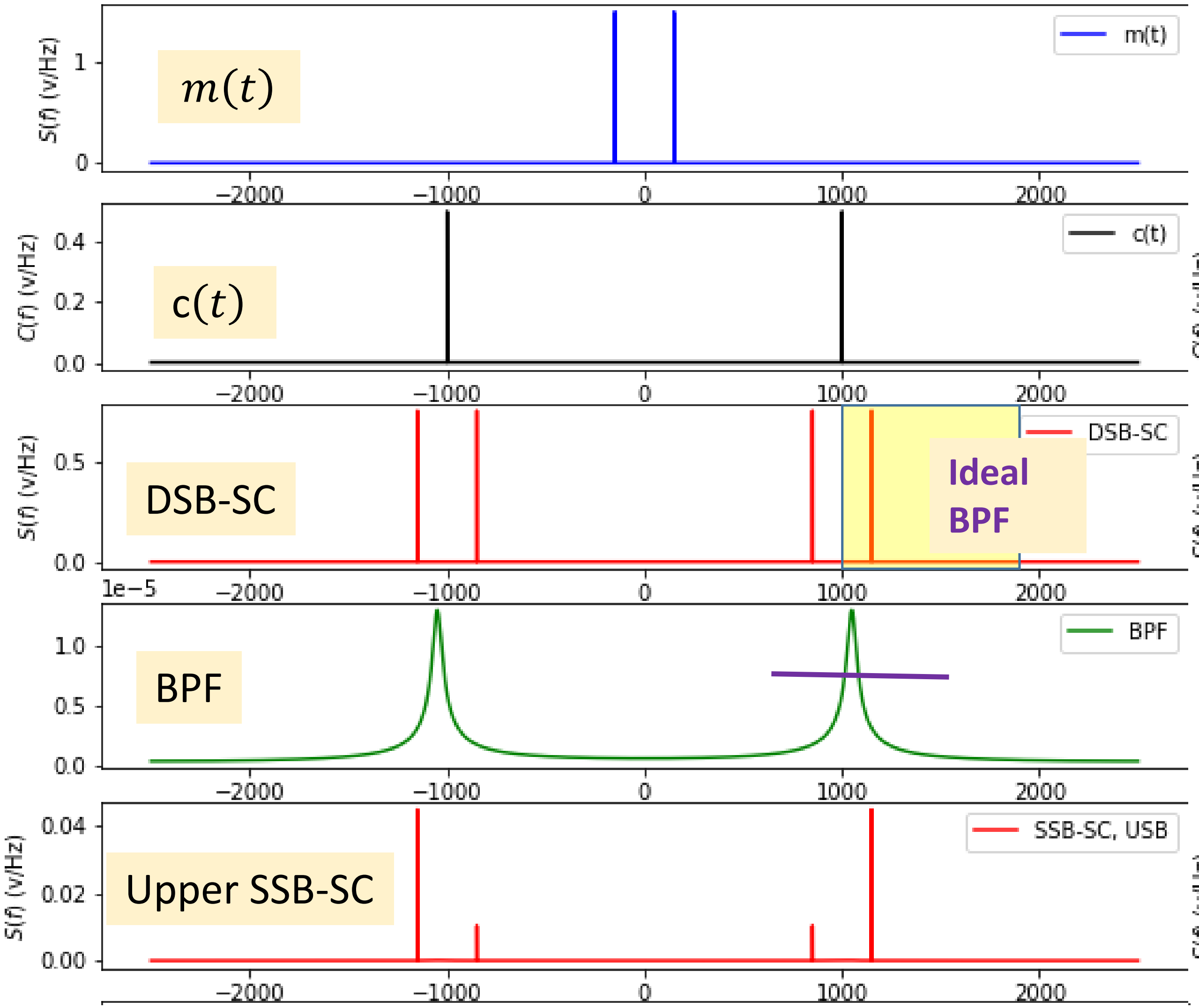




Single Tone: Filter center frequency coincides with desired upper sideband. Still part of LSB appears

