CHAPTER 4

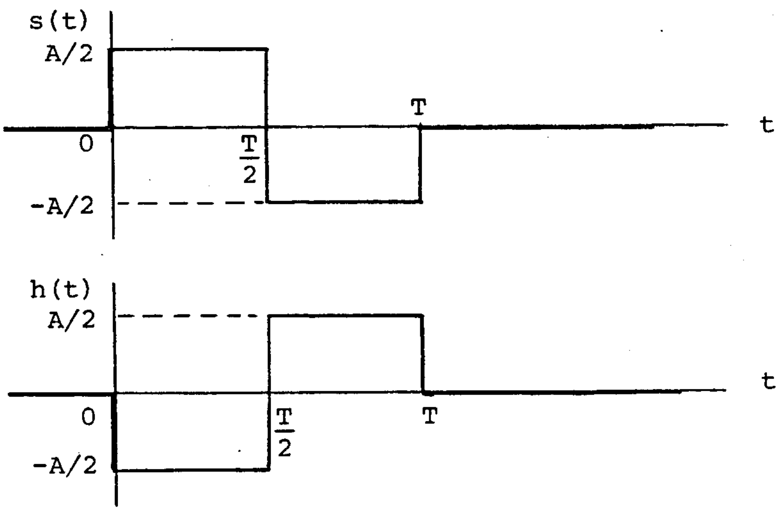
Baseband Pulse Transmission

Problem 4.1

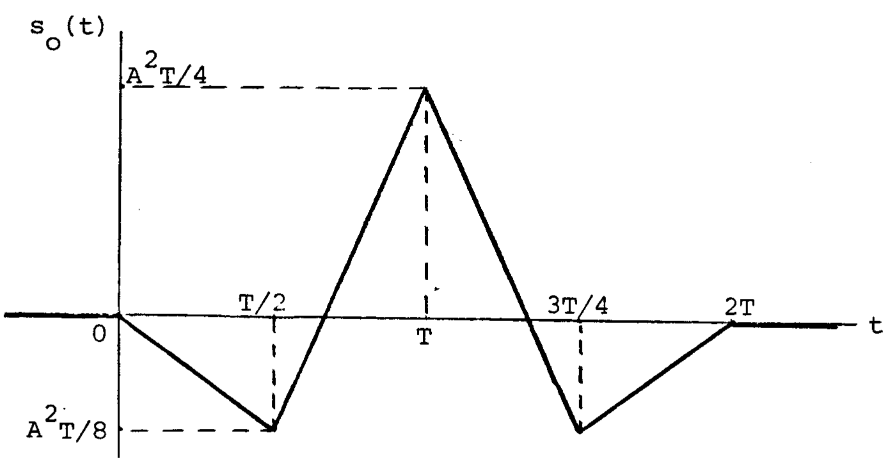
(a) The impulse response of the matched filter is

$h(t) = s(T-t)$

The s(t) and h(t) are shown below:



(b) The corresponding output of the matched filter is obtained by convolving h(t) with s(t). The result is shown below:



(c) The peak value of the filter output is equal to $A^2T/4$, occuring at t=T.

Problem 4.2

(a) The matched filter of impulse response $h_1(t)$ for pulse $s_1(t)$ is given in the solution to Problem 4.1. The matched filter of impulse response $h_2(t)$ for $s_2(t)$ is given by

$$h_2 = s_2(T-t)$$

which has the following waveform:

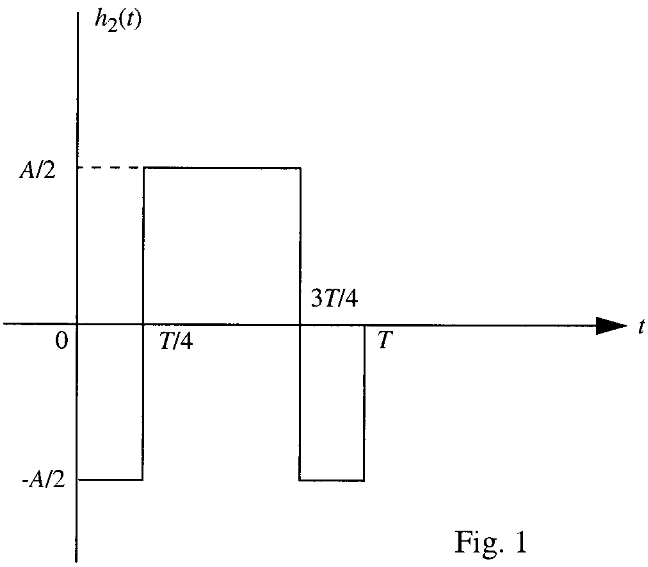


Fig. 1

(b) (i) The response of the matched filter, matched to $s_2(t)$ and due to $s_1(t)$ as input, is obtained by convolving $h_2(t)$ with $s_1(t)$, as shown by

