Frequency Modulation: Basic Principles

- To generate an angle modulated signal, the amplitude of the modulated carrier is held constant, while either the phase or the time derivative of the phase is varied linearly with the message signal m(t).
- $\mathbf{c(t) = A_c \cos(2\pi f_c t)}$; unmodulated carrier
- The expression for an angle modulated signal is
- $\mathbf{s(t) = A_c \cos\big(2\pi f_c t + \theta(t)\big)}$,
- $\theta(t)$: A phase difference that contains the information message.
- f_c is the carrier frequency in Hz.
- The instantaneous frequency of s(t) is :
- $f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\big(2\pi f_c t + \theta(t)\big)$
- $\mathbf{f_i(t) = f_c + \frac{1}{2\pi}\frac{d\theta(t)}{dt}}$

- Since the information is contained in the phase, this type of modulation is less susceptible to AWGN and interference from electrical equipment and the atmosphere.
- For **phase modulation**, the phase $\theta(t)$ is directly proportional to the modulating signal

  $$\boldsymbol{\theta(t) = k_p m(t),}$$

- k_p is the phase sensitivity measured in rad/volt.
- The time domain representation of a phase modulated signal is
- $\mathbf{s(t)_{PM} = A_c \cos\Big(2\pi f_c t + k_p m(t)\Big)}$.
- The instantaneous frequency of $\mathbf{s(t)_{PM}}$ is :
- $\mathbf{f_i(t) = f_c + \frac{k_p}{2\pi}\frac{dm(t)}{dt}}$

Frequency Modulation

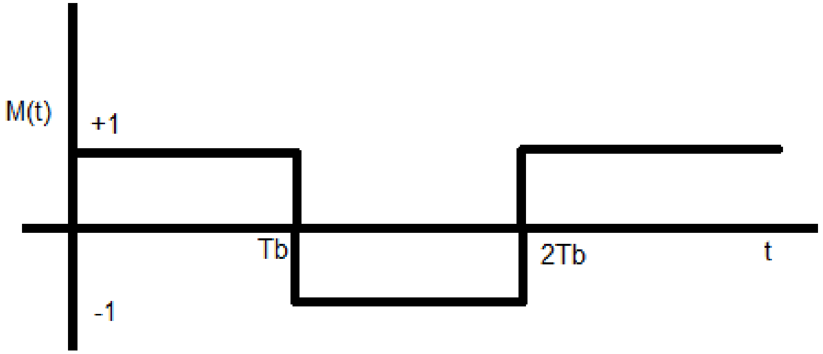
- An angle modulated signal is
- $s(t) = A_c \cos\left(2\pi f_c t + \theta(t)\right)$;
- The instantaneous frequency of s(t) is :
- $f_i(t) = f_c + \frac{1}{2\pi}\frac{d\theta(t)}{dt}$
- For **frequency modulation**, the frequency deviation of the carrier is proportional to the modulating signal:
- $\frac{1}{2\pi}\frac{d\theta(t)}{dt} = k_f m(t)$
- Integrating both sides, we get
- $\boldsymbol{\theta(t) = 2\pi k_f \int_0^t m(\alpha)d\alpha}$
- The instantaneous frequency beomes
- $\boldsymbol{f_i(t) = f_c + k_f m(t)}$;
- $f_i(t) - f_c = k_f m(t)$.
- The peak frequency deviation is
- $\Delta f = \max\{k_f m(t)\}$.
- The time domain representation of a frequency modulated signal is
- $\boldsymbol{s(t)_{FM} = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\alpha)d\alpha\right)}$.
- Where $\boldsymbol{\theta(t) = 2\pi k_f \int_0^t m(\alpha)d\alpha}$
- The average power in $s(t)$, for frequency modulation (FM) or phase modulation (PM) is: $\boldsymbol{p_{av} = \frac{(A_c)^2}{2} = constant}$.

Example: Binary Frequency Shift Keying

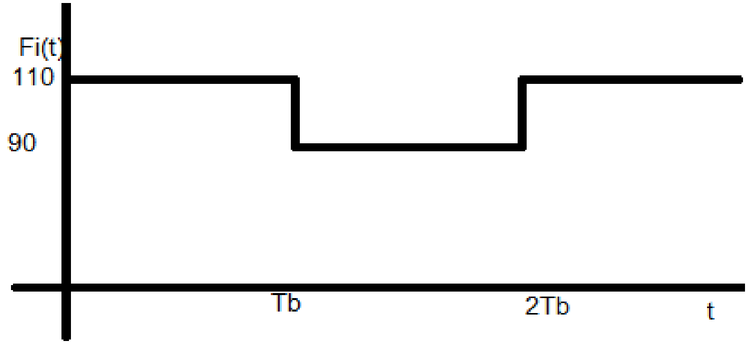
- The periodic square signal m(t), shown below, frequency modulates the carrier $c(t) = A_c \cos(2\pi 100 t)$ to produce the FM signal

$$s(t) = A_c \cos\left(2\pi 100 t + 2\pi k_f \int m(\alpha)d\alpha\right)$$ where $k_f = 10 Hz/V$.

