Generation of an FM Signal
Lecture Outline

- In this lecture, we present two methods for the generation of a frequency modulated signal:
  - The direct method, which uses a voltage controlled oscillator
  - The indirect method, in which a narrow band FM is generated first, then frequency multipliers are used to produce the desired wideband FM.
- Both methods are analyzed in detail.
- The operation of the varactor diode is briefly described.

Review: Basics of Angle Modulation

- The expression for an angle modulated signal is: $s(t) = A_c \cos\left(2\pi f_c t + \theta(t)\right)$
- The instantaneous frequency of s(t) is:

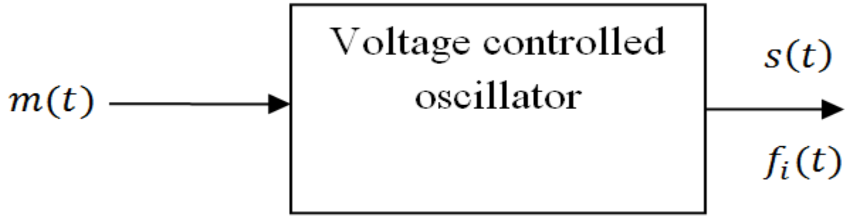
$$f_i(t) = f_c + \frac{1}{2\pi}\frac{d\theta(t)}{dt}$$

- For **phase modulation**:
  - $\theta(t) = k_p m(t)$, k_p in rad/volt.
  - $s(t)_{PM} = A_c \cos\left(2\pi f_c t + k_p m(t)\right)$
  - $f_i(t) = f_c + \frac{k_p}{2\pi}\frac{dm(t)}{dt}$
- For **frequency modulation**:
  - $f_i(t) = f_c + k_f m(t);$
  - $\theta(t) = 2\pi k_f \int_0^t m(\alpha)d\alpha;$ k_f in Hz/volt.
  - $s(t)_{FM} = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\alpha)d\alpha\right).$

- When $m(t) = A_m \cos 2\pi f_m t$
  - $f_i = f_c + A_m k_f \cos 2\pi f_m t;$
  - $s(t)_{FM} = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t A_m \cos \omega_m \alpha \, d\alpha\right)$
  - $= A_c \cos\left(2\pi f_c t + \beta \sin 2\pi f_m t\right).$
- $\beta = \frac{k_f A_m}{f_m} = \frac{peak\ frequency\ deviation}{message\ bandwidth} = \frac{\Delta f}{f_m};$
- β: is the **FM modulation index**,
- When $m(t) = A_m \cos 2\pi f_m t$, FM signal can be represented as
- $s(t) = A_c \sum_{-\infty}^{\infty} J_n(\beta) \cos(2\pi(f_c + n f_m)t)$
- Carson's rule: $B_T = 2(\beta + 1) f_m$
- When $\beta \ll 1$, the FM is termed narrow band (the BW is comparable to the BW of AM)
- Otherwise, it is termed a wideband FM. Here the BW. Is much larger than that of the AM signal.

Generation of an FM Signal

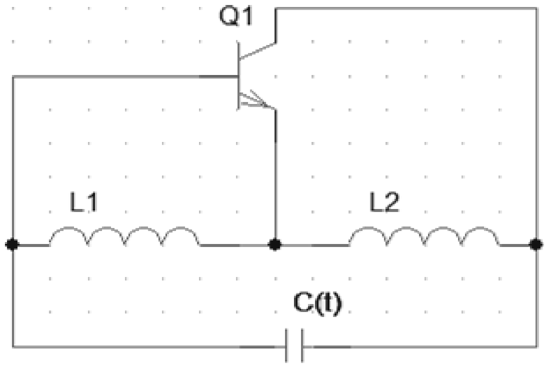
- **Direct Method for Generating an FM Signal**
- In a direct FM system, the instantaneous frequency of the carrier is varied in accordance with a message signal by means of a voltage-controlled oscillator (VCO). The voltage – frequency characteristic of a VCO is given by
- $\boldsymbol{f_i = f_c + k_f\, m(t)}$
- k_f : proportionality constant Hz/V
- A schematic diagram of a VCO is shown in the figure
- A realization of the CVO may be obtained by considering an oscillator (like the Hartley oscillator) shown on the next slide in which a varactor (voltage variable capacitor) is used. **A varactor diode is a semiconductor diode whose junction capacitance varies linearly with the applied voltage when the diode is reverse biased**



