CHAPTER 3

Pulse Modulation

Problem 3.1

Let $2W$ denote the bandwidth of a narrowband signal with carrier frequency f_c. The in-phase and quadrature components of this signal are both low-pass signals with a common bandwidth of W. According to the sampling theorem, there is no information loss if the in-phase and quadrature components are sampled at a rate higher than $2W$. For the problem at hand, we have

$f_c = 100$ kHz
$2W = 10$ kHz

Hence, $W = 5$ kHz, and the minimum rate at which it is permissible to sample the in-phase and quadrature components is 10 kHz.

From the sampling theorem, we also know that a physical waveform can be represented over the interval $-\infty < t < \infty$ by

$$g(t) = \sum_{n=-\infty}^{\infty} a_n \phi_n(t) \qquad (1)$$

where $\{\phi_n(t)\}$ is a set of orthogonal functions defined as

$$\phi_n(t) = \frac{\sin\{\pi f_s(t - n/f_s)\}}{\pi f_s(t - n/f_s)}$$

where n is an integer and f_s is the sampling frequency. If $g(t)$ is a low-pass signal band-limited to W Hz, and $f_s \geq 2W$, then the coefficient a_n can be shown to equal $g(n/f_s)$. That is, for $f_s \geq 2W$, the orthogonal coefficients are simply the values of the waveform that are obtained when the waveform is sampled every $1/f_s$ second.

As already mentioned, the narrowband signal is two-dimensional, consisting of in-phase and quadrature components. In light of Eq. (1), we may represent them as follows, respectively:

$$g_I(t) = \sum_{n=-\infty}^{\infty} g_I(n/f_s)\phi_n(t)$$

$$g_Q(t) = \sum_{n=-\infty}^{\infty} g_Q(n/f_s)\phi_n(t)$$

Hence, given the in-phase samples $g_I\left(\frac{n}{f_s}\right)$ and quadrature samples $g_Q\left(\frac{n}{f_s}\right)$, we may reconstruct the narrowband signal $g(t)$ as follows:

$$g(t) = g_I(t)\cos(2\pi f_c t) - g_Q(t)\sin(2\pi f_c t)$$

$$= \sum_{n=-\infty}^{\infty} \left[g_I\left(\frac{n}{f_s}\right)\cos(2\pi f_c t) - g_Q\left(\frac{n}{f_s}\right)\sin(2\pi f_c t)\right]\phi_n(t)$$

where f_c = 100 kHz and $f_s \geq$ 10 kHz, and where the same set of orthonormal basis functions is used for reconstructing both the in-phase and quadrature components.

Problem 3.2

(a) Consider a periodic train c(f) of rectangular pulses, each of duration T. The Fourier series expansion of c(t) (assuming that a pulse of the train is centered on the origin) is given by

$$c(t) = \sum_{n=-\infty}^{\infty} f_s \operatorname{sinc}(nf_s T)\exp(j2\pi n f_s t)$$

where f_s is the repetition frequency, and the amplitude of a rectangular pulse is assumed to be 1/T (i.e., each pulse has unit area). The assumption that $f_sT>>1$ means that the spectral lines (i.e., harmonics) of the periodic pulse train c(t) are well separated from each other.

Multiplying a message signal g(t) by c(t) yields

$$\begin{aligned} s(t) &= c(t)g(t) \\ &= \sum_{n=-\infty} f_s \operatorname{sinc}(nf_s T)\, g(t) \exp(j2\pi n f_s(t) \end{aligned} \qquad (1)$$

Taking the Fourier transform of both sides of Eq. (1) and using the frequency-shifting property of the Fourier transform:

$$S(f) = \sum_{n=-\infty}^{\infty} f_s \operatorname{sinc}(nf_s T)\, G(f-nf_s) \qquad (2)$$

where G(f) = F[g(t)]. Thus, the spectrum S(f) consists of frequency-shifted replicas of the original spectrum G(f), with the nth replica being scaled in amplitude by the factor $f_s \operatorname{sinc}(nf_sT)$.

(b) In accordance with the sampling theorem, let it be assumed that

- The signal g(t) is band-limited with

$$G(f) = 0 \quad \text{for} \quad -W < f < W$$

- The sampling frequency f_s is defined by

$$f_s > 2W$$

Then, the different frequency-shifted replicas of G(f) involved in the construction of S(f) will <u>not</u> overlap. Under the conditions described herein, the original spectrum G(f), and therefore the signal g(t), can be recovered exactly (except for a trivial amplitude scaling) by passing s(t) through a low-pass filter of bandwidth W.

<u>Problem 3.3</u>

(a) $g(t) = \text{sinc}(200t)$

This sinc pulse corresponds to a bandwidth W = 100 Hz. Hence, the Nyquist rate is 200 Hz, and the Nyquist interval is 1/200 seconds.

(b) $g(t) = \text{sinc}^2(200t)$

This signal may be viewed as the product of the sinc pulse sinc(200t) with itself. Since multiplication in the time domain corresponds to convolution in the frequency domain, we find that the signal g(t) has a bandwidth equal to twice that of the sinc pulse sin(200t); that is, 200 Hz. The Nyquist rate of g(t) is therefore 400 Hz, and the Nyquist interval is 1/400 seconds.

(c) $g(t) = \text{sinc}(200t) + \text{sinc}^2(200t)$

The bandwidth of g(t) is determined by the highest frequency component of sinc(200t) or $\text{sinc}^2(200t)$, whichever one is the largest. With the bandwidth (i.e., highest frequency component of) the sinc pulse sinc(200 t) equal to 100 Hz and that of the squared sinc pulse $\text{sinc}^2(200t)$ equal to 200 Hz, it follows that the bandwidth of g(t) is 200 Hz. Correspondingly, the Nyquist rate of g(t) is 400 Hz, and its Nyquist interval is 1/400 seconds.

Problem 3.4

(a) The PAM wave is

$$s(t) = \sum_{n=-\infty}^{\infty} [1 + \mu m'(nT_s)]g(t-nT_s),$$

where g(t) is the pulse shape, and $m'(t) = m(t)/A_m = \cos(2\pi f_m t)$. The PAM wave is equivalent to the convolution of the instantaneously sampled $[1 + \mu m'(t)]$ and the pulse shape g(t):

$$s(t) = \left\{ \sum_{n=-\infty}^{\infty} [1+\mu m'(nT_s)]\, \delta(t-nT_s) \right\} \star g(t)$$

$$= \left\{ [1+\mu m'(t)] \sum_{n=-\infty}^{\infty} \delta(t-nT_s) \right\} \star g(t)$$

The spectrum of the PAM wave is,

$$S(f) = \left\{ [\delta(f) + \mu M'(f)] \star \frac{1}{T_s} \sum_{m=-\infty}^{\infty} \delta(f - \frac{m}{T_s}) \right\} G(f)$$

$$= \frac{1}{T_s} G(f) \sum_{m=-\infty}^{\infty} [\delta(f - \frac{m}{T_s}) + \mu M'(f - \frac{m}{T_s})]$$

For a rectangular pulse g(t) of duration T=0.45s, and with AT = 1, we have:

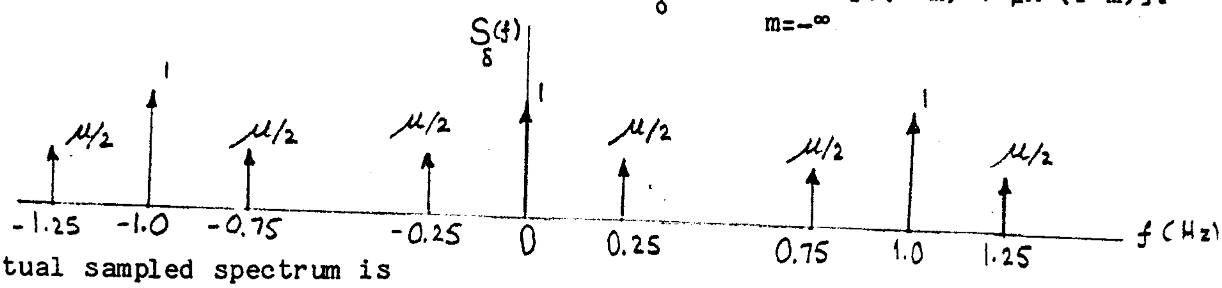
$$G(f) = AT\,\text{sinc}(fT)$$

$$= \text{sinc}(0.45f)$$

For $m'(t) = \cos(2\pi f_m t)$, and with $f_m = 0.25$ Hz, we have:

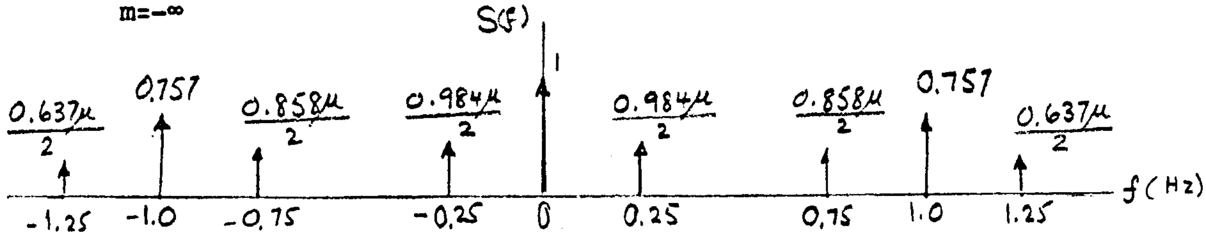
$$M'(f) = \frac{1}{2} [\delta(f-0.25) + \delta(f+0.25)]$$

For T_s = 1s, the ideally sampled spectrum is $S_\delta(f) = \sum_{m=-\infty}^{\infty} [\delta(f-m) + \mu M'(f-m)]$.

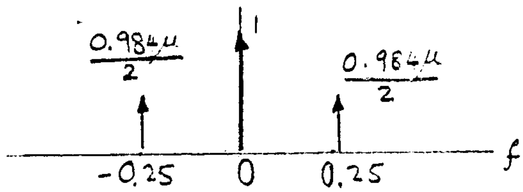


The actual sampled spectrum is

$$S(f) = \sum_{m=-\infty}^{\infty} \text{sinc}(0.45f)[\delta(f-m) + \mu M'(f-m)]$$



(b) The ideal reconstruction filter would retain the centre 3 delta functions of S(f) or:



With no aperture effect, the two outer delta functions would have amplitude $\frac{\mu}{2}$. Aperture effect distorts the reconstructed signal by attenuating the high frequency portion of the message signal.

Problem 3.5

The spectrum of the flat-top pulses is given by

$$H(f) = T \,\text{sinc}\,(fT)\exp(-j\pi fT)$$

$$= 10^{-4}\text{sinc}\,(10^{-4}f)\exp(-j\pi f 10^{-4})$$

Let $s(t)$ denote the sequence of flat-top pulses:

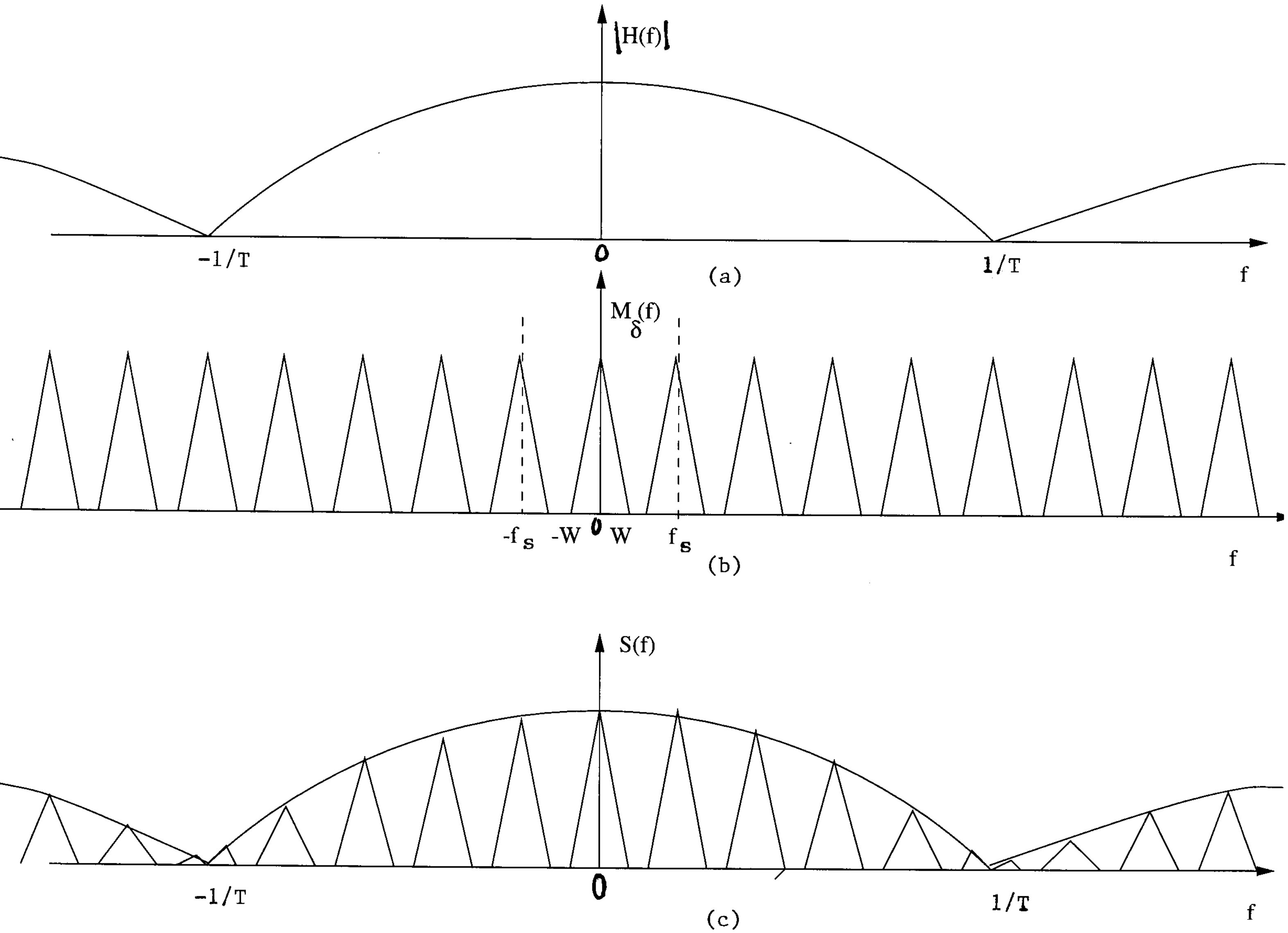
$$s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t-nT_s)$$