$$y_{21}(t) = \int_0^T s_1(\tau)h_2(t-\tau)d\tau$$

The waveform of the output $y_{21}(t)$ so computed is plotted in Figure 2. This figure also includes the corresponding waveforms of input $s_1(t)$ and impulse response $h_2(t)$.

(ii) Next, the response of the matched filter, matched to $s_1(t)$ and due to $s_2(t)$ as input, is obtained by convolving $h_1(t)$ with $s_2(t)$, as shown by

$$y_{12}(t) = \int_0^T s_2(\tau)h_1(t-\tau)d\tau$$

Figure 3 shows the waveforms of input $s_2(t)$, impulse response $h_1(t)$, and response $y_{12}(t)$.

4.2(b)

(i)

Pulse $s_1(t)$

Filter $h_2(t)$

$h_2(t) = s_2(T-t)$

Filter response

$$\int_0^T s_1(\tau)\, h_2(t-\tau)\, d\tau$$

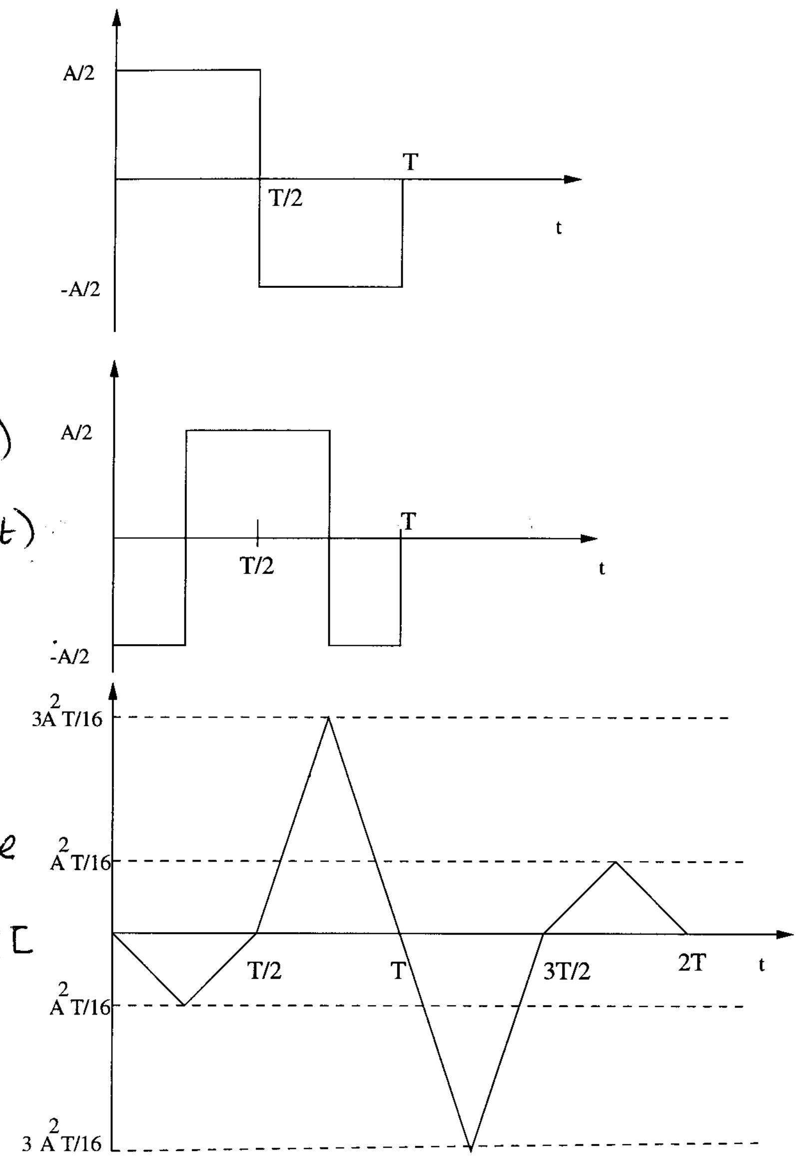


Fig. 3

4.2(b)