- Find and plot the instantaneous frequency $f_i(t)$.
- Find and sketch the time domain expression for $s(t)$.



- **Solution**: The instantaneous frequency is
- $f_i = f_c + k_f m(t)$
- f_i = 100+10 = 110 when $m(t) = +1$ $(0 < t \leq T_b)$
- f_i = 100 -10 = 90Hz when $m(t) = -1$ $(T_b \leq t \leq 2T_b)$



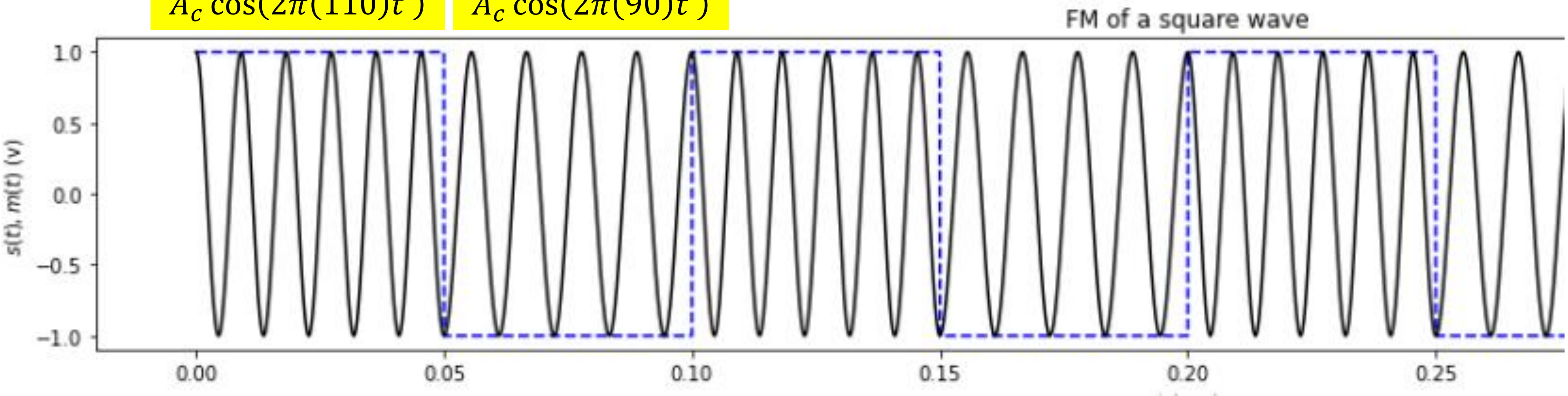
- **Remark:** In digital transmission, we will see that a binary (1) may be represent by a signal of frequency f_1 for $0 \leq t \leq T_b$ and a binary (0) by a signal of frequency f_2 for $0 \leq t \leq T_b$ (This type of digital modulation is called FSK)

Example: Binary Frequency Shift Keying

Solution: The instantaneous frequency is $\boldsymbol{f_i = f_c + k_f m(t)}$

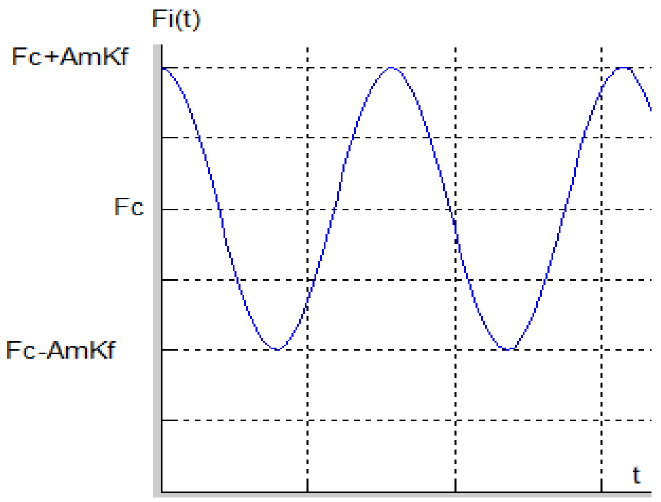
- $f_i =$ 100+10 = 110 when $m(t) = +1$ $(0 < t \leq T_b)$
- $f_i =$ 100 -10 = 90Hz when $m(t) = -1$ $(T_b \leq t \leq 2T_b)$
- The instantaneous frequency hops between the two values 110 Hz and 90 Hz as shown.
- Depending on the input binary digit, $s(t)$ may take any one of the following expressions
- $s(t) = A_c \cos(2\pi(110)t\,),\ \ when\ m(t) = +1$
- $s(t) = A_c \cos(2\pi(90)t\,),\ when\ m(t) = -1$

