Demodulation of an FM Signal
Lecture Outline

- In this lecture, we present two methods for the demodulation of a frequency modulated signal:
  - The discriminator, which is a differentiator followed by an envelope detector.
  - The phase locked loop.
- Both methods are analyzed in detail.
- The time response of the phase locked loop is analyzed in the transient and steady state conditions.
- The frequency response of a first order PLL is derived.
- The concept of pre-emphasis and de-emphasis in FM is introduced.

Review: Basics of Angle Modulation

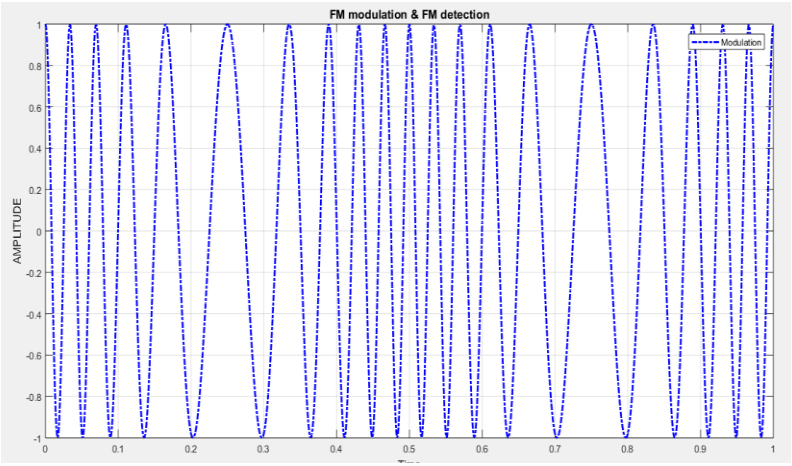
- The expression for an angle modulated signal is: $\boldsymbol{s(t) = A_c \cos\left(2\pi f_c t + \theta(t)\right)}$
- The instantaneous frequency of s(t) is: $\boldsymbol{f_i(t) = f_c + \frac{1}{2\pi}\frac{d\theta(t)}{dt}}$
- For **frequency modulation**:
  - $f_i(t) = f_c + k_f m(t)$; $\Rightarrow \boldsymbol{m(t) = (f_i(t) - f_c)/k_f}$; Key Demodulation Concept
  - $\frac{1}{2\pi}\frac{d\theta(t)}{dt} = k_f m(t)$; $\Rightarrow$ $\theta(t) = 2\pi k_f \int_0^t m(\alpha)d\alpha$; k_f in Hz/volt.
  - $s(t)_{FM} = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\alpha)d\alpha\right)$.
- For **phase modulation**:
  - $\theta(t) = k_p m(t)$, k_p in rad/volt.
  - $s(t)_{PM} = A_c \cos\left(2\pi f_c t + k_p m(t)\right)$
  - $f_i(t) = f_c + \frac{k_p}{2\pi}\frac{dm(t)}{dt}$; $\Rightarrow \boldsymbol{m(t) = \int_0^t 2\pi(f_i(\alpha) - f_c)d\alpha}$

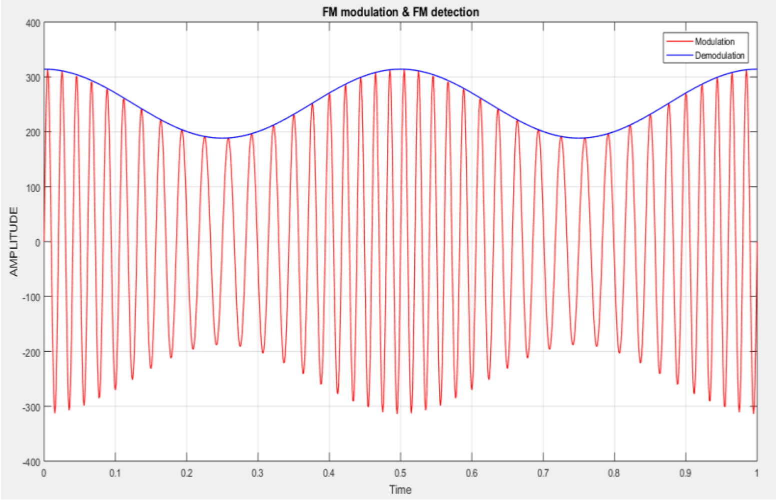
Demodulation of an FM Signal: The Discriminator

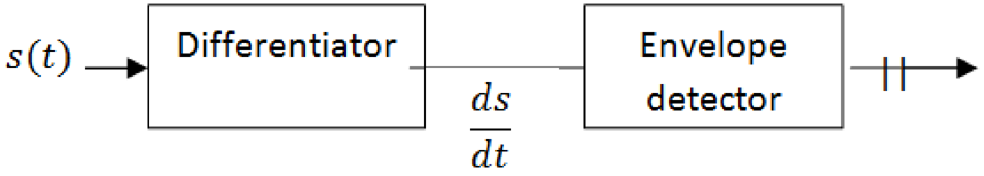
- An FM signal may be demodulated by means of what is called a ***discriminator***.
- One realization of a discriminator is a differentiator followed by an envelope detector, as illustrated in the figure. The operation of this discriminator can be explained as follows
- Let $s(t) = A_c \cos\big(\omega_c t + \theta(t)\big)$; $\theta(t) = 2\pi k_f \int_0^t m(\alpha)d\alpha$, $\theta(t) = k_p m(t)$
- $\dfrac{ds(t)}{dt} = -A_c\left(\omega_c + \dfrac{d\theta}{dt}\right)\sin\big(\omega_c t + \theta(t)\big)$
- The output of the envelope detector is $A_c\left|\left(\omega_c + \dfrac{d\theta}{dt}\right)\right| = A_c\omega_c + A_c\dfrac{d\theta}{dt}$
- The capacitor blocks the DC term and so output is:

$$\boldsymbol{V_0 = A_c \frac{d\theta}{dt}}$$

- If s(t) is an FM signal, then $\boldsymbol{V_0 = 2\pi k_f\, A_c m(t)}$
- If s(t) is a PM signal, then $\boldsymbol{V_0 = k_p \frac{dm(t)}{dt}} \Rightarrow \boldsymbol{m(t) = k_p \int V_0\,(t)dt}$
- A typical FM signal and its derivative are shown in the figure.
- Next, we review the envelope detector and explain how a differentiator is implemented.