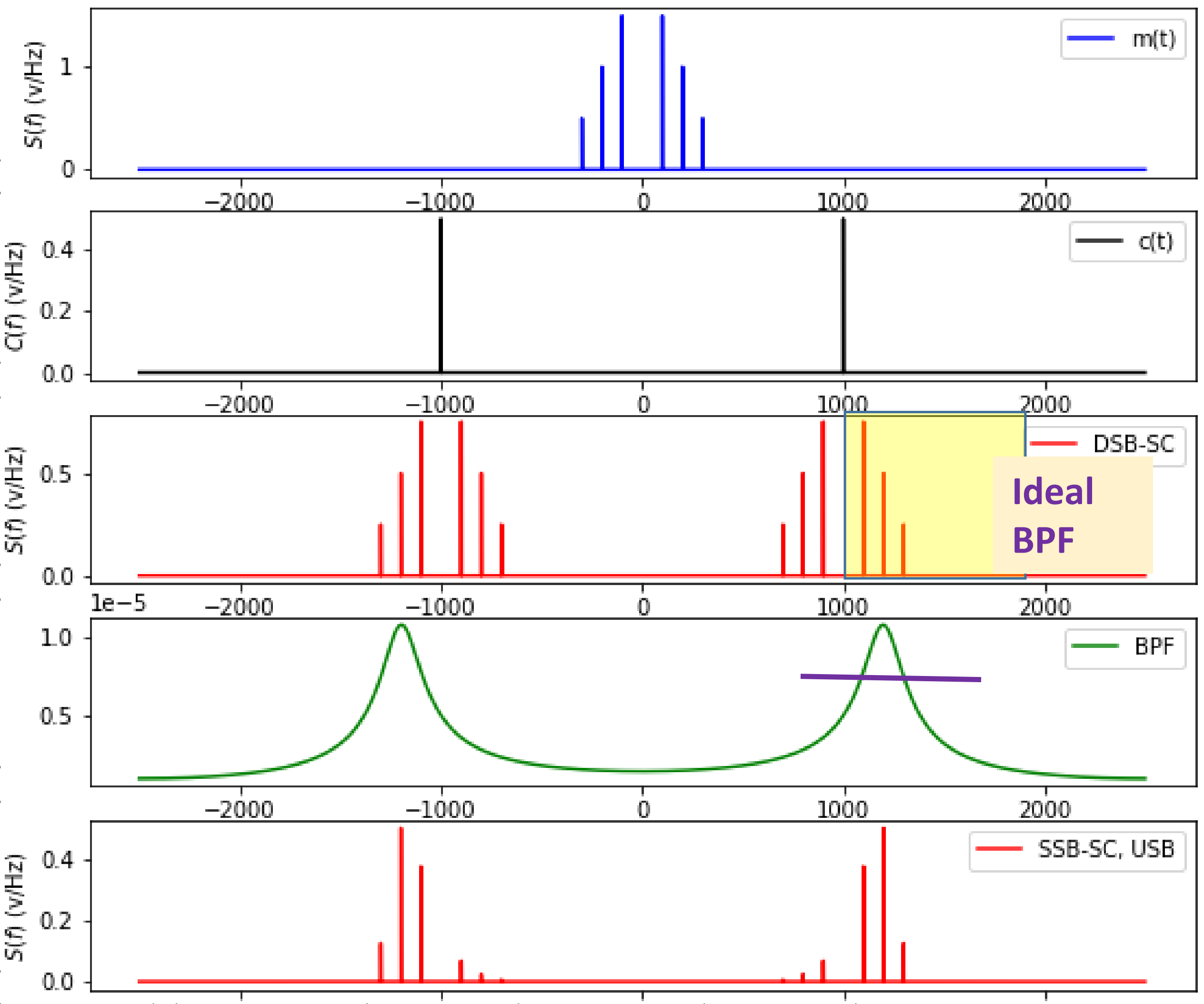


Filtering Issues in SSB Modulation



$m(t) = 3\cos(2\pi 150t);\quad \mathrm{c}(t) = \cos(2\pi 1000t);$

Single Tone: Filter center frequency does not coincide with desired USB. USB attenuates, Part of LSB appears