$$\boldsymbol{f_i = f_c + k_f\, m(t)}$$

Direct Method for Generating an FM Signal

- For the Hartley oscillator shown, the frequency of oscillation is $\boldsymbol{f_i(t) = \frac{1}{2\pi\sqrt{(L_1 + L_2)C(t)}}}$
- Let $\boldsymbol{C(t) = C_0 - k\, m(t)}$ (A varactor diode operating in the reverse bias region can act like a variable capacitor); k is a constant,
- When m(t) = 0, $C(t) = C_0$, and $f_c = \frac{1}{2\pi\sqrt{(L_1+L_2)C_0}}$
- When m(t) has a finite value, the frequency of oscillation is
- $f_i(t) = \frac{1}{2\pi\sqrt{(L_1 + L_2)(C_0 - k\, m(t))}} = \frac{1}{2\pi\sqrt{C_0(L_1 + L_2)}\sqrt{(1 - k\, m(t)/C_0)}}$
- $= f_c\left(1 - \frac{k\, m(t)}{C_0}\right)^{-1/2}$
- When $\frac{k\, m(t)}{C_0} \ll 1$, we can make the approximation (using $[(1+x)^n \cong 1 + nx]$ when x is small)
- $f_i(t) = f_c\left(1 + \frac{k\, m(t)}{2C_0}\right) = \boldsymbol{f_c + k_f\, m(t)}$
- Here it is clear that the instantaneous frequency varies linearly with the message signal.
- **Remark**: Direct method of FM generation is very simple and cheap process, but this method can't be used for broadcast application because the LC oscillator used in this method is not very stable. Its frequency depends upon various parameters such as temperature, device aging etc.

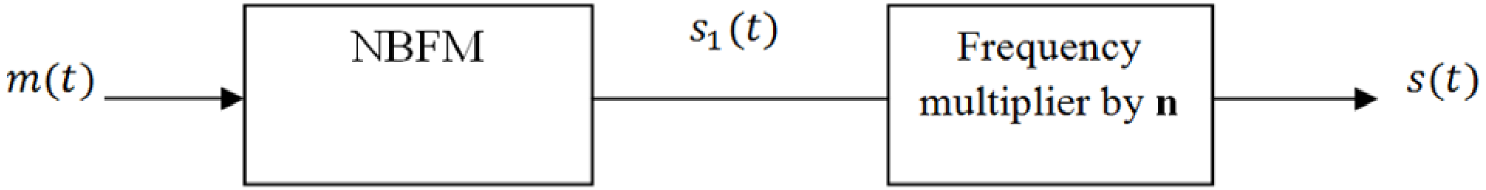


$\boldsymbol{C(t) = C_0 - k\, m(t)}$

Indirect Method for Generating an FM Signal

- A wideband FM can be generated indirectly using the block diagram below. First, a narrowband FM is generated. Then, the wideband FM is obtained by using frequency multiplication. Next, we analyze the operation of this modulator.
- Let $m(t) = A_m \cos 2\pi f_m t$ be the baseband signal, then
- $s_1(t) = A_c \cos(2\pi f_c' t + \beta' \sin 2\pi f_m t)$; $\beta' = \frac{k_f A_m}{f_m}$ is a NBFM with $\beta' \ll 1$.
- The frequency of $s_1(t)$ is $f_i' = f_c' + k_f A_m \cos 2\pi f_m t$
- Multiplying f_i by n (through frequency multiplication), we get the frequency of $s(t)$ as

$\boldsymbol{f_i' = f_c' + k_f A_m cos 2\pi f_m t}$ $\boldsymbol{f_i = n f_c' + n k_f A_m cos 2\pi f_m t}$



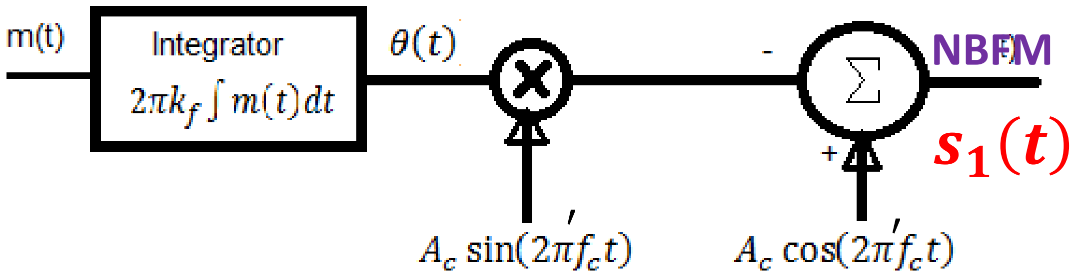
- $f_i = n f_c' + n k_f A_m cos 2\pi f_m t$
- The result is
- $s(t) = A_c \cos[2\pi(n f_c')t + n\beta' \sin 2\pi f_m t] = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$
- Where $\beta = n\beta'$ is the desired modulation index of WBFM
- $f_c = n f_c'$ is the desired carrier frequency of WBFM