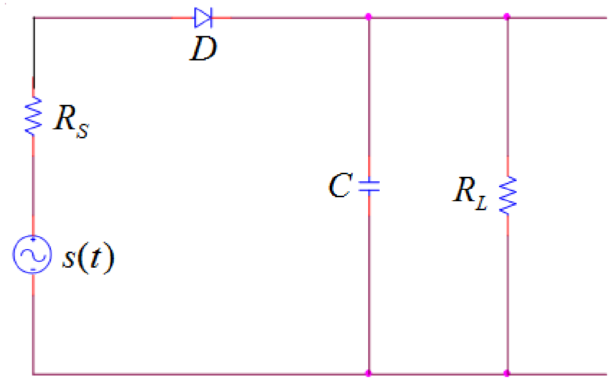
A Simple Practical Envelope Detector

The Ideal Envelope Detector: The ideal envelope detector responds to the envelope of the signal, but is insensitive to phase variation. If

$$s(t) = A_C\left(1 + k_a m(t)\right)\cos 2\pi f_C t$$

then, the output of the ideal envelope detector is $y(t) = A_C\left|1 + k_a m(t)\right|$

- A practical envelope detector consists of a diode followed by an RC circuit that forms a low pass filter.
- The operation of the envelope detector was described in a previous lecture.

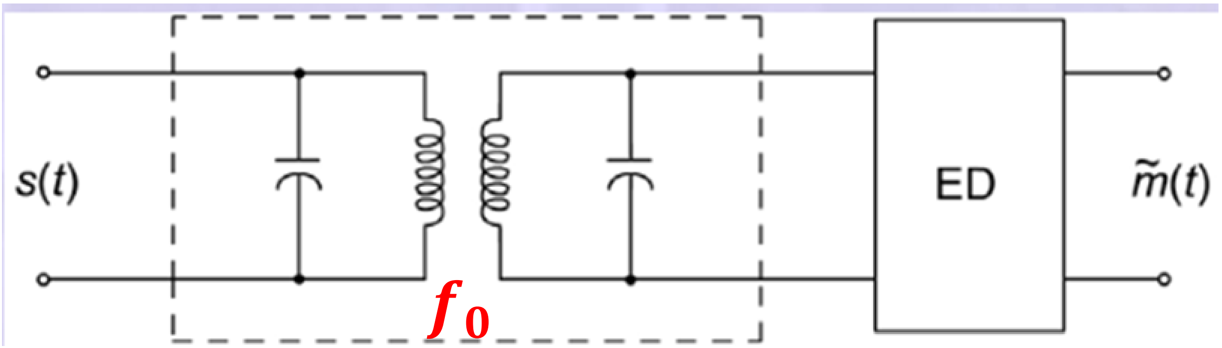


Demodulation of an FM Signal

- **Realization of the Differentiator**: From the properties of Fourier transform, we know that if

$$\Im\{g(t)\} = G(f), \quad \text{then} \quad \Im\left\{\frac{dg(t)}{dt}\right\} = j2\pi f G(f)$$

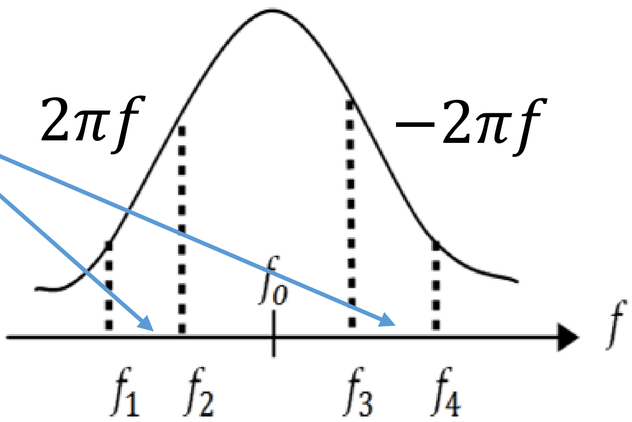
- This means that multiplication by $j2\pi f$ in the frequency domain amounts to differentiating the signal in the time-domain. Hence, we need a circuit whose frequency response is linear in f to perform time differentiation. A circuit that performs this task is a tuned circuit, provided that the signal frequency variation falls within the linear part of the characteristic, i.e., either between (f_1, f_2) or (f_3, f_4).
- The circuit below is a realization of an FM demodulator. The primary and secondary tuned circuits perform the task of differentiation, while the envelope detector extracts the envelope, which is supposed to be proportional to the message signal m(t)



Primary circuit tuned to f_c Secondary circuit tuned to $f_0 > f_c$

$$f_i(t) = f_c + k_f m(t).$$

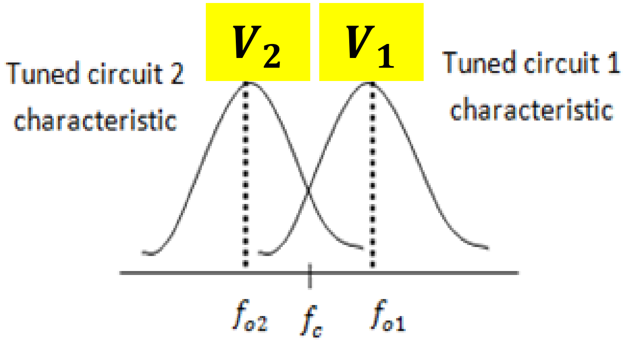
FM signal carrier should fall within these bands