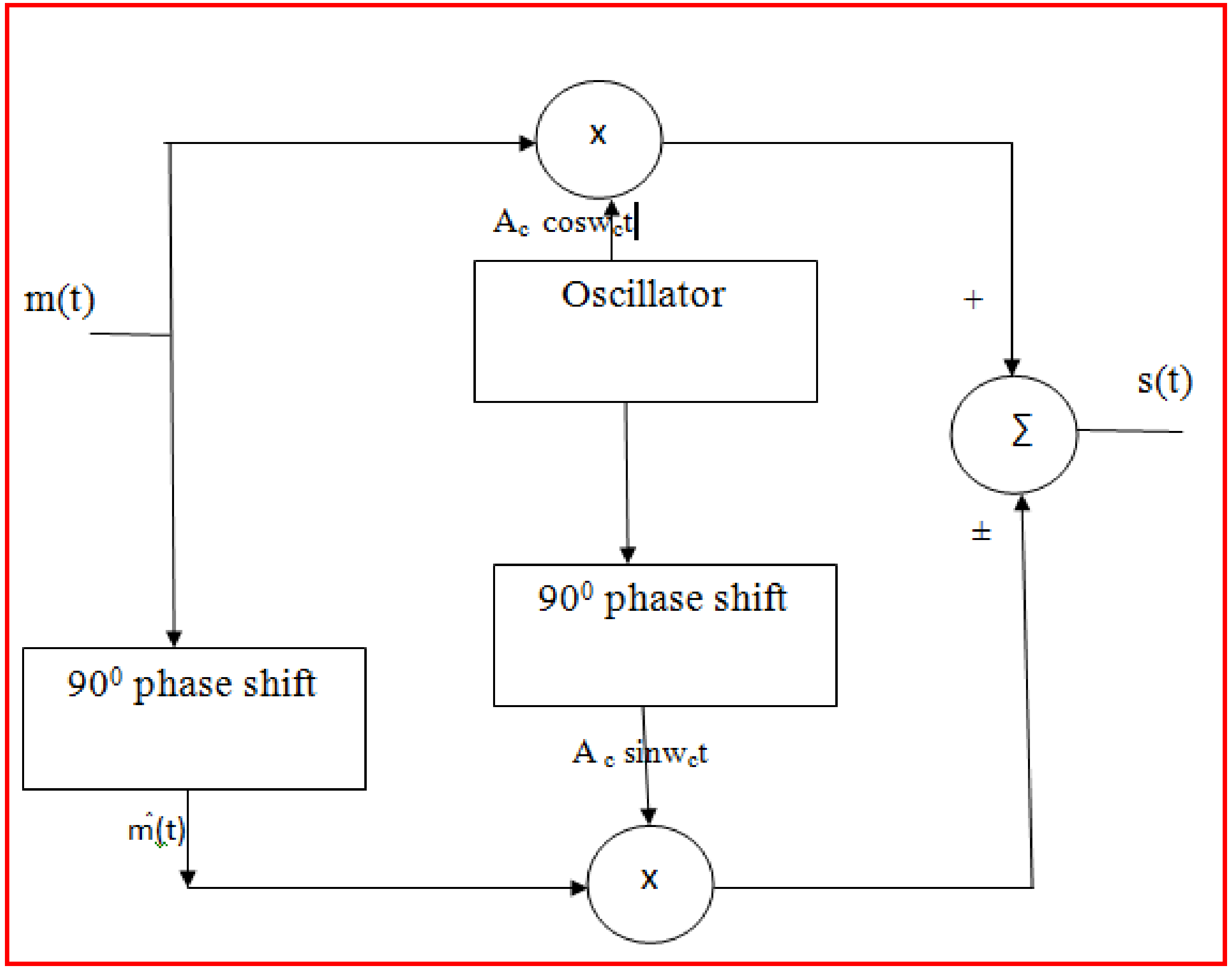


$m(t) = 3\cos(2\pi 100t) + 2\cos(2\pi 200t) + \cos(2\pi 300t)$

Multi-tone: Filter center frequency does not coincide with desired USB. Components in USB are not passed proportionately. Part of LSB appears

Generation of SSB Signal: Phase Shift Method

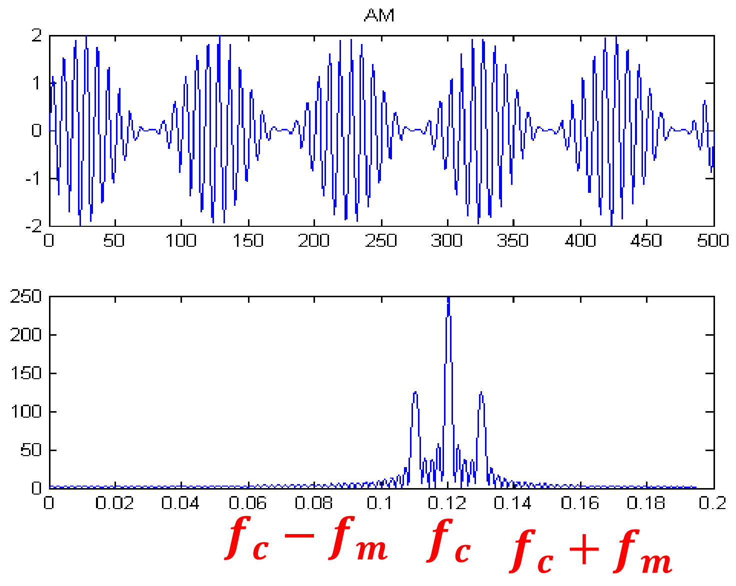
- The method is based on the time –domain representation of the SSB signal
- $$s(t) = A_C m(t) cos\omega_c t \pm A_C \hat{m}(t) sin\omega_c t$$