$\boldsymbol{f_c' = 1KHz, \beta' = 0.2}$
$\boldsymbol{f_c = 10KHz, \beta = 2 \Rightarrow n = 10}$

Generation of an FM Signal: The NBFM

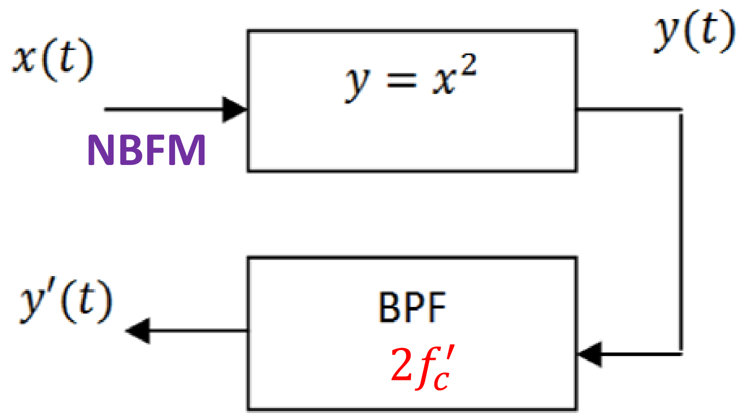
- Consider an FM signal
- $s_1(t) = A_c \cos(2\pi f_c' t + 2\pi k_f \int m(t)dt)$
- Assuming $\mathrm{m(t)} = A_m cos2\pi f_m t$,
- $\boldsymbol{s_1(t) = A_c\, cos(2\pi f'_c t + \beta' sin(2\pi f_m t))}$
- $s_1(t)$ can be expanded as
- $s_1(t) = A_c \cos(2\pi f_c' t) \cos\big(\theta(t)\big) - A_c \sin(2\pi f_c' t) \sin\big(\theta(t)\big)$
- When $|\theta(t)| = |\beta' \sin(2\pi f_m t)| \ll 1$, $cos\ \theta \cong 1$, $\sin(\theta) \cong \theta$.
- $s_1(t)$, termed narrowband, can be approximated as
- $s_1(t) \cong A_c \cos(2\pi f_c' t) - A_c \theta \sin(2\pi f_c' t)$
- $s_1(t) = A_c \cos(2\pi f_c' t) - A_c \beta' \sin(2\pi f_m t) \sin(2\pi f_c' t)$

$\boldsymbol{s_1(t) = A_c\, cos(2\pi f'_c t + \beta' sin(2\pi f_m t))}$



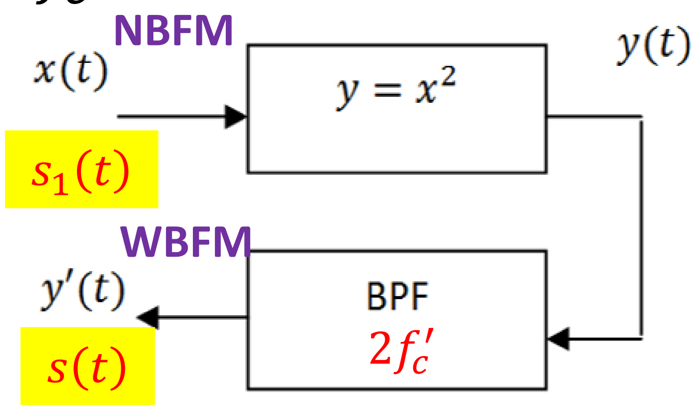
Generation of an FM Signal: Frequency Multiplication