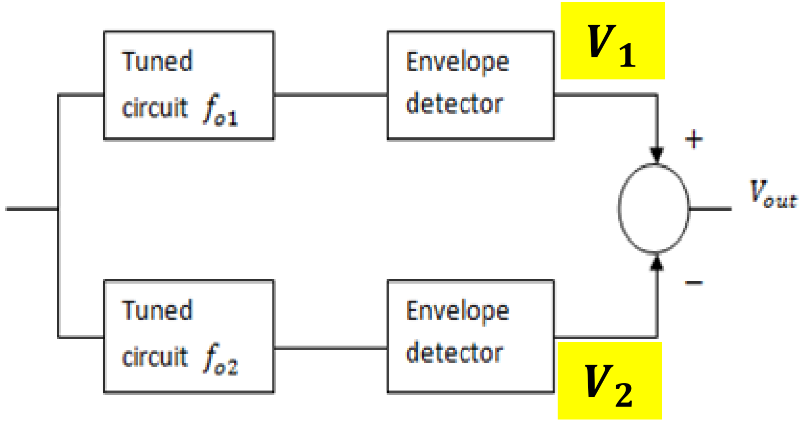


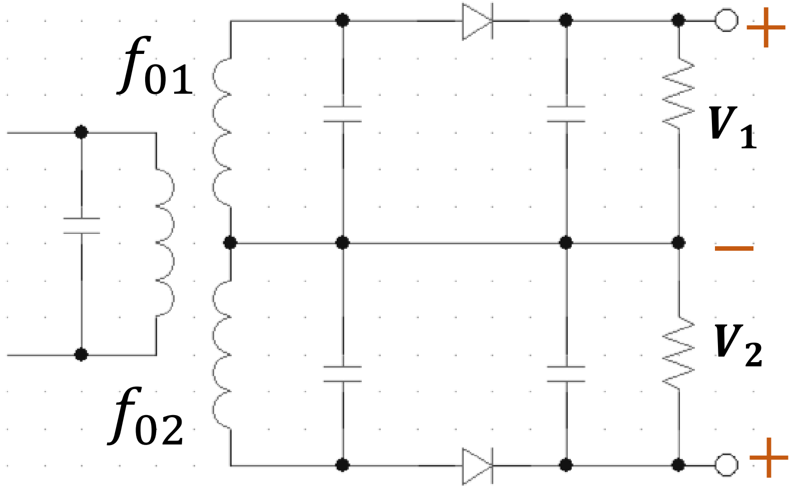
Demodulation of an FM Signal

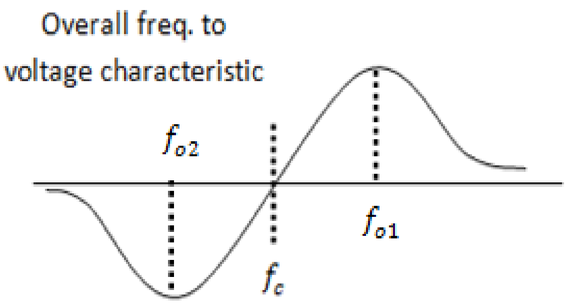
Balanced Slope Detector

- To extend the dynamic range of the differentiating circuit, two tuned circuits with center frequencies f_{01} and f_{02} are used as shown in the figure
- This circuit has a wider range of linear frequency response
- No DC blocking is necessary