(ii)

Pulse $s_2(t)$

Filter $h_1(t)$

Filter matched to Pulse $s_1(t)$

Filter response

$$\int_0^T s_2(\tau) h_1(t-\tau)\, d\tau$$

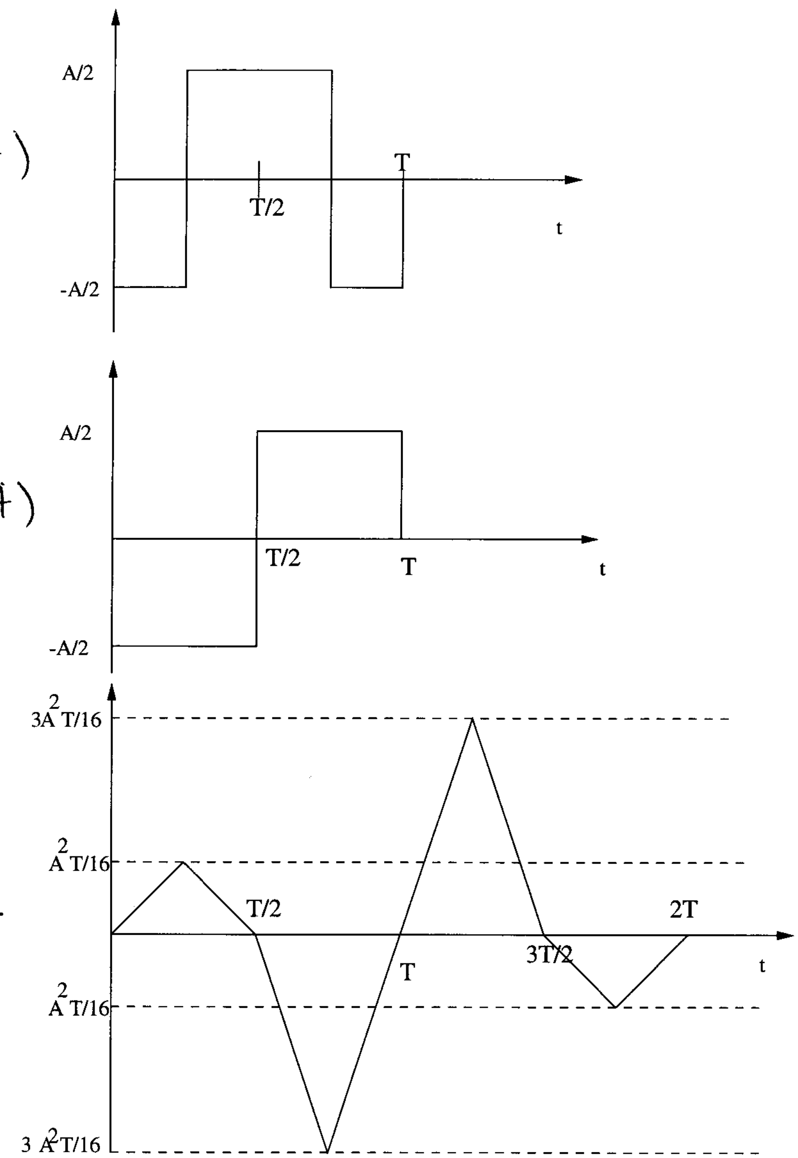


Fig. 3

Note that $y_{12}(t)$ is exactly the negative of $y_{21}(t)$. However, in both cases we find that at $t = T$, both outputs are equal to zero, as shown by

$$y_{21}(T) = y_{12}(T) = 0$$

For n pulses $s_1(t)$ $s_2(t),\ldots,s_n(t)$ that are orthogonal to each other over the interval [0,T], the n-dimensional matched filter has the following structure:

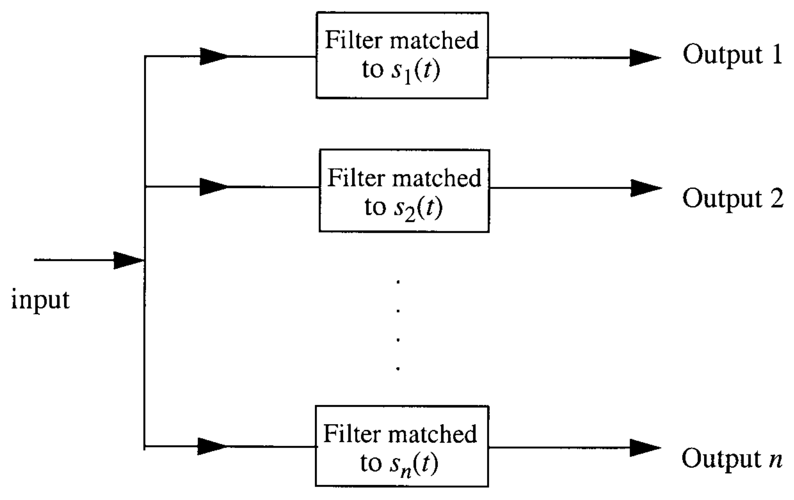


Fig. 4

Problem 4.3

Ideal low-pass filter with variable bandwidth. The transfer function of the matched filter for a rectangular pulse of duration τ and amplitude A is given by

$$H_{opt}(f) = \text{sinc}(fT)\exp(-j\pi fT) \qquad (1)$$

The amplitude response $|H_{opt}(f)|$ of the matched filter is plotted in Fig. 1(a). We wish to approximate this amplitude response with an ideal low-pass filter of bandwidth B. The amplitude response of this approximating filter is shown in Fig. 1(b). The requirement is to determine the particular value of bandwidth B that will provide the best approximation to the matched filter.

We recall that the maximum value of the output signal, produced by an ideal low-pass filter in response to the rectangular pulse occurs at $t = T/2$ for $BT \leq 1$. This maximum value, expressed in terms of the sine integral, is equal to $(2A/\pi)\text{Si}(\pi BT)$. The average noise power at the output of the ideal low-pass filter is equal to BN_0. The maximum output signal-to-noise ratio of the ideal low-pass filter is therefore

$$(\text{SNR})_0' = \frac{(2A/\pi)^2 \text{Si}^2(\pi BT)}{BN_0} \qquad (2)$$

Thus, using Eqs. (1) and (2), and assuming that AT = 1, we get

$$\frac{(\text{SNR})_0'}{(\text{SNR})_0} = \frac{2}{\pi^2 BT}\text{Si}^2(\pi BT)$$

This ratio is plotted in Fig. 2 as a function of the time-bandwidth product BT. The peak value on this curve occurs for BT = 0.685, for which we find that the maximum signal-to-noise ratio of the ideal low-pass filter is 0.84 dB below that of the true matched filter. Therefore, the " best" value for the bandwidth of the ideal low-pass filter characteristic of Fig. 1(b) is B = 0.685/T.