The spectrum $S(f) = F[s(t)]$ is as follows:

$$S(f) = f_s \sum_{k=-\infty}^{\infty} M(f-kf_s)H(f)$$

$$= f_s H(f) \sum_{k=-\infty}^{\infty} M(f-kf_s)$$

The magnitude spectrum $|S(f)|$ is thus as shown in Fig. 1c.



$1/T = 10{,}000\text{Hz}$
$f_s = 1{,}000\text{Hz}$
$W = 400\text{Hz}$

Figure 1

Problem 3.6

At $f = 1/2T_s$, which corresponds to the highest frequency component of the message signal for a sampling rate equal to the Nyquist rate, we find from Eq. (6-19) that the amplitude response

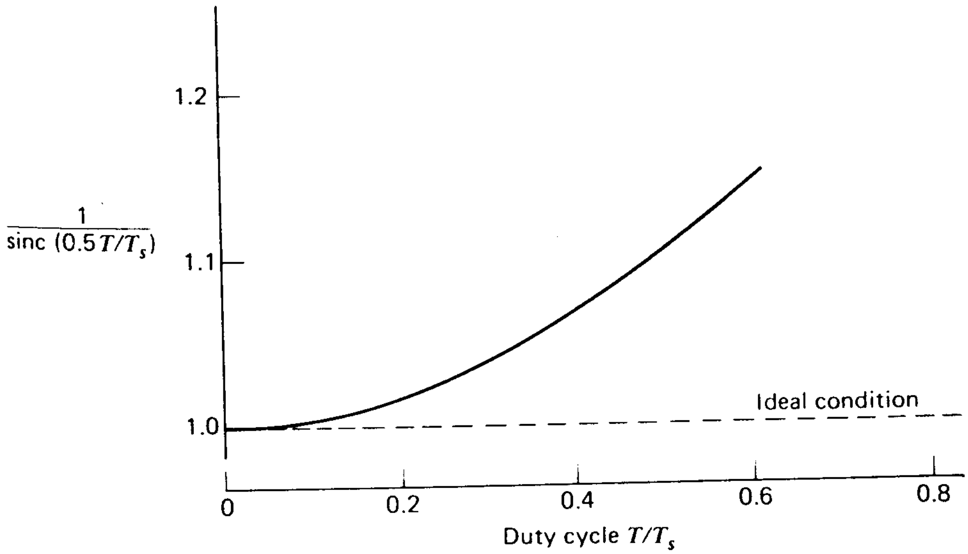


Figure 1

of the equalizer normalized to that at zero frequency is equal to

$$\frac{1}{\text{sinc}(0.5T/T_s)} = \frac{(\pi/2)(T/T_s)}{\sin[(\pi/2)(T/T_s)]}$$

where the ratio T/T_s is equal to the duty cycle of the sampling pulses. In Fig. 1, this result is plotted as a function of T/T_s. Ideally, it should be equal to one for all values of T/T_s. For a duty cycle of 10 percent, it is equal to 1.0041. It follows therefore that for duty cycles of less than 10 percent, the aperture effect becomes negligible, and the need for equalization may be omitted altogether.

Problem 3.7

Consider the full-load test tone $A\cos(2\pi f_m t)$. Denoting the kth sample amplitude of this signal by A_k, we find that the transmitted pulse is $A_k\, g(t)$, where $g(t)$ is defined by the spectrum:

$$G(f) = \begin{cases} \frac{1}{2B_T} & |f| < B_T \\ 0, & \text{otherwise} \end{cases}$$

The mean value of the transmitted signal power is

$$P = E\{\lim_{L\to\infty} \frac{1}{2LT_s} \int_{-LT_s}^{LT_s} [\sum_{k=-L}^{L} A_k\, g(t)]^2 dt\}$$

$$= E[\lim_{L\to\infty} \frac{1}{2LT_s} \int_{-LT_s}^{LT_s} \sum_{k=-L}^{L} \sum_{n=-L}^{L} A_k A_n\, g^2(t)dt]$$

$$= \lim_{L\to\infty} \frac{1}{2LT_s} \sum_{k=-L}^{L} \sum_{n=-L}^{L} E[A_k A_n] \int_{-LT_s}^{LT_s} g^2(t)dt$$

where T_s is the sampling period. However,

$$E[A_k A_n] = \begin{cases} \frac{A^2}{2}, & k = n \\ 0, & \text{otherwsie} \end{cases}$$

Therefore,

$$P = \frac{A^2}{2T_s} \int_{-\infty}^{\infty} g^2(t)dt$$

Using Rayleigh's energy theorem, we may write

$$\int_{-\infty}^{\infty} g^2(t)dt = \int_{-\infty}^{\infty} |G(f)|^2 df$$

$$= \int_{-B_T}^{B_T} (\frac{1}{2B_T})^2 df$$

$$= \frac{1}{2B_T}$$

Therefore,

$$P = \frac{A^2}{4T_s B_T}$$

The average signal power at the receiver output is $A^2/2$. Hence, the output signal-to-noise ratio is given by

$$(SNR)_O = \frac{A^2/2}{B_T N_O}$$

$$= \frac{A^2}{2B_T N_O}$$

$$= \frac{2T_s P}{N_O}$$

By choosing $B_T = 1/2T_s$, we get

$$(SNR)_O = \frac{P}{B_T N_O}$$

This shows that PAM and baseband signal transmission have the same signal-to-noise ratio for the same average transmitted power, with additive white Gaussian noise, and assuming the use of the minimum transmission bandwidth possible.

Problem 3.8

(a) The sampling interval is T_s = 125 μs. There are 24 channels and 1 sync pulse, so the time alloted to each channel is $T_c = T_s/25$ = 5 μs. The pulse duration is 1 μs, so the time between pulses is 4 μs.

(b) If sampled at the nyquist rate, 6.8 kHz, then T_s = 147 μs, T_c = 6.68 μs, and the time between pulses is 5.68 μs.

Problem 3.9

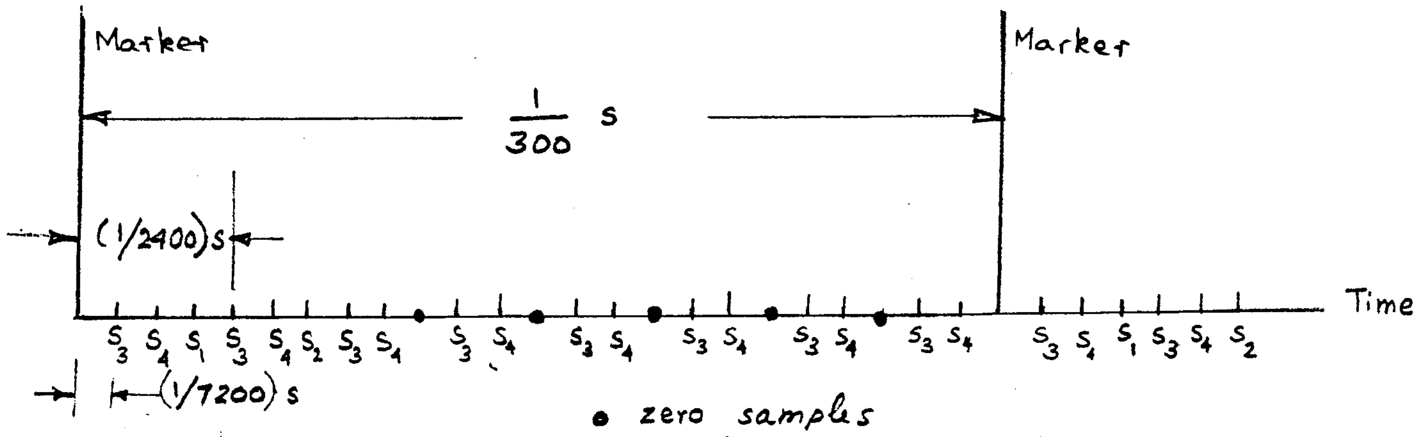
(a) The bandwidth required for each single sideband channel is 10 kHz. The total bandwidth for 12 channels is 120 kHz.

(b) The Nyquist rate for each signal is 20 kHz. For 12 TDM signals, the total data rate is 240 kHz. By using a sinc pulse whose amplitude varies in accordance with the modulation, and with zero crossings at multiples of (1/240) ms, we need a minimum bandwidth of 120 kHz.

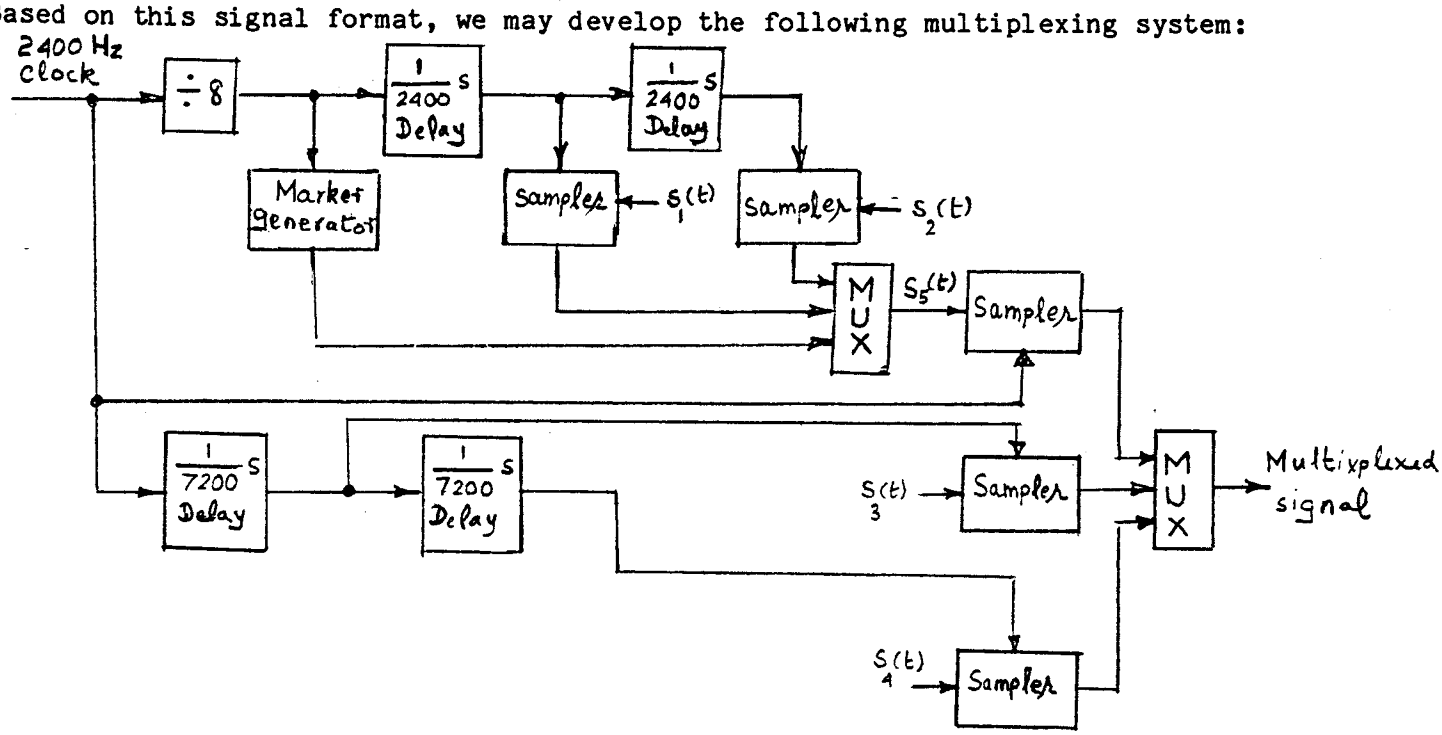
Problem 3.10

(a) The Nyquist rate for $s_1(t)$ and $s_2(t)$ is 160 Hz. Therefore, $\frac{2400}{2^R}$ must be greater than 160, and the maximum R is 3.