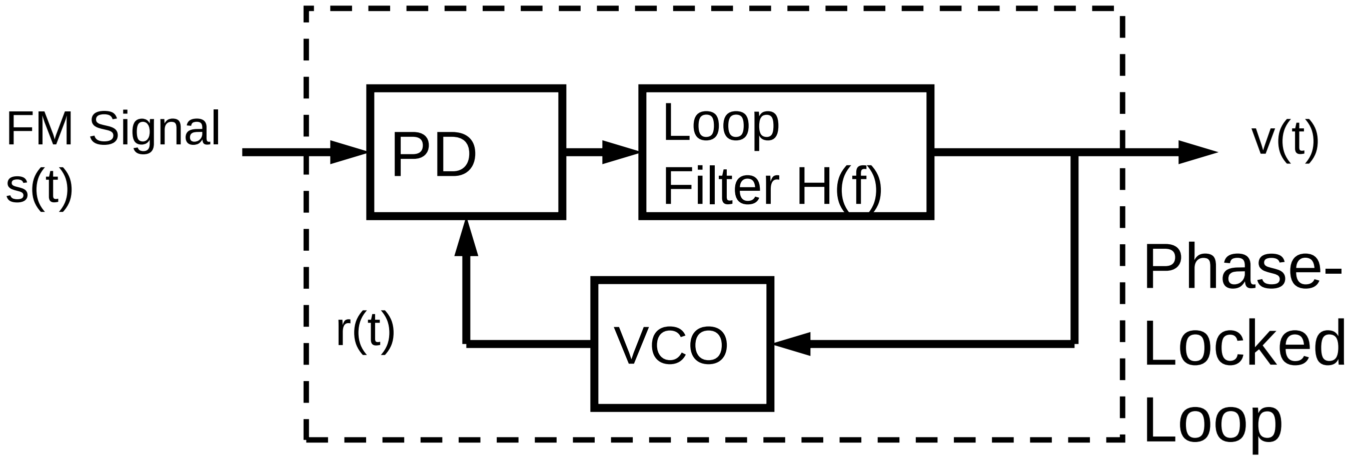
$$f_i(t) = f_c + k_f m(t)$$

$$V_0 = V_1 - V_2$$

The Phase Locked Loop

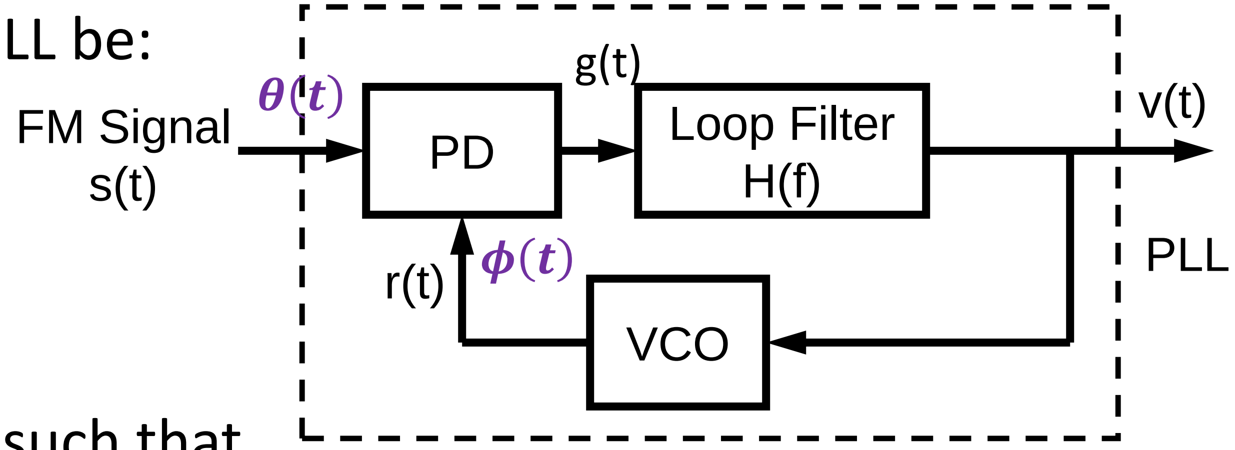
- Another implementation for the discriminator is the phase locked loop (PLL).
- The PLL is a negative feedback control system whose purpose is to force the frequency of the voltage controlled oscillator (VCO) to track the frequency and phase at its input.
- Has many applications in communications:
  - Carrier synchronization
  - Demodulation: e.g., DSB, FM
  - Frequency multiplication and division,
  - Frequency synthesis
  - Clock recovery circuits
- It consists of three main components:
  - Phase detector (PD)
  - Loop filter
  - Voltage controlled oscillator (VCO).

Functional Blocks of PLL



- Phase detector (PD): finds phase difference between the two inputs s(t) and r(t)
- Loop filter: provides appropriate control voltage for the voltage-controlled oscillator (VCO). It determines the order of the loop (first or second order).
- VCO: generates a signal r(t) with frequency determined by the control voltage v(t), hence the name VCO.

The Phased Locked Loop: Basic Operation

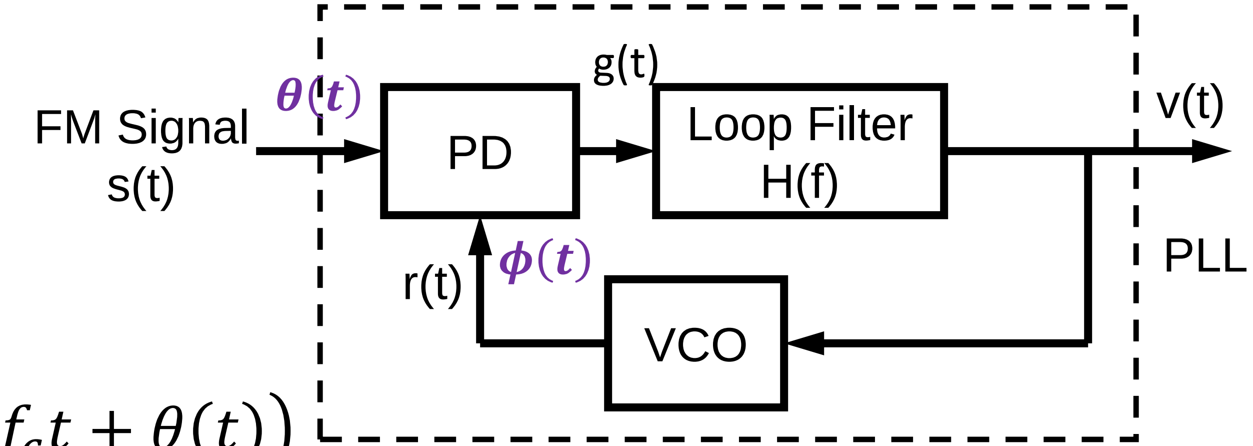


$f_r(t) = f_c + k_v v(t);$

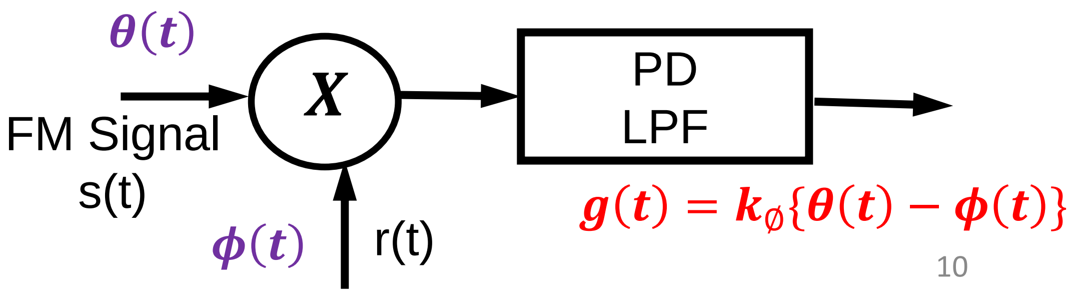
$f_r(t) = f_c + \frac{1}{2\pi}\frac{d\phi(t)}{dt};$

- **Initializing the loop:** Let the FM input to the PLL be:
- $s(t) = A_c \cos\big(2\pi f_c t + \theta(t)\big)$;
  - $\theta(t) = 2\pi k_f \int_0^t m(t)dt$; for an FM input
- Initialize the loop by setting $\theta(t) = 0$
- The frequency of the VCO will then follow f_c; such that
  - $g(t) \cong 0$; $v(t) \cong 0$
  - $r(t) = A_c' \sin(2\pi f_c t)$
- When $\theta(t) \neq 0$ the frequency of the VCO is
- $f_r(t) = f_c + k_v v(t)$; VCO is an FM modulator
- The VCO signal will then follow
  - $r(t) = A_c' \sin\big(2\pi f_c t + \phi(t)\big)$; **r(t) is an FM signal**
  - $\phi(t) = 2\pi k_v \int_0^t v(t)dt$; so that $v(t) = \left(\frac{1}{2\pi k_v}\right)\frac{d\phi(t)}{dt}$