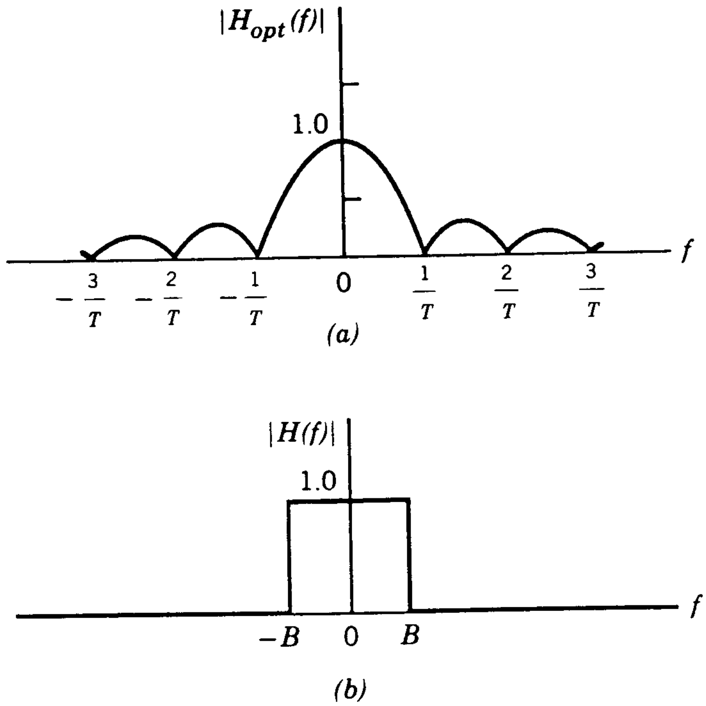


Figure 1

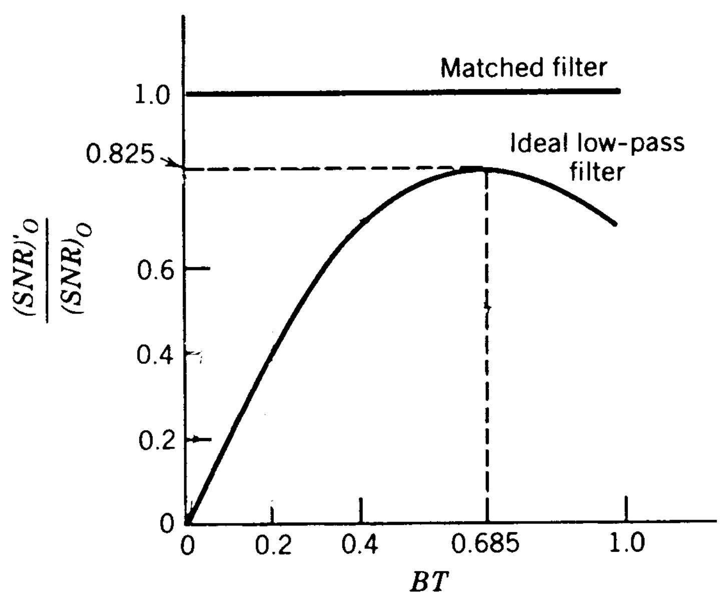
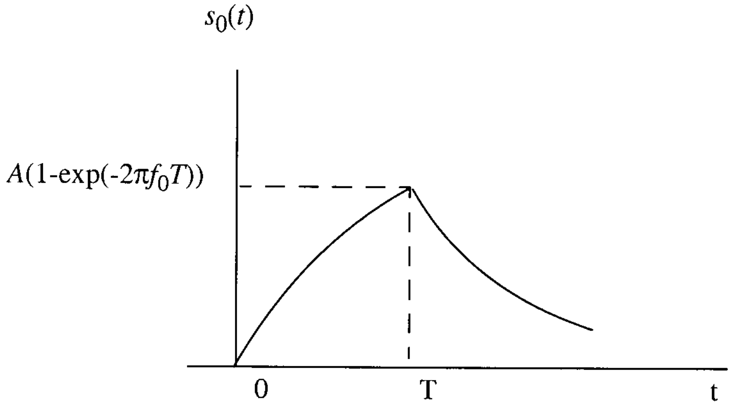


Figure 2

Problem 4.4

The output of the low-pass RC filter, produced by a rectangular pulse of amplitude A and duration T, is as shown below:



The peak value of the output pulse power is

$$P_{\text{out}} = A^2[1 - \exp(-2\pi f_0 T)]^2$$

where f_0 is the 3-dB cutoff frequency of the RC filter.

The average output noise power is

$$N_{\text{out}} = \frac{N_0}{2}\int_{-\infty}^{\infty} \frac{df}{1 + (f/f_0)^2}$$

$$= \frac{N_0 \pi f_0}{2}$$

The corresponding value of the output signal-to-noise ratio is therefore

$$(\text{SNR})_{\text{out}} = \frac{2A^2}{N_0 \pi f_0}[1 - \exp(-2\pi f_0 T)]$$

Differentiating $(\text{SNR})_0$ with respect to $f_0 T$ and setting the result equal to zero, we find that $(\text{SNR})_{\text{out}}$ attains its maximum value at

$$f_0 = \frac{0.2}{T}$$

The corresponding maximum value of $(\text{SNR})_{\text{out}}$ is

$$(\text{SNR})_{0,\max} = \frac{2A^2T}{0.2\pi N_0}[1-\exp(-0.4\pi)]^2$$

$$= \frac{1.62A^2T}{N_0}$$

For a perfect matched filter, the output signal-to-noise ratio is

$$(\text{SNR})_{0,\text{matched}} = \frac{2E}{N_0}$$

$$= \frac{2A^2T}{N_0}$$

Hence, we find that the transmitted energy must be increased by the ratio 2/1.62, that is, by 0.92 dB so that the low-pass RC filter with $f_0 = 0.2/T$ realizes the same performance as a perfectly matched filter.

Problem 4.5

(i) $p_0 > p_1$

The transmitted symbol is more likely to be 0. Hence, the average probability of symbol error is smaller when a 0 is transmitted than when a 1 is transmitted. In such a situation, the threshold λ in Figs. 4.5(a) and (b) in the textbook is moved to the right.