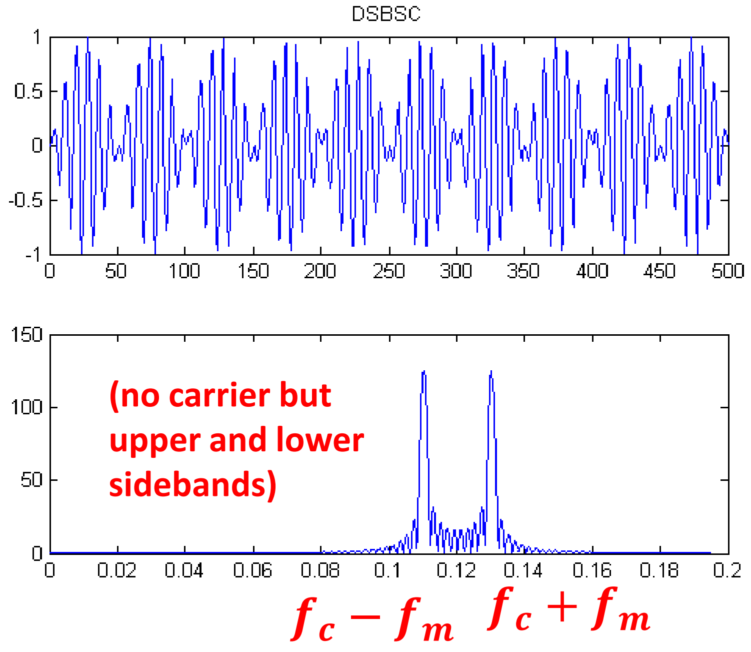
Comparison of the three types of AM modulation

Here, we show all three types of AM modulation in the time and frequency domains when $m(t) = A_m \cos(2\pi f_m t)$; $c(t) = A_c \cos(2\pi f_c t)$;

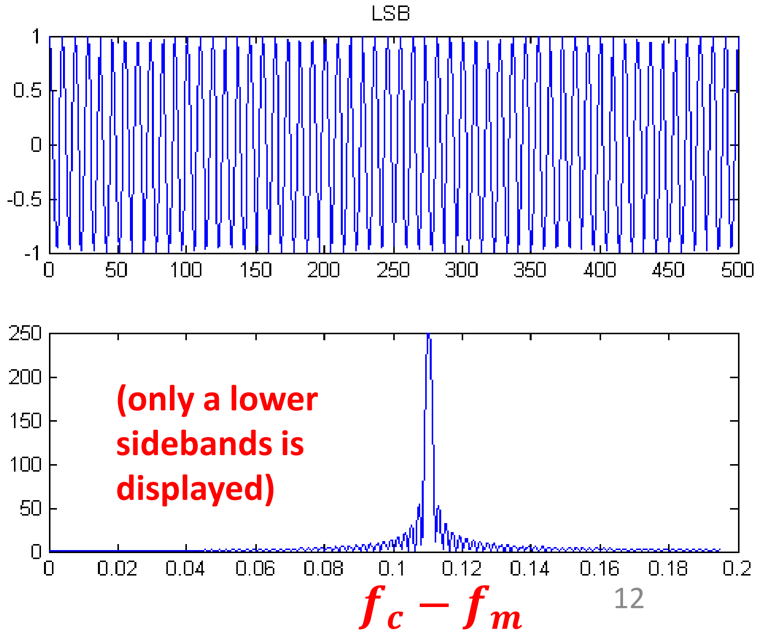
AM in time and frequency domains
(carrier + two sidebands)



DSB in time and frequency domains

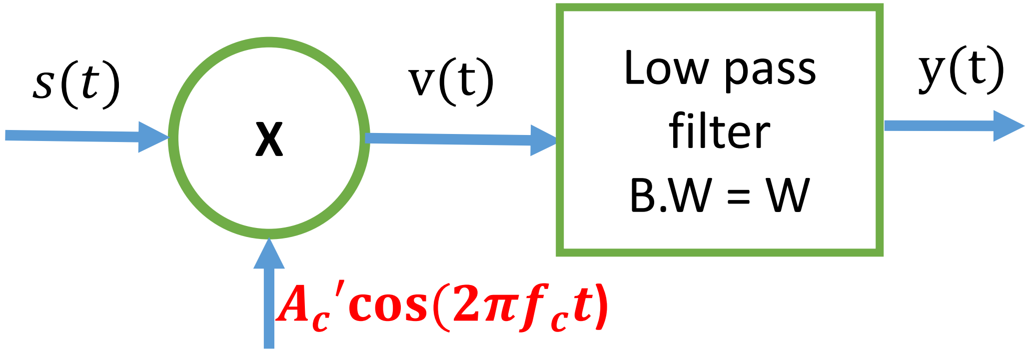


SSB in time and frequency domains



Demodulation of SSB-SC

- A SSB-SC signal is demodulated using what is known as ***coherent demodulation***. This means that the modulated signal s(t) is multiplied by a locally generated signal at the receiver which has the same frequency and phase as that of the carrier c(t) at the transmitting side