The Phased Locked Loop: Basic Operation

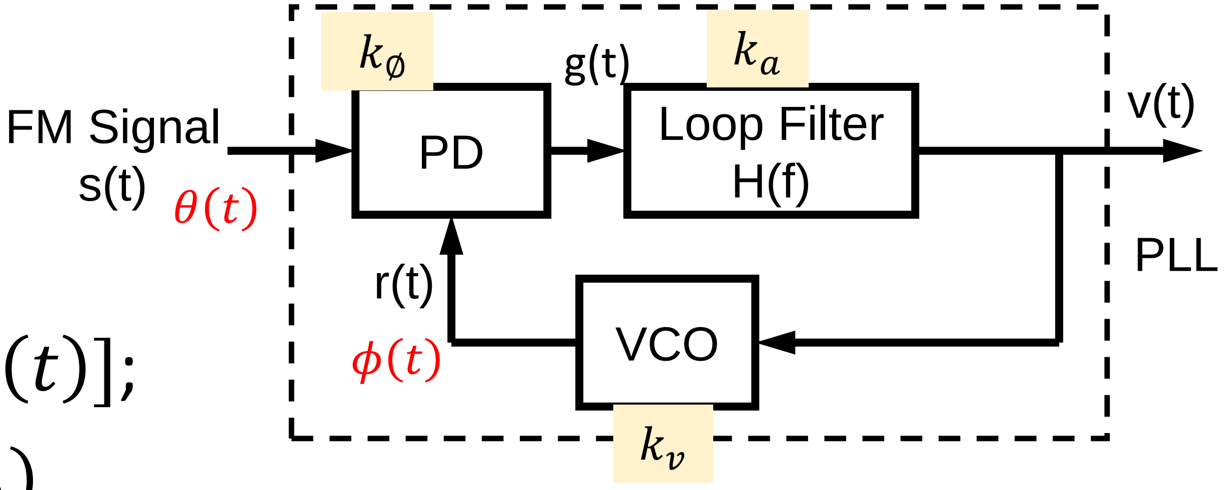


- **Phase Detector**
- Consists of a mixer followed by a LPF
- $s(t)r(t) = A_c{}'A_c\sin\big(2\pi f_c t + \phi(t)\big)\cos\big(2\pi f_c t + \theta(t)\big)$
- $= 0.5A_c{}'A_c\sin\big(4\pi f_c t + \phi(t)/2 + \theta(t)\big) + 0.5A_c{}'A_c\sin\big(\theta(t) - \phi(t)\big)$
- The output of the LPF is: $0.5A_c{}'A_c\sin\big(\theta(t) - \phi(t)\big)$
- When $\theta(t) - \phi(t)$is small, $\sin\big(\theta(t) - \phi(t)\big) \cong \theta(t) - \phi(t)$;
- $\mathbf{g}(\boldsymbol{t}) = \boldsymbol{k_\phi}[\boldsymbol{\theta}(\boldsymbol{t}) - \boldsymbol{\phi}(\boldsymbol{t})]$;



First Order PLL

- **Loop Output**
- In the simple case let $\boldsymbol{H(f) = k_a}$
- Then, the output v(t) is:
- $v(t) = k_a g(t) = (k_a)(k_\emptyset)[\theta(t) - \phi(t)];$
- $v(t) = k[\theta(t) - \phi(t)];\ k = (k_a)(k_\emptyset)$
-



$$\boldsymbol{\theta(t) = 2\pi k_f \int_0^t m(t)dt}$$

$$\boldsymbol{\phi(t) = 2\pi k_v \int_0^t v(t)dt}$$

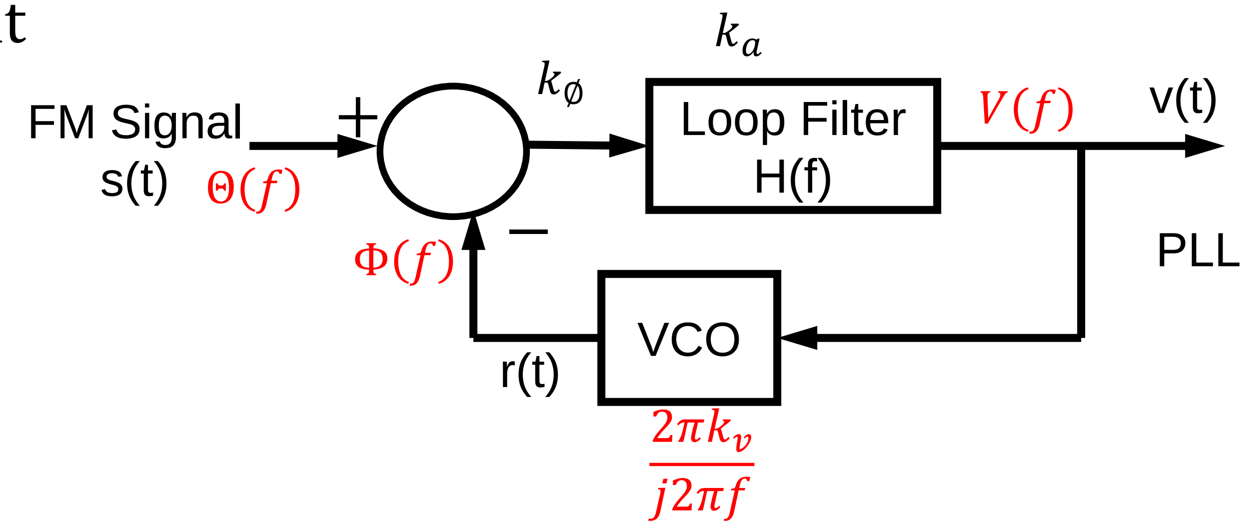
- **The two main loop equations for a first order PLL**