(ii) $p_1 > p_0$

The transmitted symbol is more likely to be 1. Hence, the average probability of symbol error is smaller when a 1 is transmitted than when a 0 is transmitted. In this second situation, the threshold λ in Figs. 4.5(a) and (b) in the textbook is moved to the left.

Problem 4.6

The average probability of error is

$$P_e = p_1 \int_{-\infty}^{\lambda} f_Y(y|1)dx + p_0 \int_{\lambda}^{\infty} f_Y(y|0)dx \tag{1}$$

An optimum choice of λ corresponds to minimum P_e. Differentiating Eq. (1) with respect to λ, we get:

$$\frac{\partial P_e}{\partial \lambda} = p_1 f_Y(\lambda|1) - p_0 f_Y(\lambda|0)$$

Setting $\frac{\partial P_e}{\partial \lambda} = 0$, we get the following condition for the optimum value of λ:

$$\frac{f_Y(\lambda_{opt}|1)}{f_Y(\lambda)_{opt}|0)} = \frac{p_0}{p_1}$$

which is the desired result.

Problem 4.7

In a binary PCM system, with NRZ signaling, the average probability of error is

$$P_e = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

The signal energy per bit is

$$E_b = A^2 T_b$$

where A is the pulse amplitude and T_b is the bit (pulse) duration. If the signaling rate is doubled, the bit duration T_b is reduced by half. Correspondingly, E_b is reduced by half.

Let $u = \sqrt{E_b/N_0}$. We may then set

$$P_e = 10^{-6} = \frac{1}{2}\,\text{erfc}(u)$$

Solving for u, we get

$$u = 3.3$$

When the signaling rate is doubled, the new value of P_e is

$$P_e' = \frac{1}{2}\,\text{erfc}\left(\frac{u}{\sqrt{2}}\right)$$

$$= \frac{1}{2}\,\text{erfc}(2.33)$$

$$= 10^{-3}.$$

Problem 4.8

(a) The average probability of error is

$$P_e = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

where $E_b = A^2T_b$. We may rewrite this formula as

$$P_e = \frac{1}{2}\,\text{erfc}\left(\frac{A}{\sigma}\right) \qquad (1)$$

where A is the pulse amplitude at $\sigma = \sqrt{N_0T_b}$. We may view σ^2 as playing the role of noise variance at the decision device input. Let

$$u = \sqrt{\frac{E_b}{N_0}} = \frac{A}{\sigma}$$

We are given that

$$\sigma^2 = 10^{-2} \text{ volts}^2, \qquad \sigma = 0.1 \text{ volt}$$

$$P_e = 10^{-8}$$

Since P_e is quite small, we may approximate it as follows:

$$\text{erfc}(u) \approx \frac{\exp(-u^2)}{\sqrt{\pi}\,u}$$

We may thus rewrite Eq. (1) as (with $P_e = 10^{-8}$)

$$\frac{\exp(-u^2)}{2}\sqrt{\pi}u = 10^{-8}$$

Solving this equation for u, we get

$$u = 3.97$$

The corresponding value of the pulse amplitude is

$$A = \sigma u = 0.1 \times 3.97$$

$$= 0.397 \text{volts}$$

(b) Let σ^2_i denote the combined variance due to noise and interference; that is

$$\sigma_T^2 = \sigma^2 + \sigma_i^2$$

where σ^2 is due to noise and σ^2_i is due to the interference. The new value of the average probability of error is 10^{-6}. Hence

$$10^{-6} = \frac{1}{2}\,\text{erfc}\left(\frac{A}{\sigma_T}\right) \quad (2)$$

$$= \frac{1}{2}\,\text{erfc}(u_T)$$

where

$$u_T = \frac{A}{\sigma_T}$$

Equation (2) may be approximated as (with $P_e = 10^{-6}$)

$$\frac{\exp(-u_T^2)}{2\sqrt{\pi}\, u_T} \approx 10^{-6}$$

Solving for u_T, we get

$$u_T = 3.37$$

The corresponding value of $\sigma^2{}_T$ is

$$\sigma_T^2 = \left(\frac{A}{u_T}\right)^2 = \left(\frac{0.397}{3.37}\right)^2 = 0.0138 \text{ volts}^2$$

The variance of the interference is therefore

$$\begin{aligned} \sigma_i^2 &= \sigma_T^2 - \sigma^2 \\ &= 0.0138 - 0.01 \\ &= 0.0038 \text{ volts}^2 \end{aligned}$$

Problem 4.9

Consider the performance of a binary PCM system in the presence of channel noise; the receiver is depicted in Fig. 1. We do so by evaluating the average probability of error for such a system under the following assumptions:

1. The PCM system uses an on-off format, in which symbol 1 is represented by A volts and symbol 0 by zero volt.
2. The symbols 1 and 0 occur with equal probability.
3. The channel noise w(t) is white and Gaussian with zero mean and power spectral density $N_0/2$.