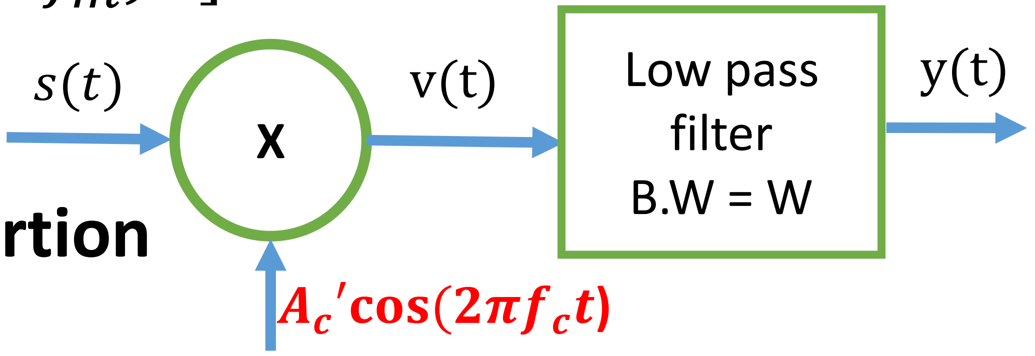


Perfect Coherent Demodulation

- Let the received signal be the upper single sideband
- $s(t) = A_C m(t) cos\omega_c t - A_C \hat{m}(t) sin\omega_c t$
- At the receiver, $s(t)$ is mixed with the carrier signal. The result is
- $v(t) = s(t) A_C' cos\omega_c t$
- $= A_C' [A_C m(t) cos\omega_c t - A_C \hat{m}(t) sin\omega_c t] cos\omega_c t$
- $= A_C A_C' m(t) (cos\omega_c t)^2 - A_C A_C' \hat{m}(t)\ sin\omega_c t\ cos\omega_c t$
- $= \frac{A_C A_C'}{2} m(t) + \frac{A_C A_C'}{2} m(t) cos2\omega_c t - \frac{A_C A_C'}{2} \hat{m}(t)\ sin2\omega_c t$
- The low pass filter admits only the first terms. The output is: $\boldsymbol{y(t) = \frac{A_C A_C'}{2} m(t)}$

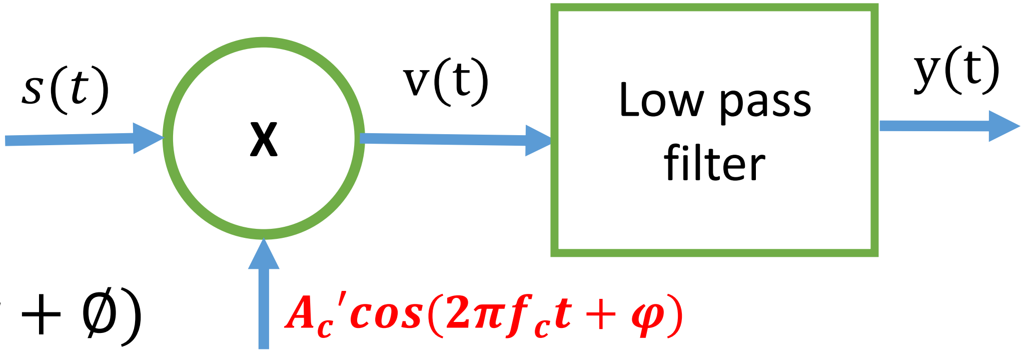
Single Tone Modulation: Why One Sideband is Sufficient

- **Example:** When $m(t) = A_m \cos(2\pi f_m t)$; $c(t) = A_c \cos(2\pi f_c t)$;
- $s_{DSB}(t) = A_c A_m \cos(2\pi f_c t)\cos(2\pi f_m t)$; DSB-SC modulation
- $s_{DSB}(t) = \frac{A_c A_m}{2}\cos(2\pi(f_c+f_m)t) + \frac{A_c A_m}{2}\cos(2\pi(f_c-f_m)t)$;
- The upper SSB signal is: $\boldsymbol{s_{SSB}(t) = \frac{A_c A_m}{2}\cos(2\pi(f_c+f_m)t)}$;
- $v(t) = s(t)A_c'\cos(2\pi f_c t) = \frac{A_c A_m A_c'}{2} cos\ (2\pi f_c t)cos(2\pi(f_c+f_m)t)$;
- $v(t) = \frac{A_c A_m A_c'}{4}[cos(2\pi(2f_c+f_m)t + \cos(2\pi f_m)\ t]$
- $\mathbf{y}(\boldsymbol{t}) = \boldsymbol{\frac{A_c A_m A_c'}{4}[cos(2\pi f_m)\ t]}$
- **Message has been recovered without distortion**