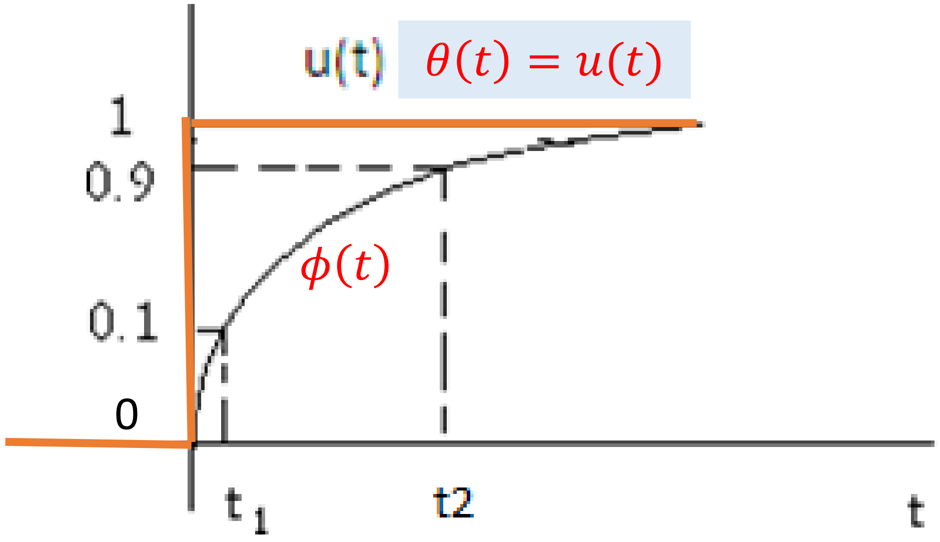
$$\boldsymbol{v(t) = k[\theta(t) - \phi(t)];}$$

- $$\boldsymbol{v(t) = (\frac{1}{2\pi k_v})\frac{d\phi(t)}{dt}}$$ OR $$\boldsymbol{\phi(t) = 2\pi k_v \int_0^t v(t)dt}$$

The Phased Locked Loop: Impulse and Step Responses

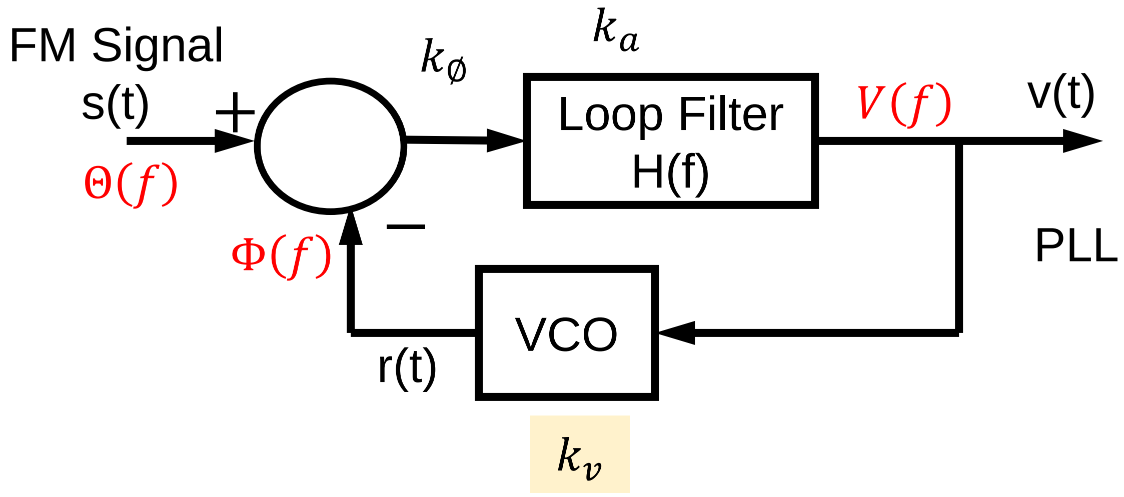
- Find loop impulse response relative to input $\boldsymbol{\theta(t)}$ **and output** $\boldsymbol{\phi(t)}$;
- $v(t) = k[\theta(t) - \phi(t)]$ (1)
- $\phi(t) = 2\pi k_v \int_0^t v(t)dt$ (2)
- Taking the Fourier transform of (1) and (2),
- $V(f) = k[\Theta(f) - \Phi(f)]$ (3)
- $\Phi(f) = \frac{2\pi k_v}{j2\pi f} V(f)$ (4)
- Combining (3), (4), the transfer function is:
- $H_1(f) = \frac{\Phi(f)}{\Theta(f))} = \frac{(2\pi)kk_v}{2\pi kk_v + j2\pi f} = \frac{K_1}{K_1 + j2\pi f}$
- $\boldsymbol{h_1(t) = K_1 e^{-K_1 t} u(t)}$; K_1: Loop Gain
- **Step Response**: If $\theta(t) = u(t)$, then
- $\boldsymbol{\phi(t) = \left(1 - e^{-K_1 t}\right) u(t)}$
- Hence, as K_1 increases, $\phi(t)$ becomes closes and closer to $\theta(t)$ for any value of t.





The Phased Locked Loop: Impulse and Step Responses

- Hence, as K_1 increases, $\phi(t)$ becomes closes and closer to $\theta(t)$ for any value of t.
- For any phase input $\theta(t)$; the steady-state condition for the loop is such that:
- $\boldsymbol{\phi(t) \cong \theta(t)}$; When the loop gain is large
- $\boldsymbol{2\pi k_v \int_0^t v(t)dt \cong 2\pi k_f \int_0^t m(t)dt}$;
- locking condition
- $\Rightarrow \boldsymbol{k_v v(t) \cong k_f m(t);}$
- $\Rightarrow \boldsymbol{v(t) \cong \frac{k_f}{k_v} m(t)}$
- Hence, FM demodulation is accomplished.