(b) With $R = 3$, we may use the following signal format to multiplex the signals $s_1(t)$ and $s_2(t)$ into a new signal, and then multiplex $s_3(t)$ and $s_4(t)$ and $s_5(t)$ including markers for synchronization:



Based on this signal format, we may develop the following multiplexing system:



Problem 3.11

In general, a line code can be represented as

$$s(t) = \sum_{n=-N}^{N} a_n g(t - nT_b)$$

Let $g(t) \rightleftharpoons G(f)$. We may then define the Fourier transform of $s(t)$ as

$$S(f) = \sum_{n=-N}^{N} a_n G(f) e^{-j\omega n T_b}$$

$$= G(f) \sum_{n=-N}^{N} a_n e^{-j\omega n T_b}$$

where $\omega = 2\pi f$. The power spectral density of $s(t)$ is

$$S_s(f) = \lim_{T \to \infty} \left[\frac{1}{T} |G(f)|^2 E \left| \sum_{n=-N}^{N} a_n e^{-j\omega n T_b} \right|^2 \right]$$

$$= |G(f)|^2 \lim_{T \to \infty} \left(\frac{1}{T} \sum_{n=-N}^{N} \sum_{m=-N}^{N} E[a_n a_m] e^{j(m-n)\omega T_b} \right)$$

where T is the duration of the binary data sequence, and E denotes the statistical expectation operator. Define the autocorrelation of the binary data sequence as

$$R(k) = E[a_n a_{n+k}]$$

By letting $m = n + k$ and $T = (2N + 1)T_b$, we may write

$$S_s(f) = |G(f)|^2 \lim_{N \to \infty} \left[\frac{1}{(2N+1)T_b} \sum_{n=-N}^{n=N} \sum_{k=-N-n}^{k=N-n} R(k) e^{jk\omega T_b} \right]$$

Replacing the outer sum over the index n by $2N+1$, we get

$$S_s(f) = \frac{|G(f)|^2}{T_b} \lim_{N \to \infty} \left[\frac{2N+1}{2N+1} \sum_{k=-N-n}^{k=N-n} R(k) e^{jk\omega T_b} \right]$$

$$= \frac{|G(f)|^2}{T_b} \sum_{k=-\infty}^{\infty} R(k) e^{jk\omega T_b} \tag{1}$$

where

$$R(k) = E[a_n a_{n+k}] = \sum_{i=1}^{I} (a_n a_{n+k}) p_i \tag{2}$$

where p_i is the probability of getting the product $(a_n\, a_{n+k})_i$ and there are I possible values for the $a_n\, a_{n+k}$ product. $G(f)$ is the spectrum of the pulse-shaping signal for representing a digital symbol. Eqs. (1) and (2) provide the basis for evaluating the spectra of the specified line codes.

(a) Unipolar NRZ signaling

For rectangular NRZ pulse shapes, the Fourier-transform pair is

$$g(t) = A\text{rect}\left(\frac{t}{T_b}\right) \rightleftharpoons G(f) = AT_b\text{sinc}(fT_b)$$

For unipolar NRZ signaling, the possible levels for a's are $+A$ and 0. For equiprobable symbols, we have the following autocorrelation values:

$$R(0) = \frac{1}{2}A^2 + \frac{1}{2} \times 0 = A^2/2$$

$$R(k) = \sum_{i=1}^{4} (a_n a_{n+k})_i p_i$$

$$= \frac{A^2}{4} + \frac{0}{4} + \frac{0}{4} + \frac{0}{4} = \frac{A^2}{4} \quad \text{for } |k| > 0$$

Thus

$$R(k) = \begin{cases} A^2/2 & \text{for } k = 0 \\ A^2/4 & \text{for } k \neq 0 \end{cases} \tag{3}$$

Therefore, the power spectral density for unipolar NRZ signals, using formulas (1) and (3), is

$$S_s(f) = \frac{|AT_b \mathrm{sinc}(fT_b)|^2}{T_b}\left[\frac{1}{4} + \frac{1}{4}\sum_{k=-\infty}^{\infty} e^{j2\pi kfT_b}\right]$$

$$= \frac{A^2T_b}{4}\mathrm{sinc}^2(fT_b)\left[1 + \sum_{k=-\infty}^{\infty} e^{j2\pi kfT_b}\right]$$

But,

$$\sum_{k=-\infty}^{\infty} e^{j2\pi kfT_b} = \frac{1}{T_b}\sum_{n=-\infty}^{\infty}\delta\left(f - \frac{n}{T_b}\right)$$

where $\delta(f)$ is a delta function in the frequency domain. Hence,

$$S_s(f) = \frac{A^2T_b}{4}\mathrm{sinc}^2(fT_b)\left[1 + \frac{1}{T_b}\sum_{n=-\infty}^{\infty}\delta\left(f - \frac{n}{T_b}\right)\right]$$

We also note that $\mathrm{sinc}(fT_b) = 0$ at $f = \frac{n}{T_b}$, $n \neq 0$; we thus get

$$S_s(f) = \frac{A^2T_b}{4}\mathrm{sinc}^2(fT_b)\left[1 + \frac{\delta(f)}{T_b}\right]$$

(b) <u>Polar Non-return-to-zero Signaling</u>

For polar NRZ signaling, the possible values for a's are $+A$ and $-A$. Assuming equiprobable symbols, we have

$$R(0) = \sum_{i=1}^{2}(a_n a_n)_i p_i$$

$$= \frac{A^2}{2} + \frac{(-A)^2}{2} = A^2$$

For $k \neq 0$, we have

$$R(k) = \sum_{i=1}^{4} (a_n a_{n+k})_i p_i$$

$$= \frac{A^2}{4} + 2\frac{(-A)(A)}{4} + 2\frac{(-A)(A)}{4} + \frac{(-A)^2}{4}$$

$$= 0$$

Thus,

$$R(k) = \begin{cases} A^2 & \text{for } k = 0 \\ 0 & \text{for } k \neq 0 \end{cases} \quad (4)$$

The power spectral density for this case, using formulas (1) and (4), is

$$S(f) = A^2 T_b \text{sinc}^2(f T_b)$$

(c) Return-to-zero Signaling

The pulse shape used for return-to-zero signaling is given by $g\left(\frac{t}{T_b/2}\right)$. We therefore have

$$G(f) = \frac{T_b}{2}\text{sinc}(fT_b/2)$$

The autocorrelation for this case is the same as that for unipolar NRZ signaling. Therefore, the power spectral density of RZ signals is

$$S_s(f) = \frac{A^2 T_b}{16}\text{sinc}^2(f T_b)\left[1 + \frac{t}{T_b} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_b}\right)\right]$$

(d) Bipolar Signals

The permitted values of level a for bipolar signals are $+A$, $-A$, and 0, where binary symbol 1 is represented alternately by $+A$ and $-A$, and binary 0 is represented by level zero. We thus have the following autocorrelation function values:

$$R(0) = \frac{A^2}{2}$$

$$R(1) = \sum_{i=1}^{4} (a_n a_{n+1})_i p_i = -\frac{A^2}{4}$$

For $k > 1$,

$$R(k) = \sum_{i=1}^{5} (a_n a_{n+k})_i p_i = \frac{A^2}{8} - \frac{A^2}{8} = 0$$

Thus,

$$R(k) = \begin{cases} \frac{A^2}{2} & \text{for } k = 0 \\ -\frac{A^2}{4} & \text{for } |k| = 1 \\ 0 & \text{for } |k| > 1 \end{cases} \qquad (5)$$

The pulse duration for this case is equal to $T_b/2$. Hence,

$$G(f) = \frac{T_b}{2}\text{sinc}\left(\frac{fT_b}{2}\right) \qquad (6)$$

Using Equations (1), (5) and (6), the power spectral density of bipolar signals is

$$S_s(f) = \frac{\left|\frac{T_b}{2}\text{sinc}\left(\frac{fT_b}{2}\right)\right|^2}{T_b}\left[\frac{A^2}{2} - \frac{A^2}{4}e^{j\omega T_b} - \frac{A^2}{4}e^{-j\omega T_b}\right]$$

$$= \frac{A^2 T_b}{8}\text{sinc}^2\left(\frac{fT_b}{2}\right)\left[1 - \frac{1}{2}(e^{j\omega T_b} + e^{-j\omega T_b})\right]$$

$$= \frac{A^2 T_b}{8}\text{sinc}^2\left(\frac{fT_b}{2}\right)[1 - \cos(2\pi f T_b)]$$

$$= \frac{A^2 T_b}{8} \text{sinc}^2\left(\frac{fT_b}{2}\right) \sin^2(\pi f T_b)$$

(e) Manchester Code

The permitted values of a's in the Manchester code are $+A$ and $-A$. Hence,

$$R(0) = \frac{1}{4}A^2 + \frac{1}{4}(-A)^2 + \frac{1}{4}(-A)^2 + \frac{1}{4}(A^2)$$

$$= A^2$$

For $k \neq 0$,

$$R(k) = \sum_{i=1}^{4} (a_n a_{n+k})_i p_i = \frac{A^2}{4} + \frac{(-A)(A)}{4} + \frac{A(-A)}{4} + \frac{(-A)^2}{4}$$

$$= 0$$

Thus,

$$R(k) = \begin{cases} A^2 & \text{for } k = 0 \\ 0 & \text{for } k \neq 0 \end{cases}$$

The pulse shape of Manchester signaling is given by

$$g(t) = \text{rect}\left(\frac{t + T_b/4}{T_b/2}\right) - \text{rect}\left(\frac{t - T_b/4}{T_b/2}\right)$$

The pulse spectrum is therefore

$$G(f) = \frac{T_b}{2}\text{sinc}\left(\frac{fT_b}{2}\right)e^{j\omega T_b/4} - \frac{T_b}{2}\text{sinc}\left(\frac{fT_b}{2}\right)e^{-j\omega T_b/4}$$

$$= jT_b \text{sinc}\left(\frac{fT_b}{2}\right)\sin\left(\frac{2\pi f T_b}{4}\right)$$

Therefore, the power spectral density of Manchester NRZ has the form

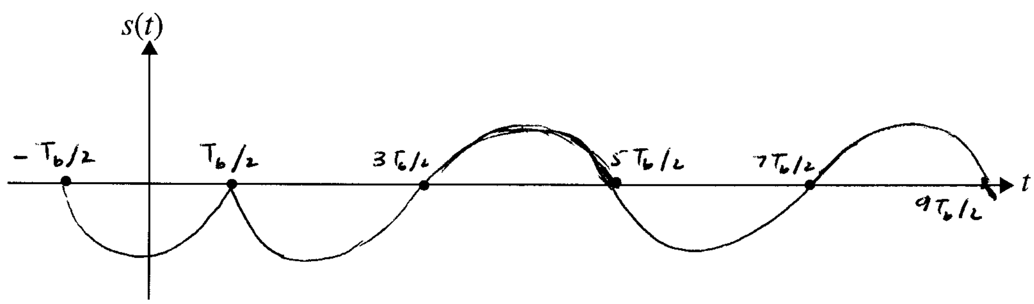
$$S_s(f) = A^2 T_b \operatorname{sinc}^2\left(\frac{fT_b}{2}\right)\sin^2\left(\frac{\pi f T_b}{2}\right)$$

Problem 3.12

Power spectral density of a binary data stream will not be affected by the use of differential encoding. The reason for this statement is that differential encoding uses the same pulse shaping functions as ordinary encoding methods. If the number of bits is high, then the probability of a symbol one and symbol zero are the same for both cases.

Problem 3.13

(a)



(b) $$g(t) = \begin{cases} \cos\left(\frac{\pi t}{T_b}\right), & \frac{-T_b}{2} < t \le \frac{T_b}{2} \\ 0, & \text{otherwise} \end{cases}$$

Equivalently, we may write

$$g(t) = \cos\left(\frac{\pi t}{T_b}\right) A \operatorname{rect}\left(\frac{t}{T_b}\right)$$

where rect(t) is a rectangular function of unit amplitude and unit duration. The Fourier transform of $g(t)$ is given by

$$G(f) = \frac{AT_b}{2}\left[\delta\left(f - \frac{2}{T_b}\right) + \delta\left(f + \frac{2}{T_b}\right)\right] \ast \operatorname{sinc}(fT_b)$$

where A denotes the pulse amplitude and $\ast$ denotes convolution in the frequency domain.