$A_c \cos(2\pi(110)t\,)$ $A_c \cos(2\pi(90)t\,)$



Single Tone Frequency Modulation

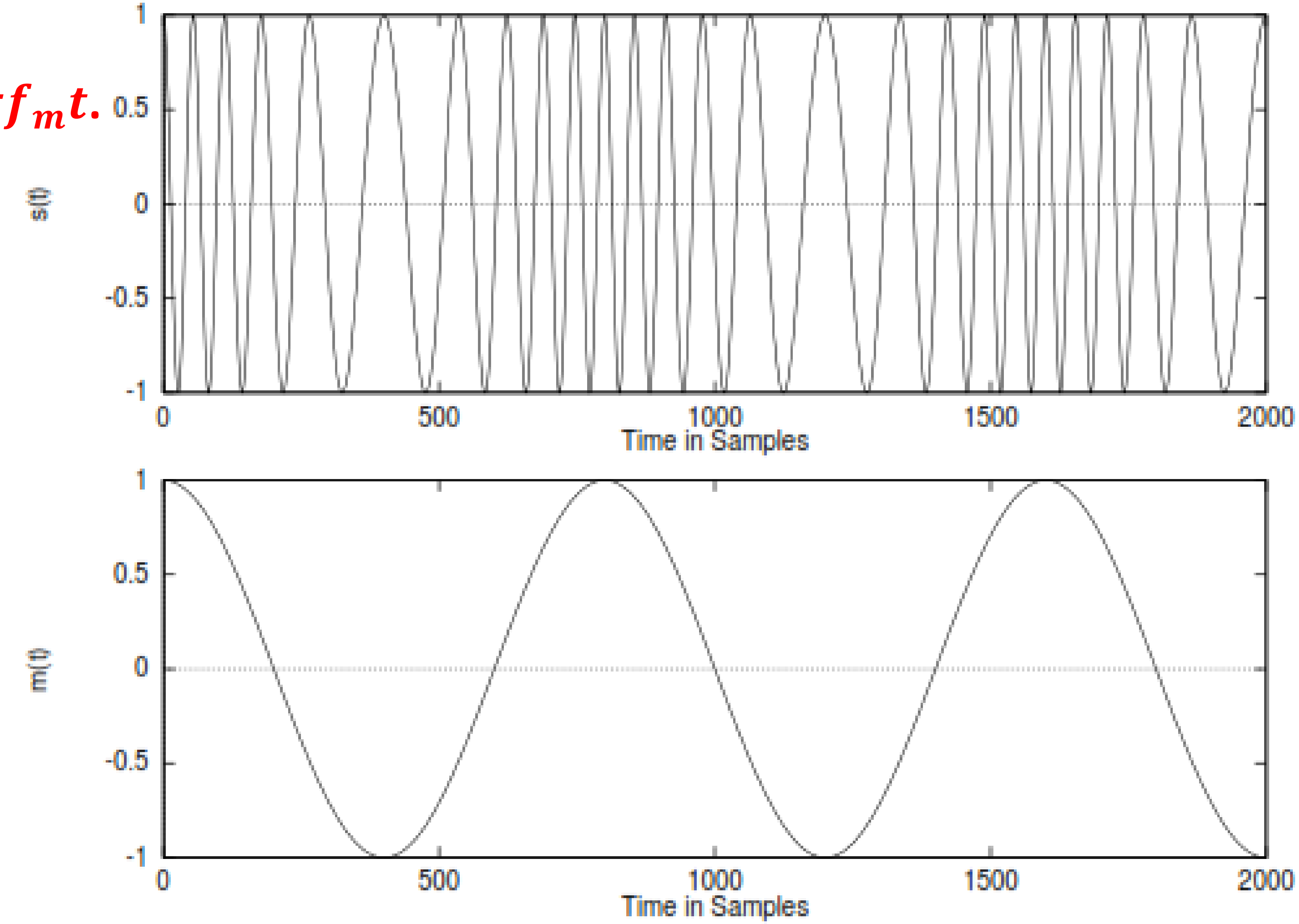
- Assume that the message $m(t) = A_m \cos \omega_m t$.
- The instantaneous frequency is: $\boldsymbol{f_i = f_c + k_f m(t) = f_c + A_m k_f \cos 2\pi f_m t.}$
- This frequency is plotted in the figure.
- The peak frequency deviation (from the un-modulated carrier) is : $\Delta f = k_f A_m$.
- The FM signal is: $\boldsymbol{s(t)_{FM} = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\alpha) d\alpha\right)}$
- $s(t)_{FM} = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t A_m \cos \omega_m \alpha \, d\alpha\right)$
- $s(t) = A_c \cos \left(2\pi f_c t + \frac{k_f A_m}{f_m} \sin 2\pi f_m t\right)$.
- $\boldsymbol{s(t) = A_c \cos \left(2\pi f_c t + \beta \sin 2\pi f_m t\right)}$.
- Where β is the **FM modulation index**, defined as
- $\beta = \frac{k_f A_m}{f_m} = \frac{peak\ frequency\ deviation}{message\ bandwidth} = \frac{\Delta f}{f_m}$
- In the figure below, we show a sinusoidal message signal $m(t)$ and the resulting FM signal $s(t)$.



Single Tone Frequency Modulation

$$f_i = f_c + A_m k_f \cos 2\pi f_m t.$$

$$T = \frac{1}{f}$$



fc = 1 KHz,
fm = 100 Hz.