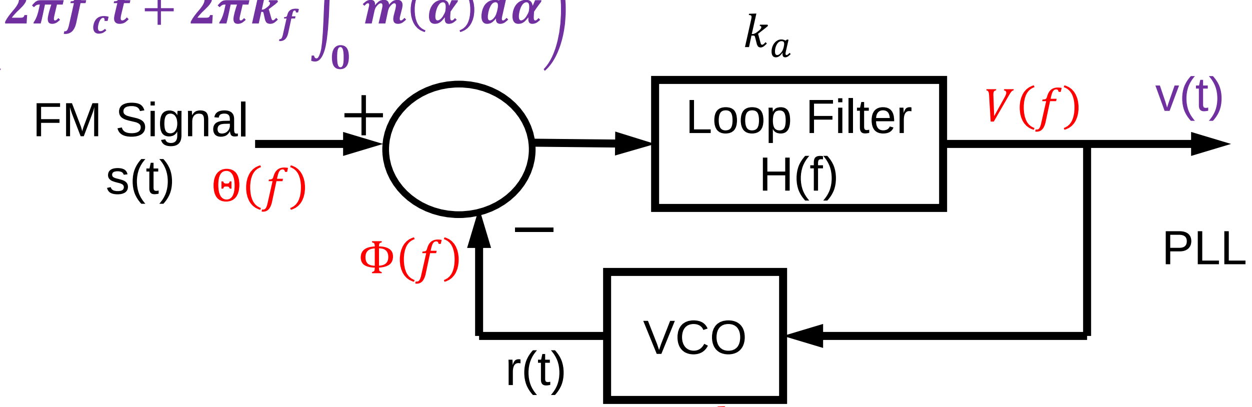
$$\mathbf{r}(\boldsymbol{t}) = \boldsymbol{A_c}' \boldsymbol{sin}\left(\boldsymbol{2\pi f_c t + 2\pi k_v \int_0^t v(t)dt}\right)$$

$$\boldsymbol{s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(t)dt\right)}$$

The Phased Locked Loop: Steady-State Frequency Response

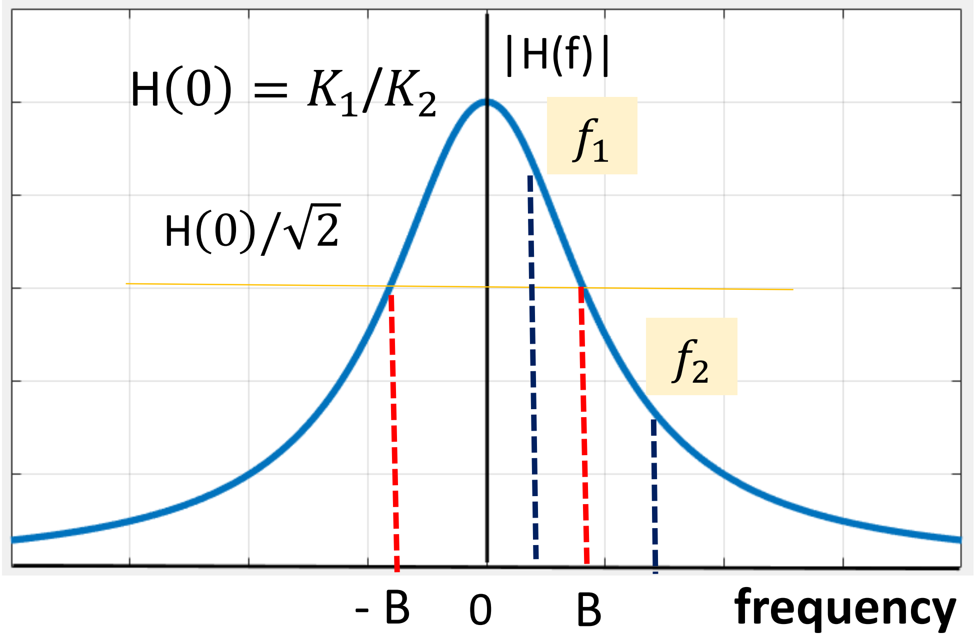
- *Here,* $m(t) = A_m \cos(2\pi f t)$;
- **Find V(f)/M(f)**
- $v(t) = k[\theta(t) - \phi(t)]$ (1)
- $\phi(t) = 2\pi k_v \int_0^t v(t)dt$ (2)
- $\theta(t) = 2\pi k_f \int_0^t m(t)dt$ (3)
- Taking the Fourier transform of (1), (2), and (3)
- $V(f) = k[\Theta(f) - \Phi(f)]$ (4)
- $\Phi(f) = \frac{2\pi k_v}{j2\pi f} V(f)$ (5)
- $\Theta(f) = \frac{2\pi k_f}{j2\pi f} M(f)$ (6)
- Combining (4), (5), and (6), the transfer function is:
- $\mathrm{H}(f) = \frac{V(f)}{M(f))} = \frac{(2\pi)kk_f}{2\pi kk_v + j2\pi f} = \frac{K_2}{K_1 + j2\pi f}$
- **PLL** acts like a LPF with 3-dB bandwidth of $B = K_1/\sqrt{2}$.
- High frequencies are attenuated more than low ones.
- $m(t) = (1)[cos(2\pi f_1(t)] + (1)[cos(2\pi f_2(t)]$
- $v(t) = A_1[cos(2\pi f_1(t - t_1)] + A_2[cos(2\pi f_2(t - t_2)]$