Using the replication property of the delta function $\delta(f)$, we get

$$G(f) = \frac{AT_b}{2}\left[\operatorname{sinc}\left(T_b\left(f - \frac{2}{T_b}\right)\right) + \operatorname{sinc}\left(T_b\left(f + \frac{2}{T_b}\right)\right)\right]$$

Using Eq. (1.52) of the textbook, the power spectral density of the binary data stream is

$$S(f) = \frac{|G(f)|^2}{T_b}$$

$$= \frac{A^2 T_b}{4}\left[\operatorname{sinc}^2\left(T_b\left(f - \frac{2}{T_b}\right)\right) + \operatorname{sinc}^2\left(T_b\left(f + \frac{2}{T_b}\right)\right)\right.$$

$$\left. + 2\operatorname{sinc}\left(T_b\left(f - \frac{2}{T_b}\right)\right)\operatorname{sinc}\left(T_b\left(f + \frac{2}{T_b}\right)\right)\right] \quad (1)$$

Note that the two spectral components $\operatorname{sinc}\left(T_b\left(f - \frac{2}{T_b}\right)\right)$ and $\operatorname{sinc}\left(T_b\left(f + \frac{2}{T_b}\right)\right)$ overlap in the frequency interval $-(1/T_b) \le f \le (1/T_b)$, hence the presence of cross-product terms in Eq. (1).

Figure 1 plots the normalized power spectral density $S(f)/(A^2T_b/4)$ versus the normalized frequency fT_b. The interesting point to note in this figure is the significant reduction in the power spectrum of the pulse-shaped data stream $x(t)$ in the interval $-1/T_b \le f \le 1/T_b$.

(c) The power spectral density of the standard form of polar NRZ signaling is

$$S(f) = A^2 T_b \operatorname{sinc}^2(fT_b) \quad (2)$$

Comparing this expression with that of Eq. (1), we observe the following differences:

	Polar NRZ signals using cosine pulses	Polar NRZ signals using rectangular pulses
$f = 0$	0	A^2T_b
$f = \pm 2/T_b$	$A^2T_b/4$	0

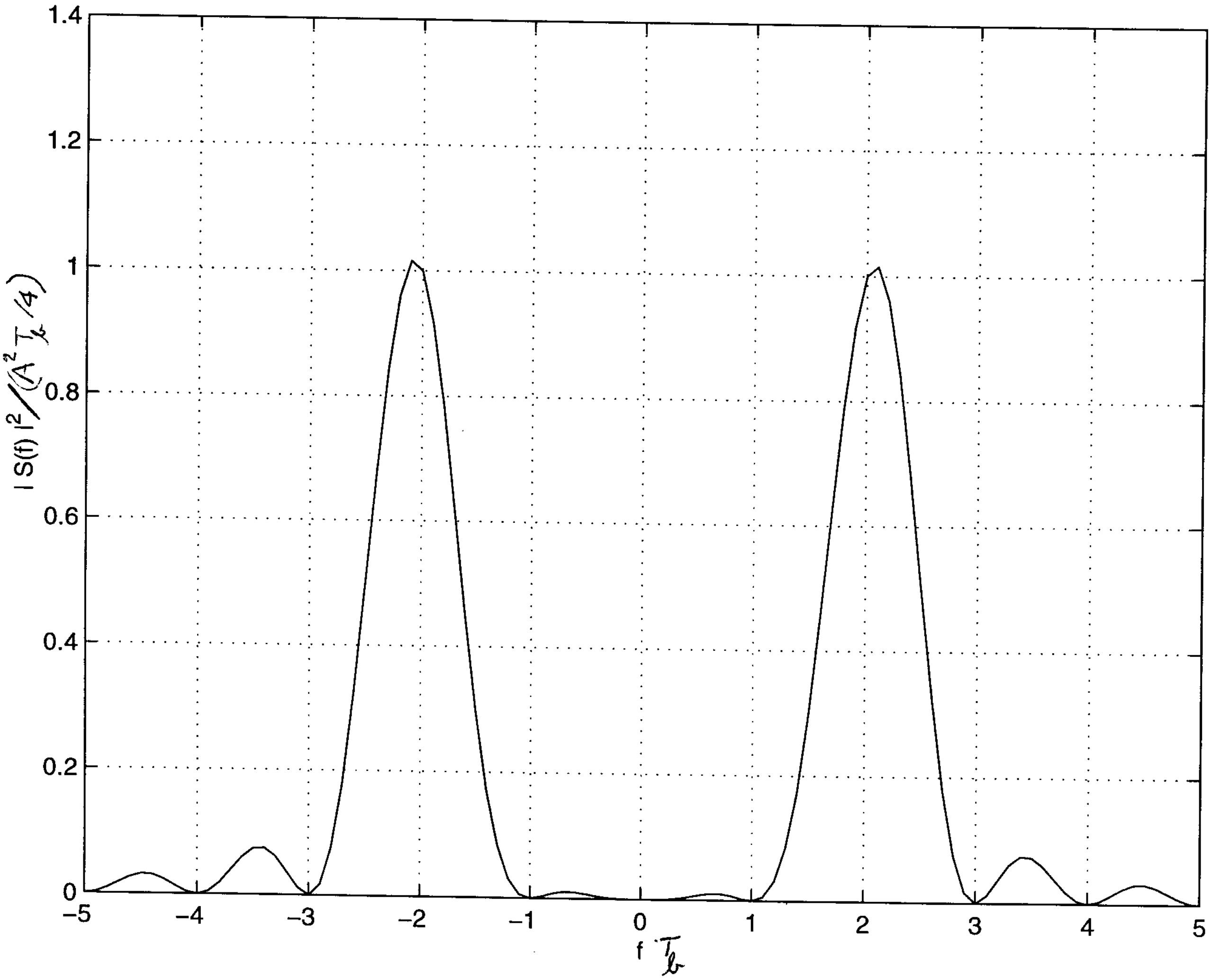
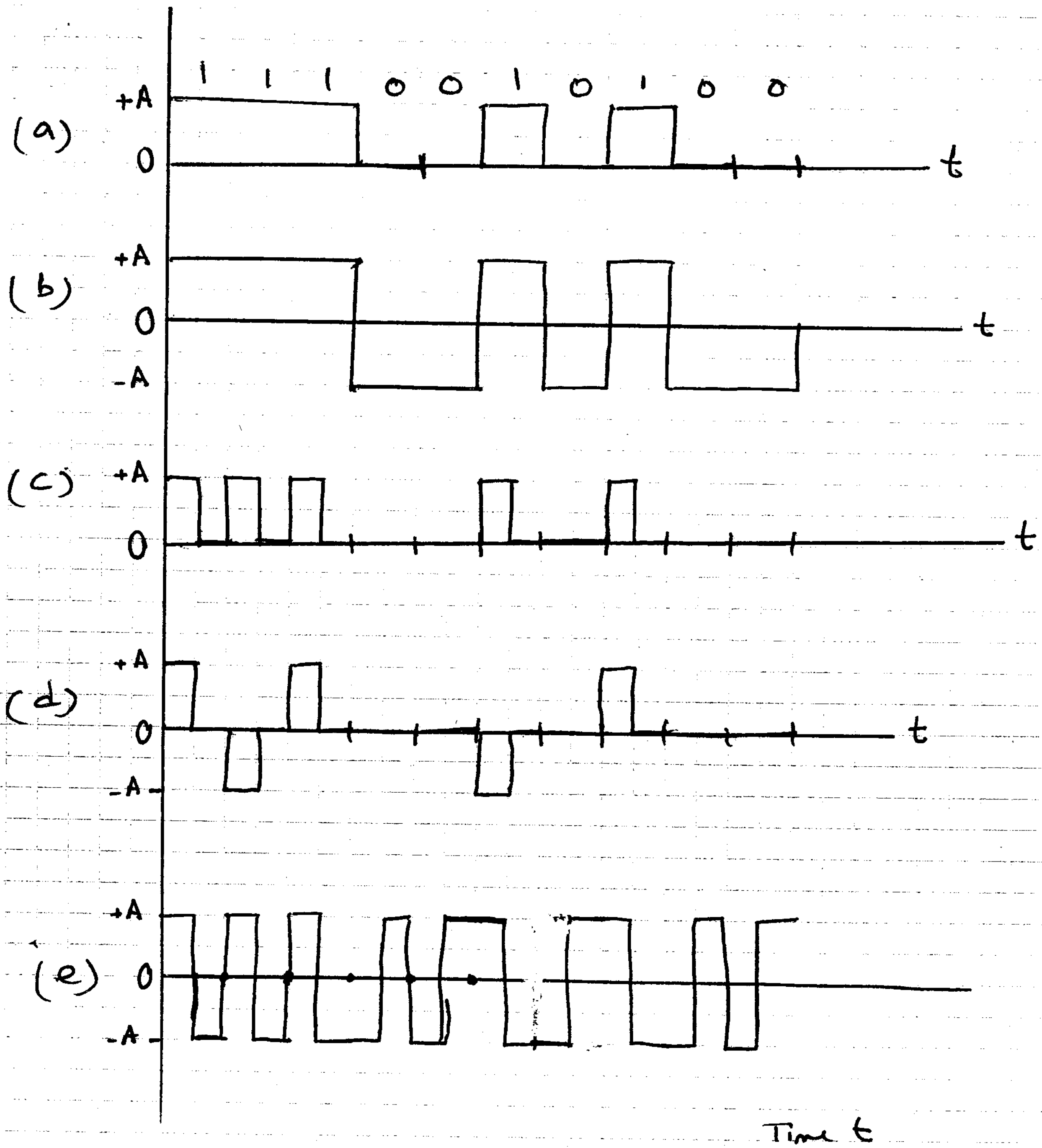
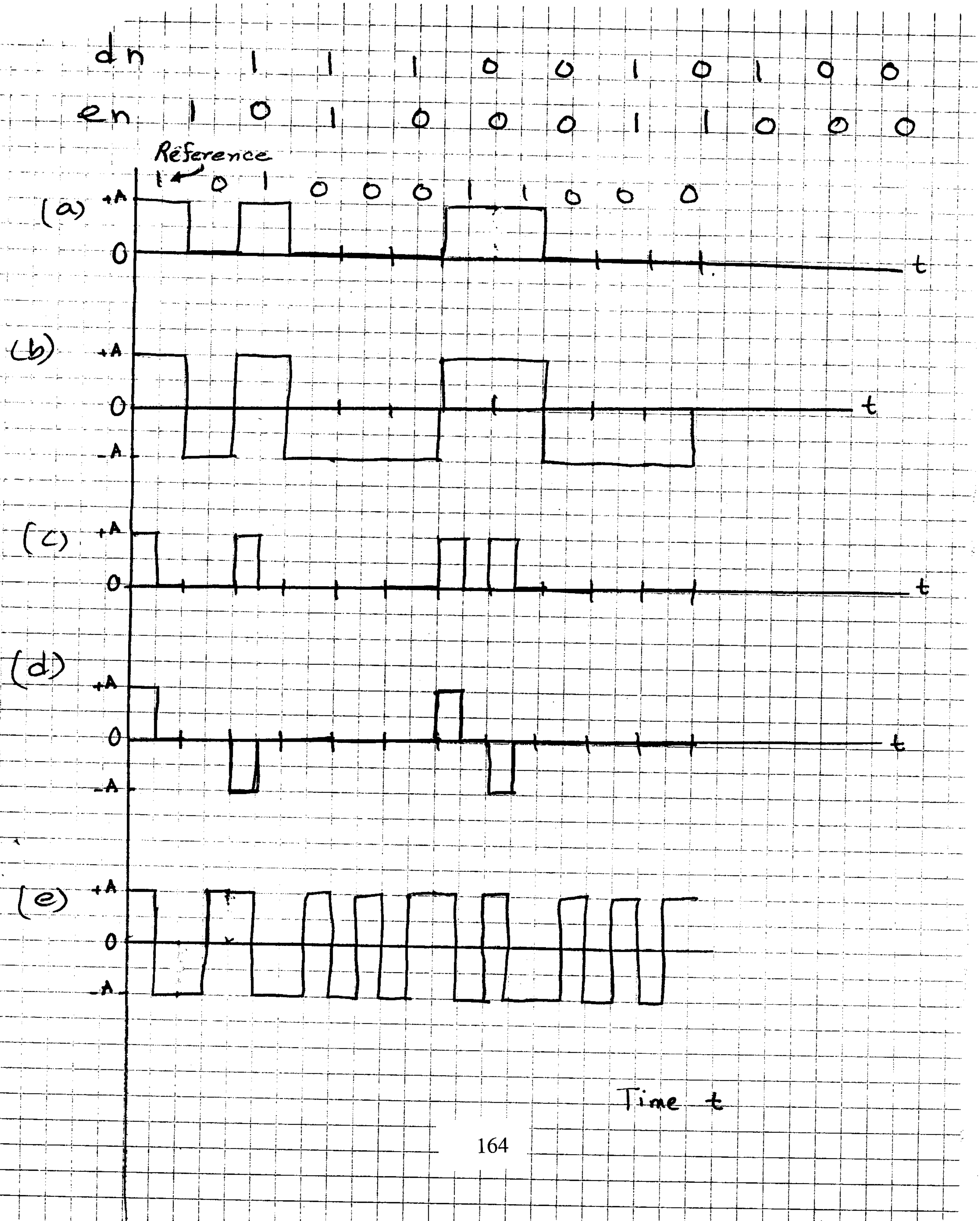