Effect of Carrier Non-Coherence on Demodulated Signal: Constant Phase Shift

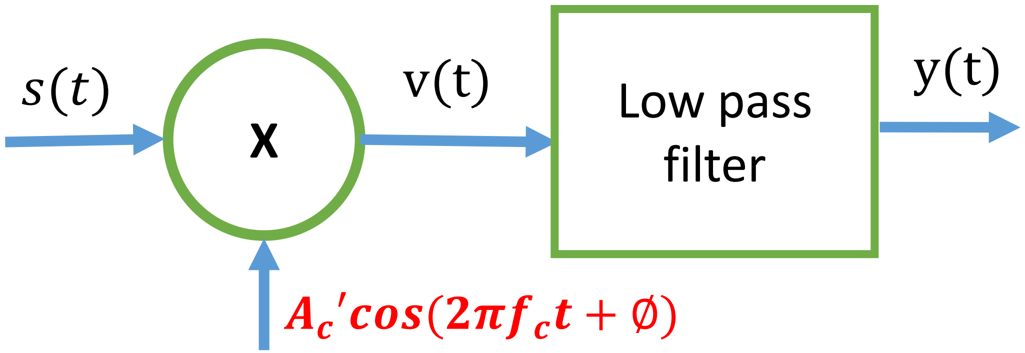
A constant phase difference between c(t) and $c'(t)$



- The local oscillator takes the form
- $\acute{c}(t) = \acute{A}_c \cos(\omega_c t + \emptyset);$
- $v(t) = [A_c m(t)\cos\omega_c t - A_c \hat{m}(t)\sin\omega_c t]\acute{A}_c \cos(\omega_c t + \emptyset)$
- $= A_c \acute{A}_c m(t)\cos\omega_c t\cos(\omega_c t + \emptyset) - A_c \acute{A}_c \hat{m}(t)\sin\omega_c t\cos(\omega_c t + \emptyset)$
- $= \frac{A_c \acute{A}_c}{2} m(t)\cos(2\omega_c t + \emptyset) + \frac{A_c \acute{A}_c}{2} m(t)\cos(\emptyset)$
- $- \frac{A_c \acute{A}_c}{2} \hat{m}(t)\cos(2\omega_c t + \emptyset) + \frac{A_c \acute{A}_c}{2} \hat{m}(t)\sin(\emptyset)$
- $\boldsymbol{y(t) = \frac{A_c \acute{A}_c}{2} m(t)\cos(\emptyset) + \frac{A_c \acute{A}_c}{2} \hat{m}(t)\sin(\emptyset)}$
- Note that there is a distortion due to the appearance of the Hilbert transform of the message signal at the output.

Single Tone Modulation: Effect of a Constant Phase Shift of the Carrier at Receiver

- **Example**: When $m(t) = A_m \cos(2\pi f_m t)$; $c(t) = A_c \cos(2\pi f_c t)$;
- The upper SSB signal is: $\boldsymbol{s_{SSB}(t) = \frac{A_c A_m}{2} \cos(2\pi(f_c + f_m)t)}$;
- $v(t) = s(t) A_c' \cos(2\pi f_c t + \emptyset) = \frac{A_c A_m A_c'}{2} \cos(2\pi f_c t + \emptyset) \cos(2\pi(f_c + f_m)t)$;
- $v(t) = \frac{A_c A_m A_c'}{4} [\cos(4\pi f_c t + 2\pi f_m t + \emptyset) + \cos(2\pi f_m t - \emptyset)]$
- $\boldsymbol{y(t) = \frac{A_c A_m A_c'}{4} [\cos(2\pi f_m t - \emptyset)]}$
- If the message consists of multiple tones
- $y(t) = \frac{A_c A_1 A_c'}{4} [\cos(2\pi f_1 (t - \emptyset/2\pi f_1))]$
$$+ \frac{A_c A_2 A_c'}{4} [\cos(2\pi f_2 (t - \emptyset/2\pi f_2))] + \frac{A_c A_3 A_c'}{4} [\cos(2\pi f_3 (t - \emptyset/2\pi f_3))]$$