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\alpha)d\alpha\right)$$



$$|H(f)| = \frac{K_2}{\sqrt{{K_1}^2 + (2\pi f)^2}}$$

$\frac{2\pi k_v}{j2\pi f}$

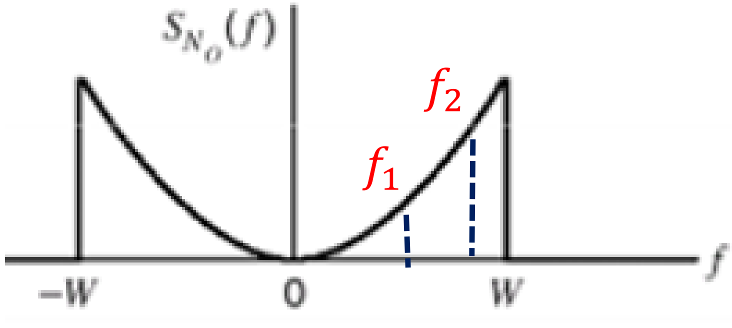


Pre-emphasis and De-emphasis in FM

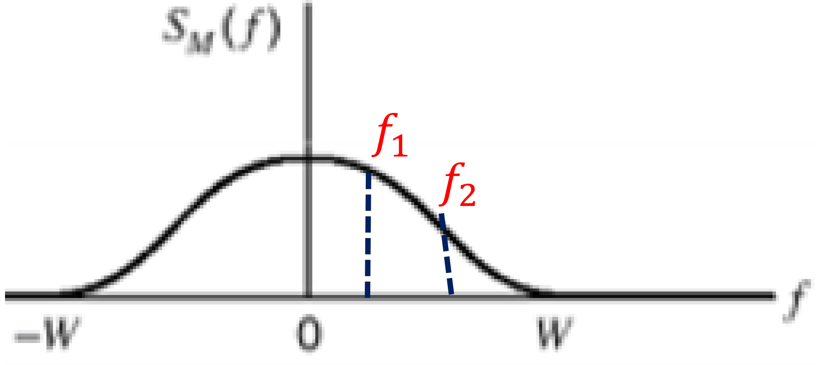
Three factors affect the quality of high frequencies at the FM demodulator output:

1. Power spectral density of message (in practice) falls off for higher frequencies.
2. Power spectral density of noise at demodulator output is proportional to the square of the frequency (will not be derived here).
3. The PLL behaves as a low pass filter, in the sense that higher frequencies will be attenuated more than low frequencies.

- These three effects severely affect the high frequencies of the signal resulting in a lower signal to noise ratio (SNR) for these frequencies. Hence, high frequencies will be distorted to a higher degree than the low frequencies.
- Pre-emphasis and de-emphasis are used to maintain an almost constant SNR over all frequencies.



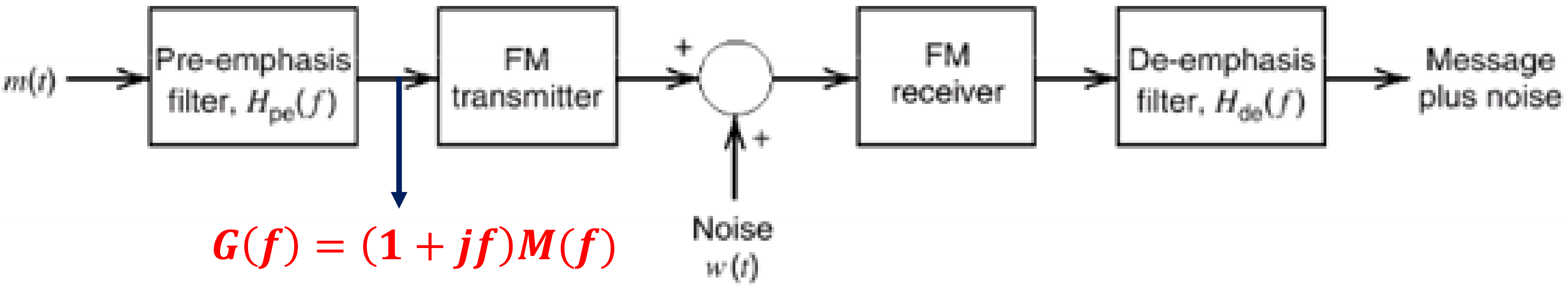
Noise PSD at PLL output



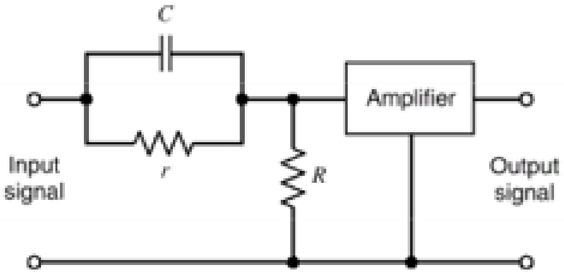
Message PSD

Pre-emphasis and De-emphasis in FM

- The pre-emphasis filter $H_{pe}(f)$ is used to emphasize the high frequency components of the message prior to modulation, and hence before noise is introduced.
- The de-emphasis filter $H_{de}(f)$ used at the receiver restores the original message signal
- $\boldsymbol{H_{pe}(f)H_{de}(f) = constant}$; for a distortion-less transmission).
- In theory, $\boldsymbol{H_{pe}(f) \propto f}$ and $\boldsymbol{H_{de}(f) \propto 1/f}$



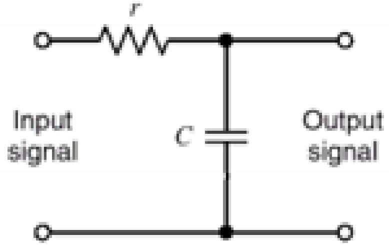
$\boldsymbol{G(f) = (1 + jf)M(f)}$



(a) Pre-emphasis filter

$H_{pe}(f) \cong 1 + jf / f_0$

$f_0 = 1/(2\pi rC), \quad R \ll r, \quad 2\pi f rC \ll 1$



(b) De-emphasis filter

$$H_{de}(f) = \frac{1}{1 + jf / f_0}$$

AM versus FM

- Normal AM requires simple circuits, and is very easy to generate and demodulate
- It is simple to tune, and is used in almost all short wave broadcasting.
- The area of coverage of AM is greater than FM (longer wavelengths ; lower frequencies).
- AM is power inefficient, and is susceptible to static and other forms of electrical noise.
- The main advantage of FM is its audio quality and immunity to noise. Most forms of static and electrical noise affect the amplitude, and an FM receiver will not respond significantly to such an amplitude noise.
- The audio quality of an FM signal increases as the frequency deviation increases (deviation from the center frequency), which is why FM broadcast stations use such large deviation.
- The main disadvantage of FM is the larger bandwidth it requires