Figure 1

Problem 3.14



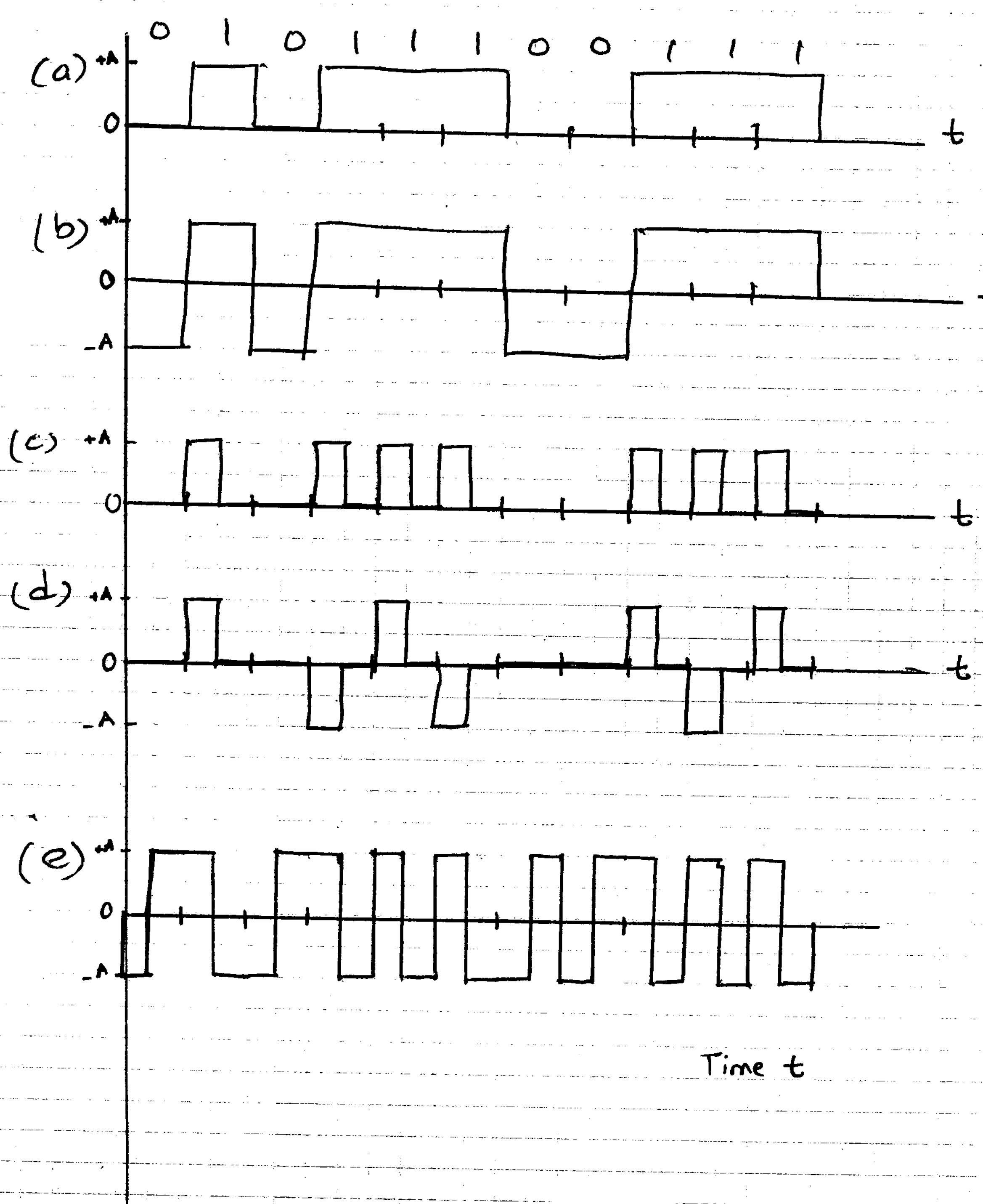
Problem 3.15(a)



Problem 3.15(b)

dn 1 1 1 0 0 1 0 1 0 0

en 0 1 0 1 1 1 0 0 1 1 1.



Problem 3.16

The minimum number of bits per sample is 7 for a signal-to-quantization noise ratio of 40 dB. Hence,

$$\begin{pmatrix}\text{The number of samples} \\ \text{in a duration of } 10s\end{pmatrix} = 8000 \times 10$$
$$= 8 \times 10^4 \text{ samples}$$

The minimum storage is therefore

$$= 7 \times 8 \times 10^4$$
$$= 5.6 \times 10^5$$
$$= 560 \text{ kbits}$$

Problem 3.17

Suppose that baseband signal m(t) is modeled as the sample function of a Gaussian random process of zero mean, and that the amplitude range of m(t) at the quantizer input extends from $-4A_{rms}$ to $4A_{rms}$. We find that samples of the signal m(t) will fall outside the amplitude range $8A_{rms}$ with a probability of overload that is less than 1 in 10^4. If we further assume the use of a binary code with each code word having a length n, so that the number of quantizing levels is 2", we find that the resulting quantizer step size is

$$\delta = \frac{8A_{rms}}{2^{R}} \tag{1}$$

Substituting Eq. (1) to the formula for the output signal-to-quantization noise ratio, we get

$$(\text{SNR})_o = \frac{3}{16}(2^{2R}) \tag{2}$$

Expressing the signal-to-noise ratio in decibels:

$$10\log_0(\text{SNR})_o = 6R - 7.2 \tag{3}$$

This formula states that each bit in the code word of a PCM system contributes 6dB to the signal-to-noise ratio. It gives a good description of the noise performance of a PCM system, provided that the following conditions are satisfied:

1. The system operates with an average signal power above the error threshold, so that the effect of transmission noise is made negligible, and performance is thereby limited essentially by quantizing noise alone.
2. The quantizing error is uniformly distributed.
3. The quantization is fine enough (say $R > 6$) to prevent signal-correlated patterns in the quantizing error waveform.
4. The quantizer is aligned with the amplitude range from $-4A_{rms}$ to $4A_{rms}$.

In general, conditions (1) through (3) are true of toll quality voice signals. However, when demands on voice quality are not severe, we may use a coarse quantizer corresponding to $R \leq 6$. In such a case, degradation in system performance is reflected not only by a lower signal-to-noise ratio, but also by an undesirable presence of signal-dependent patterns in the waveform of quantizing error.

Problem 3.18

(a) Let the message bandwidth be W. Then, sampling the message signal at its Nyquist rate, and using an R-bit code to represent each sample of the message signal, we find that the bit duration is

$$T_b = \frac{T_s}{R} = \frac{1}{2WR}$$

The bit rate is

$$\frac{1}{T_b} = 2WR$$

The maximum value of message bandwidth is therefore

$$W_{max} = \frac{50 \times 10^6}{2 \times 7}$$

$$= 3.57 \times 10^6 \text{ Hz}$$

(b) The output signal-to-quantizing noise ratio is given by (see Example 2):

$$10 \log_{10}(\text{SNR})_0 = 1.8 + 6R$$

$$= 1.8 + 6 \times 7$$

$$= 43.8 \text{ dB}$$

Problem 3.19

Let a signal amplitude lying in the range

$$x_i - \frac{1}{2}\delta_i \leq x \leq x_i + \frac{1}{2}\delta_i$$

be represented by the quantized amplitude x_i. The instantaneous square value of the error is $(x-x_i)^2$. Let the probability density function of the input signal be $f_X(x)$. If the step size δ_i is small in relation to the input signal excursion, then $f_X(x)$ varies little within the quantum step and may be approximated by $f_X(x_i)$. Then, the mean-square value of the error due to signals falling within this quantum is

$$E[Q_i^2] = \int_{x_i - \frac{1}{2}\delta_i}^{x_i + \frac{1}{2}\delta_i} (x-x_i)^2 f_X(x)\,dx$$

$$\simeq \int_{x_i - \frac{1}{2}\delta_i}^{x_i + \frac{1}{2}\delta_i} (x-x_i)^2 f_X(x_i)\,dx$$

$$= f_X(x_i) \int_{x_i - \frac{1}{2}\delta_i}^{x_i + \frac{1}{2}\delta_i} (x-x_i)^2 dx$$

$$= f_X(x_i) \int_{-\frac{1}{2}\delta_i}^{\frac{1}{2}\delta_i} x^2\,dx$$

$$= \frac{1}{12}\delta_i^3 f_X(x_i) \qquad (1)$$

The probability that the input signal amplitude lies within the ith interval is

$$p_i = \int_{x_i - \frac{1}{2}\delta_i}^{x_i + \frac{1}{2}\delta_i} f_X(x)\,dx \simeq f_X(x_i) \int_{x_i - \frac{1}{2}\delta_i}^{x_i + \frac{1}{2}\delta_i} dx = f_X(x_i)\delta_i \qquad (2)$$

Therefore, eliminating $f_X(x_i)$ between Eqs. (1) and (2), we get

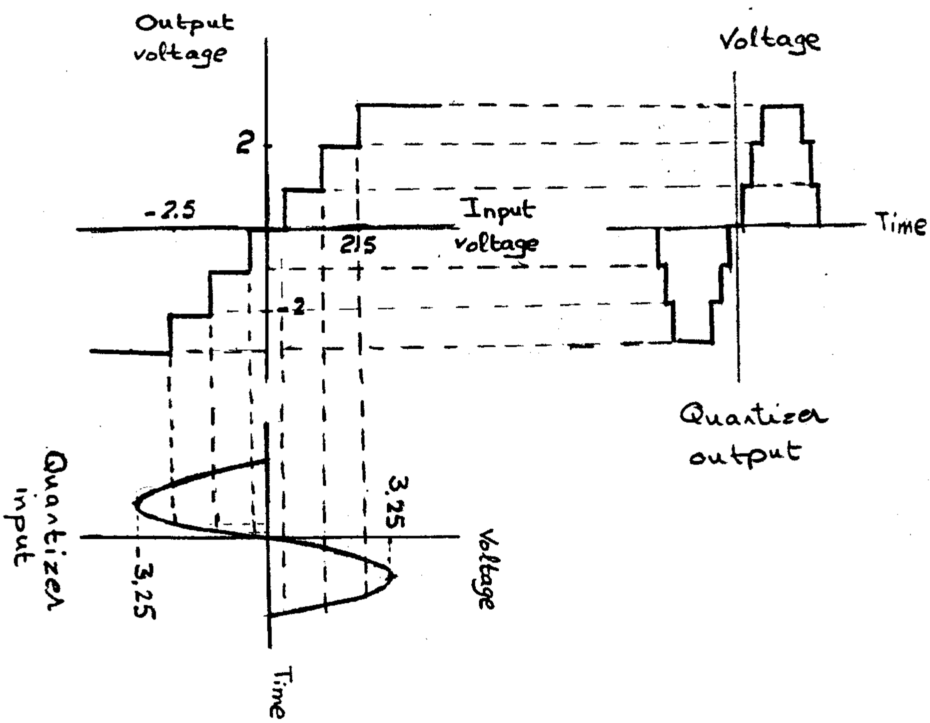
$$\therefore E[Q_i^2] = \frac{1}{12} p_i \delta_i^2$$

The total mean-square value of the quantizing error is the sum of that contributed by each of the several quanta. Hence,

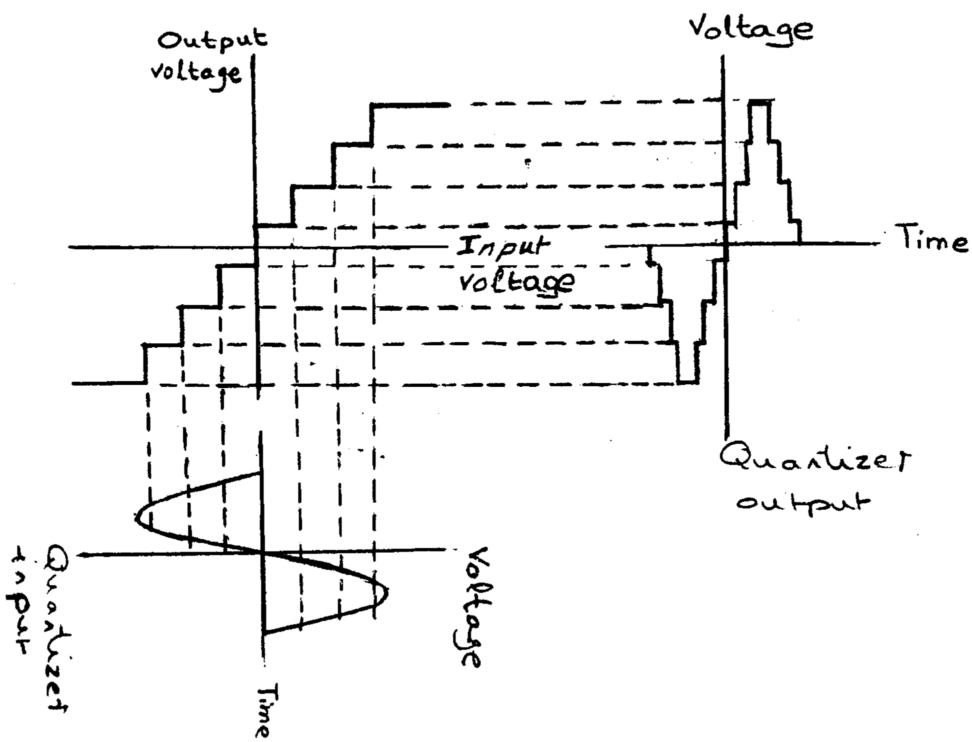
$$\sum_i E[Q_i^2] = \frac{1}{12} \sum_i p_i \delta_i^2$$