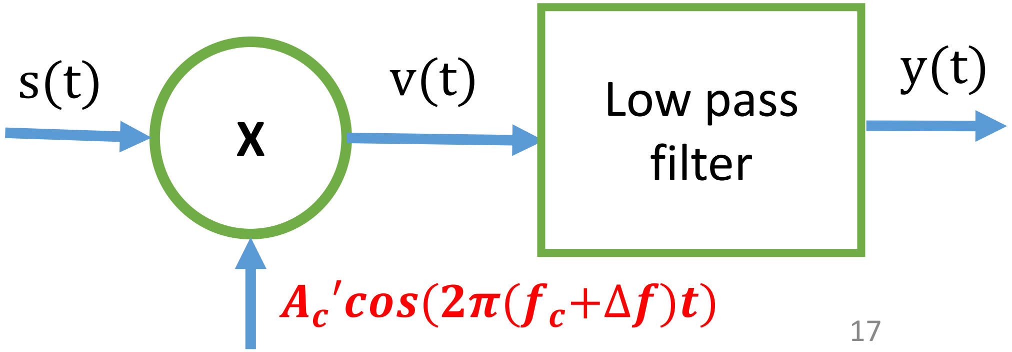


Here, phase distortion becomes more apparent since we **cannot** write $y(t) = kx(t - t_d)$

Effect of Carrier Non-Coherence on Demodulated Signal: Constant Frequency Difference

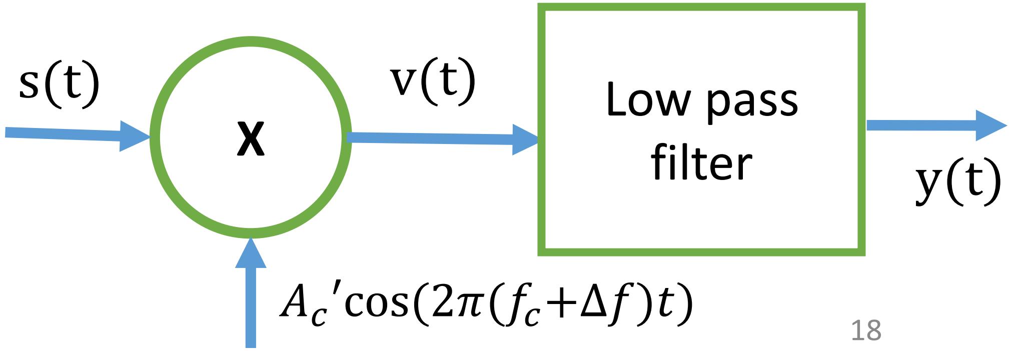
Constant Frequency Difference between c(t) and $c'(t)$

- $\acute{c}(t) = A'_c \cos\ 2\pi(f_c + \Delta f)t;$ Constant frequency shift
- $v(t) = [A_c m(t) \cos \omega_c t - A_c \hat{m}(t) \sin \omega_c t] \acute{A}_c \cos\ 2\pi(f_c + \Delta f)t$
- $= \frac{A_c \acute{A}_c}{2} m(t)[\cos(2\omega_c + \Delta\omega)t + \cos 2\pi \Delta f t]$
- $- \frac{A_c \acute{A}_c}{2} \hat{m}(t)[\sin(2\omega_c + \Delta\omega)t - \sin 2\pi \Delta f t]$
- $\boldsymbol{y(t) = \frac{A_c \acute{A}_c}{2} m(t) \cos 2\pi \Delta f t + \frac{A_c \acute{A}_c}{2} \hat{m}(t) \sin 2\pi \Delta f t}$
- Once again we have distortion and m(t) appears as if single sideband modulated on a carrier frequency = Δf



Single Tone Modulation: Effect of a Constant Frequency of the Carrier at Receiver

- **Example**: When $m(t) = A_m\cos(2\pi f_m t)$; c$(t) = A_c\cos(2\pi f_c t)$;
- The upper SSB signal is: $\boldsymbol{s_{SSB}(t) = \frac{A_cA_m}{2}\cos(2\pi(f_c+f_m)t)}$;
- $v(t) = s(t)A_c'\cos(2\pi f_c t + 2\pi\Delta f t)$
- $v(t) = \frac{A_cA_mA_c'}{2}cos\,(2\pi f_c t + 2\pi\Delta f t)cos(2\pi(f_c+f_m)t)$;
- $v(t) = \frac{A_cA_mA_c'}{4}[cos\,(4\pi f_c t + 2\pi f_m t + 2\pi\Delta f t) + \cos(2\pi f_m t - 2\pi\Delta f t)]$
- $\boldsymbol{y(t) = \frac{A_cA_mA_c'}{4}[cos(2\pi(f_m - \Delta f)t)]}$
- So, when $\Delta f = 100$, a message component with **f = 1000Hz appears as a 900Hz** component at the demodulator output. Again, distortion occurs because of failing to synchronize the transmitter and receiver carrier frequencies.