- **Frequency Multiplier**: It is a device for which the frequency of the output signal is an integer multiple of the frequency of the input signal. It is primarily a nonlinear characteristic followed by a band pass filter. Now we illustrate the operation of this device.
- **The Square Law Device:** Let the input be an FM signal of the form:
- $$x(t) = A_c \cos(2\pi f_c' t + \beta' \sin 2\pi f_m t) = A_c \cos(\phi)$$
- The output of the square law characteristic is:
- $y(t) = x(t)^2 = A_c^2 cos^2(\phi) = \frac{A_c^2}{2}[1 + \cos(2\phi)]$
- $= \frac{A_c^2}{2} + \frac{A_c^2}{2}\cos(2\phi)$
- $= \frac{A_c^2}{2} + \frac{A_c^2}{2} cos[2\pi(2f_c')t + 2\beta' \sin(2\pi f_m t)]$



Generation of an FM Signal: Frequency Multiplication

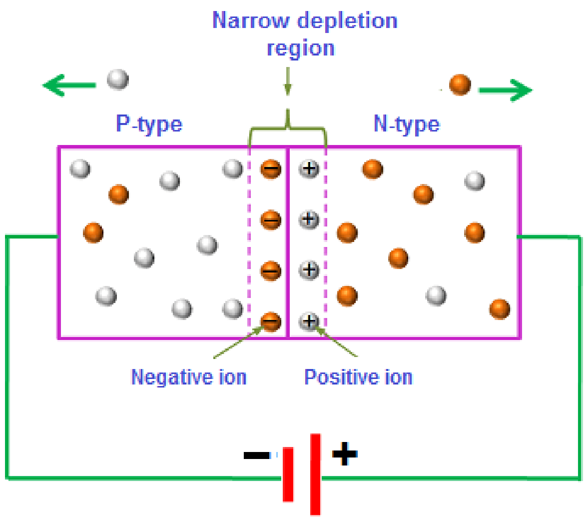
- $y(t) = \frac{A_c^2}{2} + \frac{A_c^2}{2} cos[2\pi(2f_c')t + 2\beta' \sin(2\pi f_m t)];$
- If $y(t)$ is passed through a BPF of center frequency $2f_c'$, then the DC term will be suppressed and the filter output is



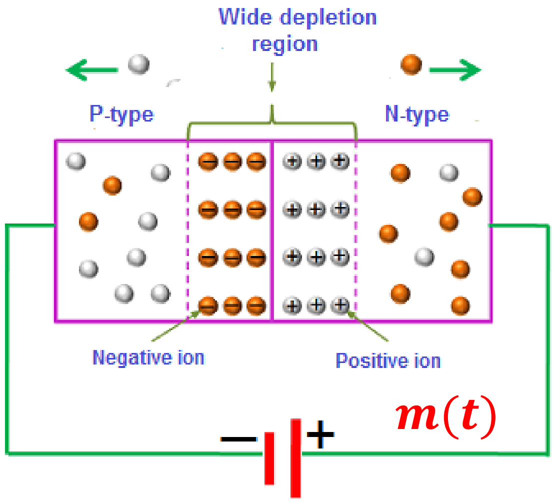
- $y'(t) = \frac{A_c^2}{2} cos[2\pi(2f_c')t + 2\beta' \sin(2\pi f_m t)]$
- $\boldsymbol{y'(t) = \frac{A_c^2}{2} cos[2\pi(f_c)t + \beta sin(2\pi f_m t)]}$
- As can be seen from this result, the output is a signal with twice the frequency of the input signal and a modulation index twice that of the input.
- $\boldsymbol{f_c = 2f_c'; \beta = 2\beta'}$
- To get frequency multiplication higher than two, a cascade of units, similar to what was described above, can be formed with the number of stages that achieve the desired carrier frequency and modulation index.

The Varactor Diode

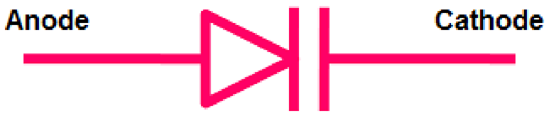
- **Definition:** The diode whose internal capacitance varies with the variation of the reverse voltage is known as the Varactor diode. The varactor diode always works in reverse bias, and it is a voltage-dependent semiconductor device.
- The Varactor diode is made up of n-type and p-type semiconductor material. In an n-type semiconductor material, the electrons are the majority charge carrier and in the p-type material, the holes are the majority carriers. When the p-type and n-type semiconductor material are joined together, the p-n junction is formed, and the depletion region is created at the PN-junction. The positive and negative ions make the depletion region.
- **Reference**: https://circuitglobe.com/varactor-diode.html



Large C: Low reverse bias voltage



Small C: High reverse bias voltage



Symbol

$$C = \frac{\varepsilon A}{d}$$

$$C(t) = C_0 - k\, m(t)$$