Problem 3.20

(a)

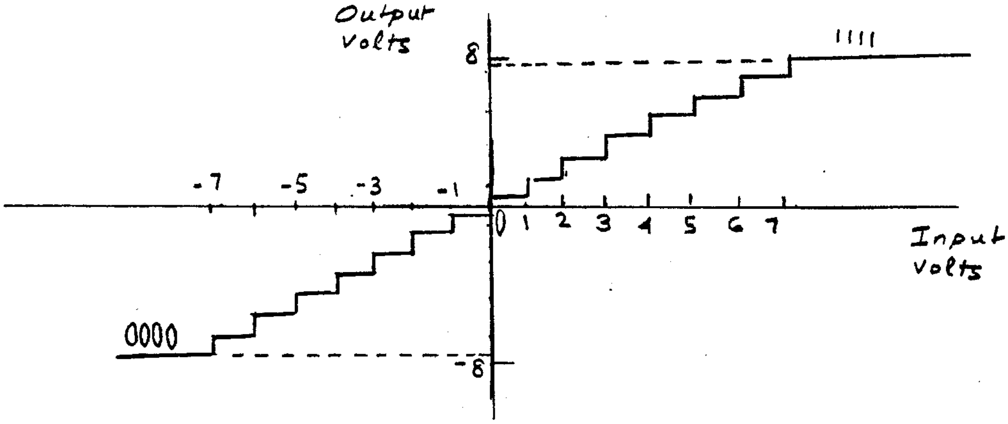


(b)



Problem 3.21

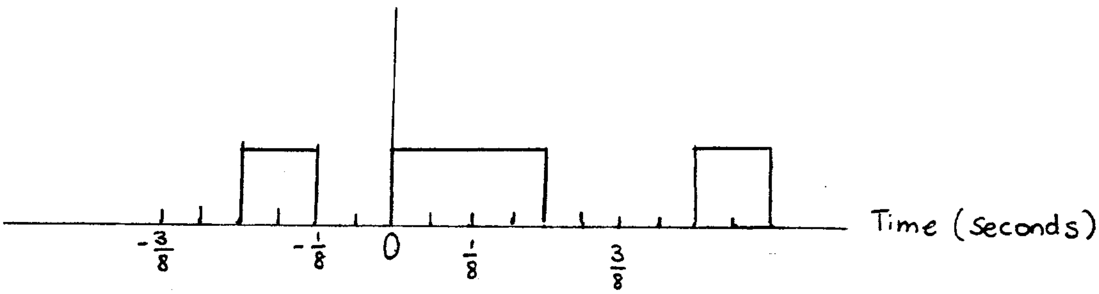
The quantizer has the following input-output curve:



At the sampling instants we have:

t	m(t)	code
-3/8	$-3\sqrt{2}$	0011
-1/8	$-3\sqrt{2}$	0011
+1/8	$3\sqrt{2}$	1100
+3/8	$3\sqrt{2}$	1100

And the coded waveform is (assuming on-off signaling):

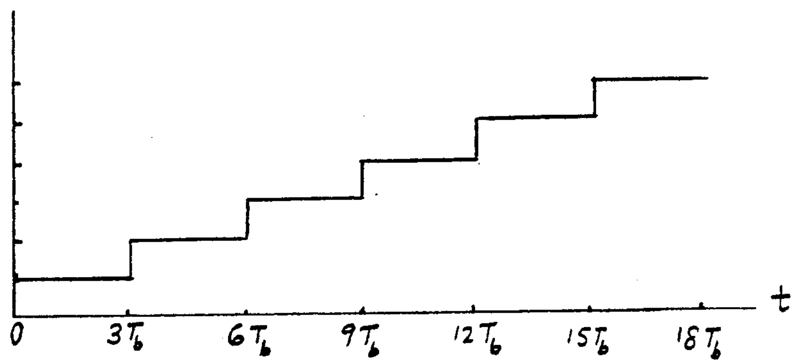


Problem 3.22

The transmitted code words are:

t/T_b	code
1	001
2	010
3	011
4	100
5	101
6	110

The sampled analog signal is



Problem 3.23

(a) The probability p_1 of any binary symbol being inverted by transmission through the system is usually quite small, so that the probability of error after n regenerations in the system is very nearly equal to n p_1. For very large n, the probability of more than one inversion must be taken into account. Let p_n denote the probability that a binary symbol is in error after transmission through the complete system. Then, p_n is also the probability of an odd number of errors, since an even number of errors restores the original value. Counting zero as an even number, the probability of an even number of errors is $1-p_n$. Hence

$$p_{n+1} = p_n(1-p_1)+(1-p_n)p_1$$
$$= (1-2p_1)p_n+p_1$$

This is a linear difference equation of the first order. Its solution is

$$p_n = \frac{1}{2}[1-(1-2p_1)^n]$$

(b) If p_1 is very small and n is not too large, then

$$(1-2p_1)^n \approx 1-2p_1n$$

and

$$p_n \approx \frac{1}{2}[1-(1-2p_1n)]$$
$$= p_1n$$

Problem 3.24 - Regenerative repeater for PCM

Three basic functions are performed by regenerative repeaters: equalization, timing and decision-making.

Equalization: The equalizer shapes the incoming pulses so as to compensate for the effects of amplitude and phase distortion produced by the imperfect transmission characteristics of the channel.

Timing: The timing circuitry provides a periodic pulse train, derived from the received pulses, for sampling the equalized pulses at the instants of time where the signal-to-noise ratio is maximum.

Decision-making: The extracted samples are compared to a predetermined threshold to make decisions. In each bit interval, a decision is made whether the received symbol is 1 or 0 on the basis of whether the threshold is exceeded or not.

Problem 3.25

$m(t) = A\tanh(\beta t)$

To avoid slope overload, we require

$$\frac{\Delta}{T_s} \geq \max\left|\frac{dm(t)}{dt}\right| \tag{1}$$

$$\frac{dm(t)}{dt} = A\beta\,\text{sech}^2(\beta t) \tag{2}$$

Hence, using Eq. (2) in (1):

$$\Delta \geq \max(A\beta\,\text{sech}^2(\beta t)) \times T_s \tag{3}$$

$$\text{Since } \text{sech}(\beta t) = \frac{1}{\cosh(\beta t)}$$

$$= \frac{2}{e^{+\beta t} + e^{-\beta t}}$$

it follows that the maximum value of sech(βt) is 1, which occurs at time $t = 0$. Hence, from Eq. (3) we find that $\Delta \geq A\beta T_s$.

Problem 3.26

The modulating wave is

$$m(t) = A_m \cos(2\pi f_m t)$$

The slope of m(t) is

$$\frac{dm(t)}{dt} = -2\pi f_m A_m \sin(2\pi f_m t)$$

The maximum slope of m(t) is equal to $2\pi f_m A_m$.

The maximum average slope of the approximating signal $m_a(t)$ produced by the delta modulator is δ/T_s, where δ is the step size and T_s is the sampling period. The limiting value of A_m is therefore given by

$$2\pi f_m A_m > \frac{\delta}{T_s}$$

or

$$A_m > \frac{\delta}{2\pi f_m T_s}$$

Assuming a load of 1 ohm, the transmitted power is $A_m^2/2$. Therefore, the maximum power that may be transmitted without slope-overload distortion is equal to $\delta^2/8\pi^2 f_m^2 T_s^2$.

Problem 3.27

$f_s = 10 f_{\text{Nyquist}}$

$f_{\text{Nyquist}} = 6.8$ kHz

$f_s = 10 \times 6.8 \times 10^3 = 6.8 \times 10^4$ Hz

$$\frac{\Delta}{T_s} \geq \max\left|\frac{dm(t)}{dt}\right|$$

For the sinusoidal signal $m(t) = A_m \sin(2\pi f_m t)$, we have

$$\frac{dm(t)}{dt} = 2\pi f_m A_m \cos(2\pi f_m t)$$

Hence,

$$\left|\frac{dm(t)}{dt}\right|_{\max} = |2\pi f_m A_m|_{\max}$$

or, equivalently,

$$\frac{\Delta}{T_s} \geq |2\pi f_m A_m|_{\max}$$

Therefore,

$$|A_m|_{\max} = \frac{\Delta}{T_s \times 2\pi \times f_m}$$

$$= \frac{\Delta f_s}{2\pi f_m}$$

$$= \frac{0.1 \times 6.8 \times 10^4}{2\pi \times 10^3}$$

$$= 1.08\text{V}$$

Problem 3.28

(a) From the solution to Problem 3.27, we have

$$A = \frac{\Delta f_s}{2\pi f_m} \quad \text{or} \quad \Delta = \frac{2\pi f_m A}{f_s} \tag{1}$$

The average signal power $= \frac{A^2}{2}$

$$= \frac{1}{2}\left(\frac{\Delta f_s}{2\pi f_m}\right)^2$$

With slope overload avoided, the only source of quantization of noise is granular noise. Replacing $\Delta/2$ for PCM with Δ for delta modulation, we find that the average quantization noise power is $\Delta^2/3$; for more details, see the solution to part (b) of Problem 3.30. The waveform of the reconstruction error (i.e., granular quantization noise) is a pattern of bipolar binary pulses characterized by (1) duration $= T_s = 1/f_s$, and (2) average power $= \Delta/3$. Hence, the autocorrelation function of the quantization noise is triangular in shape with a peak value of $\Delta^2/3$ and base $2T_s$, as shown in Fig. 1:

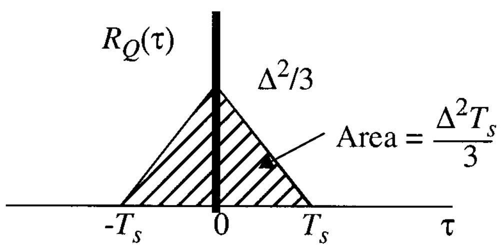


Fig. 1

From random process theory, we recall that

$$S_Q(f)\big|_{f=0} = \int_{-\infty}^{\infty} R_Q(\tau)d\tau$$

which, for the problem at hand, yields

$$S_Q(0) = \frac{\Delta^2 T_s}{3} = \frac{\Delta^2}{3f_s}$$

Typically, in delta modulation the sampling rate f_s is very large compared to the highest frequency component of the original message signal. We may therefore approximate the power spectral density of the granular quantization noise as

$$S_Q(f) \approx \begin{cases} \Delta^2/3f_s & -W \le f \le W \\ 0, & \text{otherwise} \end{cases}$$

where W is the bandwidth of the reconstruction filter at the demodulator output. Hence, the average quantization noise power is

$$N = \int_{-W}^{W} S_Q(f)df = \frac{2\Delta^2 W}{3f_s} \tag{2}$$

Substituting Eq. (2) into (1), we get

$$N = 2\left(\frac{2\pi f_m A}{f_s}\right)^2 \frac{W}{3f_s}$$

$$= \frac{8\pi^2 f_m^2 A^2 W}{3f_s^3}$$

(b) Correspondingly, output signal-to-noise ratio is

$$\text{SNR} = \frac{\left(\frac{1}{2}\right)A^2}{(8\pi^2 f_m^2 A^2 W)/3f_s^3}$$

$$= \frac{3f_s^3}{16\pi^2 f_m^2 W}$$

Problem 3.29

(a) $A \le \dfrac{\Delta f_s}{2\pi f_m}$

$$\Delta \ge \frac{2\pi f_m A}{f_s}$$

$$\Delta \geq \frac{2 \times \pi \times 10^3 \times 1}{50 \times 10^3}$$

$$= 0.126\text{V}$$

(b) $$(\text{SNR})_{\text{out}} = \frac{3}{8\pi^2} \frac{f_s^3}{f_m^2 W}$$

$$= \frac{3}{16\pi^2} \times \frac{(50 \times 10^3)^3}{10^6 \times 5 \times 10^3}$$