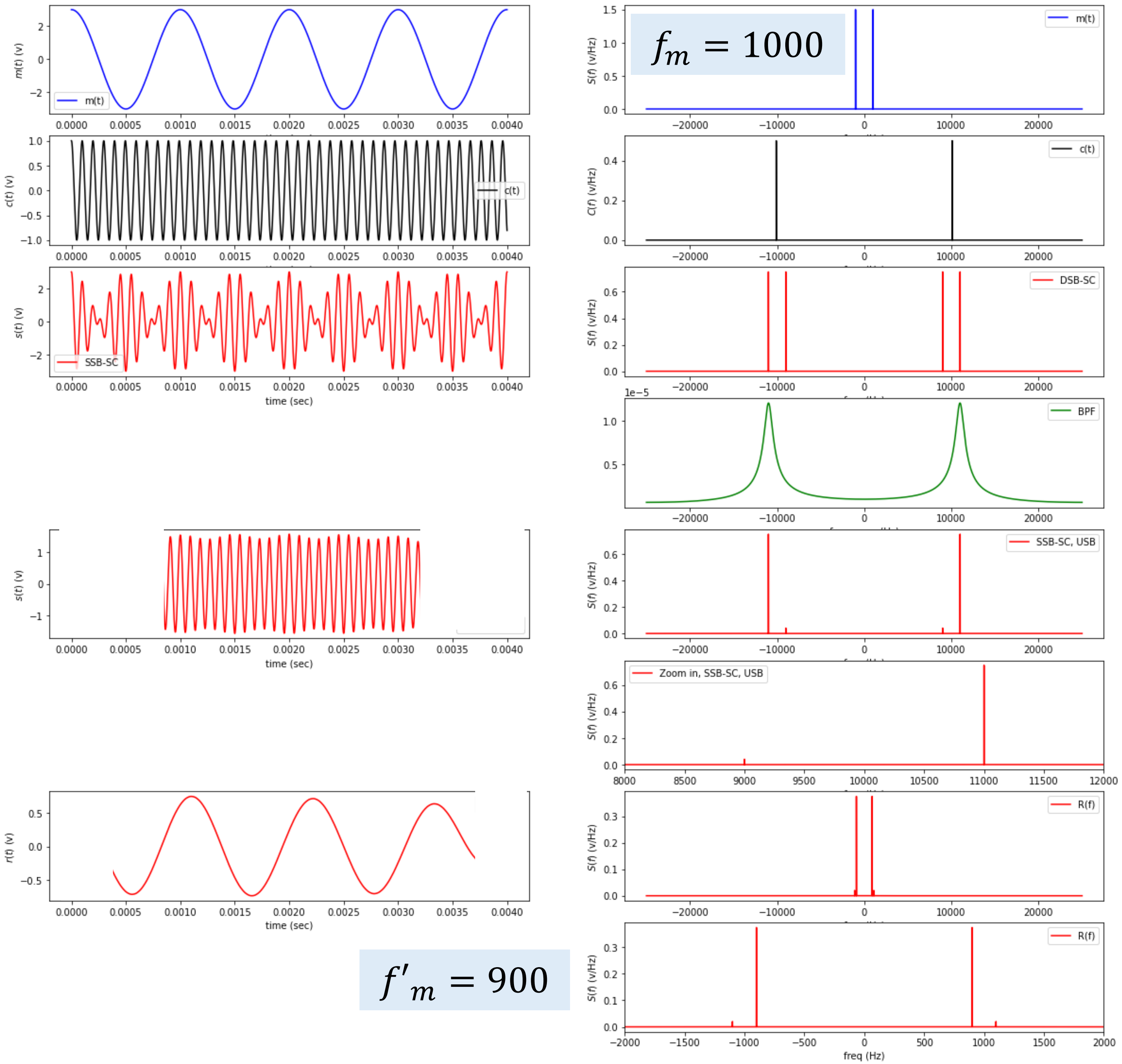
Single Tone Modulation: Effect of a Constant Frequency of the Carrier at Receiver

$$m(t) = 3\cos(2\pi 1000t)$$

$$\mathrm{c}(t) = \cos(2\pi 10000t)$$

$$\mathrm{c}'(t) = \cos(2\pi 10100t)$$

$$\Delta f = 100$$



m(t)

c(t)

DSB-SC

BPF

SSB-SC

SSB-SC

y(t)

y(t)