$$= 475$$

In decibels,

$$(\text{SNR})_{\text{out}} = 10\log_{10}475$$

$$= 26.8 \quad \text{dB}$$

Problem 3.30

(a) For linear delta modulation, the maximum amplitude of a sinusoidal test signal that can be used without slope-overload distortion is

$$A = \frac{\Delta f_s}{2\pi f_m}$$

$$= \frac{0.1 \times 60 \times 10^3}{2\pi \times 1 \times 10^3} \qquad f_s = 2 \times 3 \times 10^3$$

$$= 0.95\text{V}$$

(b) (i) Under the pre-filtered condition, it is reasonable to assume that the granular quantization noise is uniformly distributed between -Δ and +Δ. Hence, the variance of the quantization noise is

$$\sigma_Q^2 = \int_{-\Delta}^{\Delta} \frac{1}{2\Delta} q^2 dq$$

$$= \frac{1}{6\Delta}[q^3]_{-\Delta}^{\Delta}$$

$$= \frac{\Delta^2}{3}$$

The signal-to-noise ratio under the pre-filtered condition is therefore

$$(\text{SNR})_{\text{prefiltered}} = \frac{A^2/2}{\Delta^2/3}$$

$$= \frac{3A^2}{2\Delta^2}$$

$$= \frac{3 \times 0.95^2}{2 \times 0.1^2}$$

$$= 135$$
$$= 21.3 \text{ dB}$$

(ii)The signal-to-noise ratio under the post-filtered condition is

$$\left(\frac{S}{N}\right)_{\text{postfiltered}} = \frac{3}{16\pi^2} \times \frac{f_s^3}{f_m^2 W}$$

$$= \frac{3}{16\pi^2} \times \frac{(60)^3}{(1)^2 \times 3}$$

$$= 1367$$

$$= 31.3 \text{ dB}$$

The filtering gain in signal-to-noise ratio due to the use of a reconstruction filter at the demodulator output is therefore 31.3 - 21.3 = 10 dB.

Problem 3.31

Let the sinusoidal signal $m(t) = A\sin\omega_0 t$, where $\omega_0 = 2\pi f_0$

The autocorrelation of the signal is

$$R_m(\tau) = \frac{A^2}{2}\cos(\omega_0\tau)$$

$$R_m(0) = \frac{A^2}{2}$$

$$R_m(1) = \frac{A^2}{2}\cos\left(\omega_0 \times \frac{1}{10\omega_0}\right)$$

$$= \frac{A^2}{2}\cos(0.1)$$

For this problem, we thus have

$$\mathbf{R}_m = [R_m(0)], \quad \mathbf{r}_m = [R_m(1)]$$

(a) The optimum solution is given by

$$\mathbf{w}_0 = \mathbf{R}_m^{-1}\mathbf{r}_m$$

$$= \frac{\frac{A^2}{2}\cos(0.1)}{\frac{A^2}{2}} = \cos(0.1)$$

$$= 0.995$$

(b) $J_{\min} = R_m(0) - \mathbf{r}_m^T\mathbf{R}_m^{-1}\mathbf{r}_m$

$$= \frac{A^2}{2} - \frac{A^2}{2}\cos(0.1) \times \frac{A^2}{2}\cos(0.1)/(A^2/2)$$

$$= \frac{A^2}{2}(1 - \cos^2(0.1))$$

$$= 0.005A^2$$

Problem 3.32

$$\mathbf{R}_x = \begin{bmatrix} 1 & 0.8 & 0.6 \\ 0.8 & 1 & 0.8 \\ 0.6 & 0.8 & 1 \end{bmatrix}$$

$$\mathbf{r}_x = [0.8,\ 0.6,\ 0.4]^T$$

(a) $\mathbf{w}_0 = \mathbf{R}_x^{-1}\mathbf{r}_x$

$$= \begin{bmatrix} 1 & 0.8 & 0.6 \\ 0.8 & 1 & 0.8 \\ 0.6 & 0.8 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0.8 \\ 0.6 \\ 0.4 \end{bmatrix}$$

$$= \begin{bmatrix} 0.875 \\ 0 \\ -0.125 \end{bmatrix}$$

(b) $J_{\min} = R_x(0) - \mathbf{r}_x^T\mathbf{R}_x^{-1}\mathbf{r}_x$

$$= R_x(0) - \mathbf{r}_x^T\mathbf{w}_0$$

$$= 1 - [0.8,\ 0.6,\ 0.4] \begin{bmatrix} 0.875 \\ 0 \\ -0.125 \end{bmatrix}$$

$$= 1 - (0.8 \times 0.875 - \ 0.4 \times \ 0.125)$$

$$= 1 - 0.7 + 0.05$$

$$= 0.35$$

Problem 3.33

$$\mathbf{R}_x = \begin{bmatrix} 1 & 0.8 \\ 0.8 & 1 \end{bmatrix}$$

$$\mathbf{r}_x = [0.8, \quad 0.6]^T$$

(a) $\mathbf{w}_0 = \mathbf{R}_x^{-1}\mathbf{r}_x$

$$= \begin{bmatrix} 0.8889 \\ -0.1111 \end{bmatrix}$$

(b) $J_{\min} = R_x(0) - \mathbf{r}_x^T\mathbf{R}_x^{-1}\mathbf{r}_x$

$$= 1 - 0.6444$$

$$= 0.3556$$

which is slightly worse than the result obtained with a linear predictor using three unit delays (i.e., three coefficients). This result is intuitively satisfying.

Problem 3.34

Input signal variance = $R_x(0)$

The normalized autocorrelation of the input signal for a lag of one sample interval is

$$\rho_x(1) = \frac{R_x(1)}{R_x(0)} = 0.75$$

$$\text{Error variance} = R_x(0) - R_x(1)R_x^{-1}(0)R_x(1)$$

$$= R_x(0)(1 - \rho_x^2(1))$$

$$\text{Processing gain} = \frac{R_x(0)}{R_x(0)(1-\rho_x^2(1))}$$

$$= \frac{1}{1-\rho_x^2(1)}$$

$$= \frac{1}{1-(0.75)^2}$$

$$= 2.2857$$

Expressing the processing gain in dB, we have

$$10\log_{10}(2.2857) = 3.59 \text{ dB}$$

Problem 3.35

$$\text{Processing gain} = \frac{R_x(0)}{R_x(0)\left(1-\frac{\mathbf{r}_x^T\mathbf{R}_x^{-1}\mathbf{r}_x}{R_x(0)}\right)}$$

(a) Three-tap predictor:
Processing gain = 2.8571
= 4.56 dB

(b) Two-tap predictor:
Processing gain = 2.8715
= 4.49 dB

Therefore, the use of a three-tap predictor in the DPCM system results an improvement of 4.56 - 4.49 = 0.07 dB over the corresponding system using a two-tap predictor.

Problem 3.36

(a) For DPCM, we have $10\log_{10}(\text{SNR})_0 = \alpha + 6n$ dB

For PCM, we have $10\log_{10}(\text{SNR})_0 = 4.77 + 6n - 20\log_{10}(\log(1+\mu))$

where n is the number of quantization levels
SNR of DPCM

SNR = $\alpha + 6n$, where $-3 < \alpha < 15$

For n=8, the SNR is in the range of 45 to 63 dBs.

<u>SNR of PCM</u>

$$\begin{aligned} SNR &= 4.77 + 6n - 20\log_{10}(\log(2.56)) \\ &= 4.77 + 48 - 14.8783 \\ &= 38 \text{ dB} \end{aligned}$$

Therefore, the SNR improvement resulting from the use of DPCM is in the range of 7 to 25 dB.

(b) Let us assume that n_1 bits/sample are used for DPCM and n bits/sample for PCM

If $\alpha = 15$ dB, then we have

$15 + 6n_1 = 6n - 10.0$

Rearranging: $(n - n_1) = \dfrac{10 + 15}{6}$

$= 4.18$

which, in effect, represents a saving of about 4 bits/sample due to the use of DPCM.

If, on the other hand, we choose $\alpha = -3$ dB, we have

$-3 + 6n_1 = 6n - 10$

Rearranging: $(n - n_1) = \dfrac{10 - 3}{6}$

$= \dfrac{7}{6}$

$= 1.01$

which represents a saving of about 1 bit/sample due to the use of DPCM.

Problem 3.37

The transmitting prediction filter operates on exact samples of the signal, whereas the receiving prediction filter operates on quantized samples.

Problem 3.38

Matlab codes

```
% Problem 3.38, CS: Haykin
%flat-topped PAM signal
%and magnitude spectrum
% Mathini Sellathurai

%data
fs=8000; % sample frequency
ts=1.25e-4; %1/fs
pulse_duration=5e-5; %pulse duration

% sinusoidal sgnal;
td=1.25e-5; %sampling frequency of signal
fd=80000;
t=(0:td:100*td);
fm=10000;
s=sin(fm*t);

% PAM signal generation
pam_s=PAM(s,td,ts,pulse_duration);
figure(1);hold on
```

```
plot(t,s,'--');
plot(t(1:length(pam_s)),pam_s);
xlabel('time')
ylabel('magnitude')
legend('signal','PAM-signal');

% Computing magnitude spectrum S(f) of the signal
a=((abs(fft(pam_s)).^2));
a=a/max(a);
f=fs*(fs/fd:fs*(fs/fd):(length(a))*fs*(fs/fd);
figure(2)
plot(f,a);
xlabel('frequency');
ylabel('magnitude')

% finding the zeros
index=find(a<1e-5);

% finding the first zero
fprintf('Envelopes goes through zero for the first time = %6d\n', min(index)*fs*(fs/fd))
```

```
function pam_s=PAM(s,td,ts,pulse_duration)

% Problem 3.38, CS: Haykin
%flat-topped PAM signal
%used in   Problem 3.38, CS: Haykin
% Mathini Sellathurai

potd=pulse_duration/td;
tsotd=ts/td;

y=zeros(1,length(s));
tt=1:(tsotd):length(s);

for kk=1:length(tt);
y(tt(kk):tt(kk)+potd-1)=s(tt(kk)).*ones(1,potd);
end

pam_s=y(1:length(s)-potd);
```

Answers: 3.38

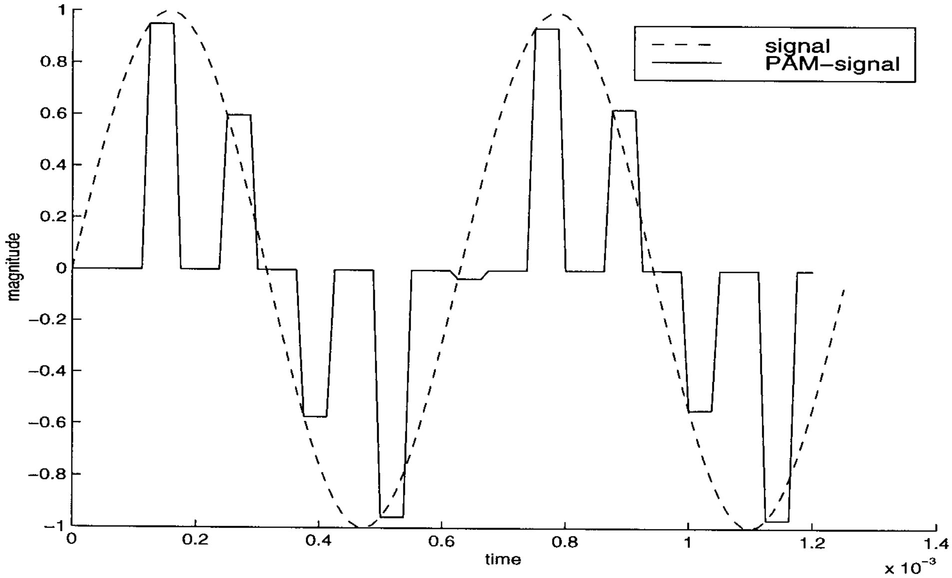


Figure 1: Flat-topped PAM signal

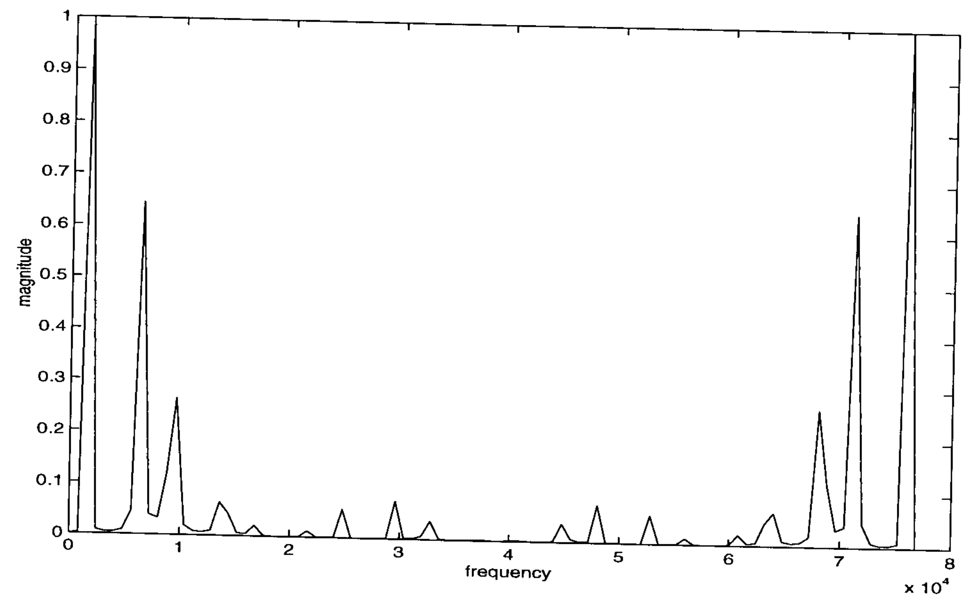


Figure .2: Magnitude spectrum of flat-topped PAM signal

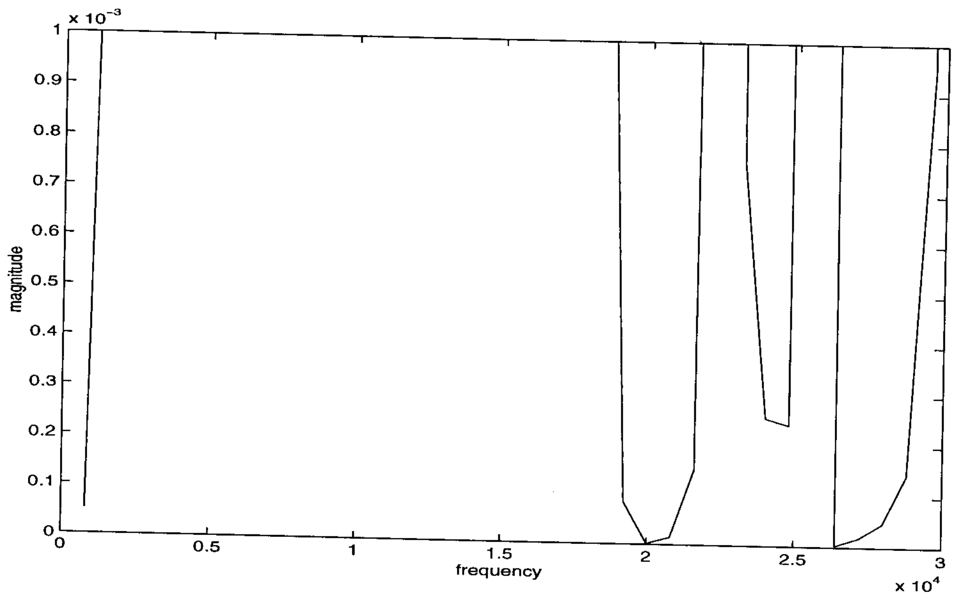


Figure .3: Zoomed magnitude spectrum of flat-topped PAM signal

Problem 3.39

Matlab codes

```
%problem 3.39, CS: Haykin
%mue-law pCM and uniform quantizing
%Mathini Sellathurai
clear all

%sinusoidal signal
t=[0:2*pi/100:2*pi];
a=sin(t);

% input signal to noise ratio in db
SNRdb=[-20 -15 -10 -5 0 5 10 15 20 25 ];

for nEN=1:10
  sqnrfm=0; sqnrfu=0;
for k=1:100
  snr = 10^(SNRdb(nEN)/10);
  wn= randn(1,length(a))/sqrt(snr); % noise
  a1=a+wn; %signal plus noise

   [a_quanu,codeu,sqnr_u]=u_pcm(a1,256); %call u-PCM
   [a_quanm,codem,sqnr_m]=mue_pcm(a1,256,255); %call mue-PCM

   sqnrfm=sqnrfm+sqnr_m;
   sqnrfu=sqnrfu+sqnr_u;
end
  SNROm(nEN)=sqnrfm/k; %bin-SNR-MUE-PCM
  SNROu(nEN)=sqnrfu/k;  %bin-SNR-U-PCM
end

%plots
figure;hold on;
plot(SNRdb,SNROu,'-+')
plot(SNRdb,SNROm,'-o')
xlabel('input signal-to-noise-ration in db')
 ylabel('output signal-to-noise-ration in db')
legend('uniform  PCM, 256 levels','mue-law PCM, mue=255')
```

```
function [a_q,snr]=u_pcm(a,n)
% function to generate uniform PCM for sinwave
%used in problem 3.39, CS: Haykin
%Mathini Sellathurai

n=length(a);
amax=max(abs(a));
a_q=a;
b_q=a_q;
d=2/n;
q=d.*[0:n-1];
q=q-((n-1)/2)*d;
for i=1:n
a_q(find((q(i)-d/2<= a_q) & (a_q <=q(i)+d/2)))=...
q(i).*ones(1,length(find((q(i)-d/2 <=a_q) & (a_q<=q(i)+d/2))));
b_q(find(a_q==q(i)))=(i-1).*ones(1,length(find(a_q==q(i))));
end
a_q =a_q*amax;

snr=20*log10(norm(a)/norm(a-a_q));
```

```
function [a_q,snr]=mue_pcm(s,n,mue)
% function to generate mue-law PCM for sinwave
%used in problem 3.39, CS: Haykin
%Mathini Sellathurai

a=max(abs(s));

% mue-law
y=(log(1+mue*abs(s/a))./log(1+mue)).*sign(s);
[y_q,code,sqn]=u_pcm(y,n);

%inverse mue-law
a_q=(((1+mue).^(abs(y_q))-1)./mue).*sign(y_q);
a_q=a_q*a;

%SNR
snr=20*log10(norm(s)/norm(s-a_quan));
```

Answer to Problem 3.39

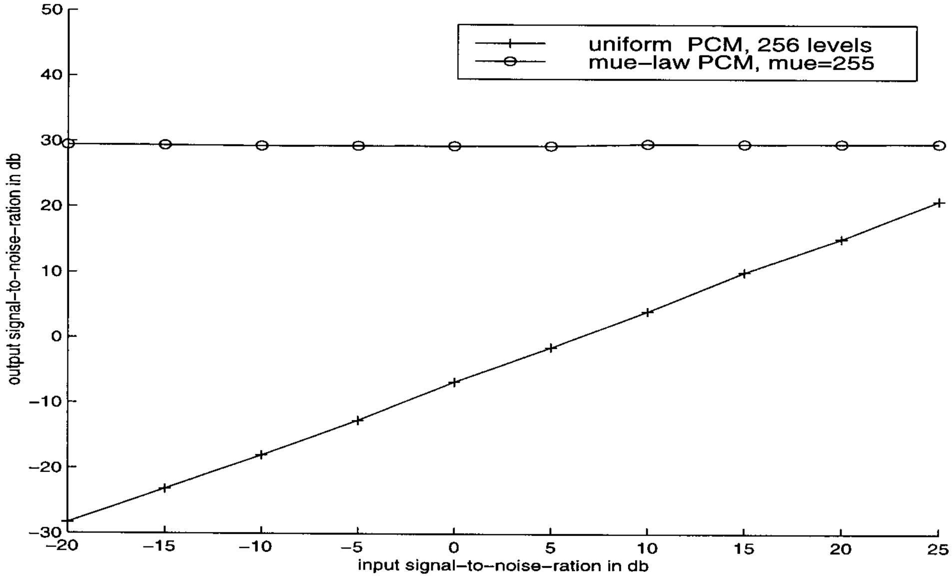


Figure 1 . input signal-to-noise ratio Vs. output signal-to-noise ratio for μ-law PAM and uniform PCM

Problem 3.40

Matlab codes

```
% Problem 3.40, CS: Haykin
%Normalized LMS- prediction
%of AR process/ speech signal
% Mathini Sellathurai

clear all
mue=0.05; % step size parameter, a value between 0 ans 2
p=2; % filter order
N=10; % size of data
M=1;% number of realizations

% initializing counters
err1=zeros(1,N-p);
xhat1=zeros(1,N-p);
x=zeros(1,N);

for m=1:M % 100 realizations

x(1:2)= [0.1 0.2];

%AR process
for k=3:N
x(k)=(0.8*x(k-1)-0.1*x(k-2))+0.1*rand(1);
end

% LMS prediction
[err, xhat]=LMS(x,mue,p);
err1=err1+err.^2;
xhat1=xhat1+xhat;
end

plot(err1/m,'-');
```

```
function [err, xhat]=LMS(xx,mue,p)
% function Normalized LMS
%p-order of the filter
%mue-step size parameter
%used in problem 3.40, CS: Haykin
%Mathini Sellathurai

% length of the data
N=length(xx);

% initializing weights and erros
w=zeros(p,N-p);
err=ones(1,N-p);
xhat=zeros(1,N-p);

%prediction
 l=1;
        for k=1:N-p
                h=xx(k:p+k-1);
                err(l)=(xx(k+p)-h*w(:,l));
                xhat(l)=h*w(:,l);
                xxx=xx(l+p-1)+xx(l+p-2);
                w(:,l+1)=w(:,l)+(mue/xxx)*h'*err(l);
              l=l+1;
        end
```

Answer to Problem 3.40

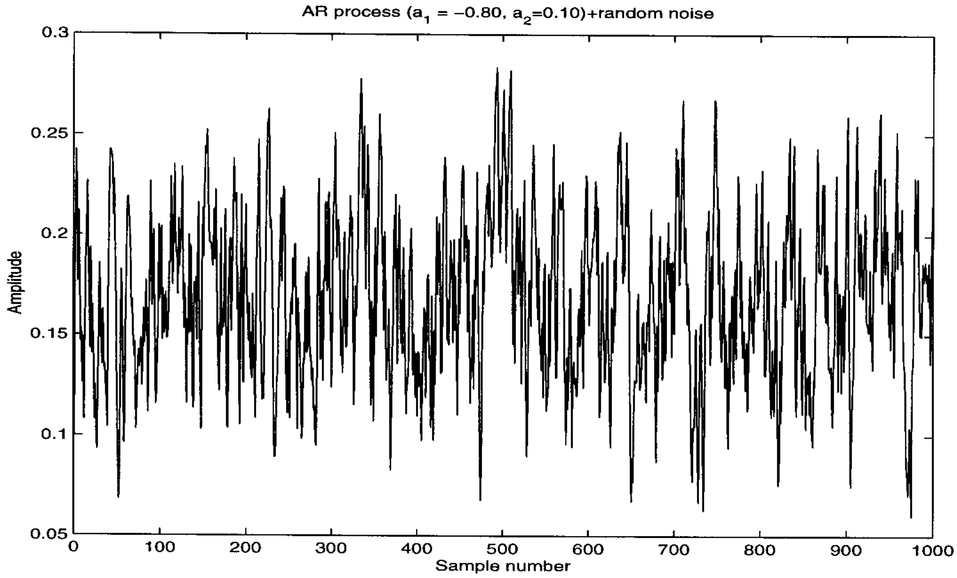


Figure 1 : Noisy-AR-process, $a_0 = -0.80$, $a_1 = 0.10$

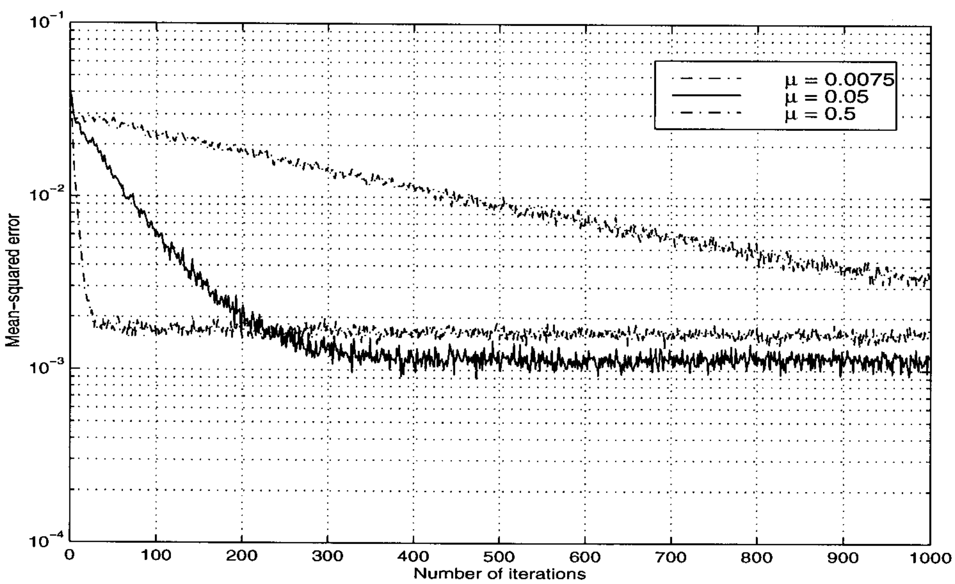


Figure 2 : Learning curves for $\mu = 0.0075, 0.05, 0.5$