To determine the average probability of error, we consider the two possible kinds of error separately. We begin by considering the first kind of error that occurs when symbol 0 is sent and the receiver chooses symbol 1. In this case, the probability of error is just the probability that the correlator output in Fig. 1 will exceed the threshold λ owing to the presence of noise, so the transmitted symbol 0 is mistaken for symbol 1. Since the a priori probabilities of symbols 1 and 0 are equal, we have $p_0 = p_1$ Correspondingly, the expression for the threshold λ simplifies as follows:

$$\lambda = \frac{A^2T_b}{2} \tag{1}$$

where T_b is the bit duration, and A^2T_b is the signal energy consumed in the transmission of symbol 1. Let y denote the correlator output:

$$y = \int_0^{T_b} s(t)x(t)dt \tag{2}$$

Under hypothesis H_0, corresponding to the transmission of symbol 0, the received signal x(t) equals the channel noise w(t). Under this hypothesis we may therefore describe the correlator output as

$$H_0 : y = A \int_0^{T_b} w(t)dt \tag{3}$$

Since the white noise w(t) has zero mean, the correlator output under hypothesis H_0 also has zero mean. In such a situation, we speak of a *conditional mean*, which (for the situation at hand) we describe by writing

$$\mu_0 = E[Y \mid H_0] = E\left[\int_0^{T_b} W(t)dt \right] = 0 \tag{4}$$

where the random variable Y represents the correlator output with y as its sample value and W(t) is a white-noise process with w(t) as its sample function. The subscript 0 in the conditional mean μ_0 refers to the condition that hypothesis H_0 is true. Correspondingly, let $\sigma^2{}_0$ denote the *conditional variance* of the correlator output, given that hypothesis H_0 is true. We may therefore write

$$\begin{aligned} \sigma_0^2 &= E[Y^2 \mid H_0] \\ &= E\left[\int_0^{T_b} \int_0^{T_b} W(t_1)W(t_2)dt_1\, dt_2 \right] \end{aligned} \tag{5}$$

The double integration in Eq. (5) accounts for the squaring of the correlator output. Interchanging the order of integration and expectation in Eq. (5), we may write

$$\begin{aligned} \sigma_0^2 &= \int_0^{T_b} \int_0^{T_b} E[W(t_1)W(t_2)]dt_1\, dt_2 \\ &= \int_0^{T_b} \int_0^{T_b} R_w(T_1 - t_2)dt_1\, dt_2 \end{aligned} \tag{6}$$

The parameter $R_w(t_1 - t_2)$ is the *ensemble-averaged autocorrelation function* of the white-noise process W(t). From random process theory, it is recognized that the autocorrelation function and power spectral density of a random process form a Fourier transform pair. Since the white-noise process W(t) is assumed to have a constant power spectral density of $N_0/2$, it follows that the autocorrelation function of such a process consists of a delta function weighted by $N_0/2$. Specifically, we may write

$$R_w(t_1 - t_2) = \frac{N_0}{2}\,\delta(\tau - t_1 + t_2) \tag{7}$$

Substituting Eq. (7) in (6), and using the property that the total area under the Dirac delta function $\delta(\tau - t_1 + t_2)$ is unity, we get

$$\sigma_0^2 = \frac{N_0 T_b A^2}{2} \tag{8}$$

The statistical characterization of the correlator output is computed by noting that it is Gaussian distributed, since the white noise at the correlator input is itself Gaussian (by assumption). In summary, we may state that under hypothesis H_0 the correlator output is a Gaussian random variable with zero mean and variance $N_0T_bA^2/2$, as shown by

$$f_0(y) = \frac{1}{\sqrt{\pi N_0 T_b}\, A} \exp\left(- \frac{y^2}{N_0 T_b A^2} \right) \tag{9}$$

where the subscript in $f_0(y)$ signifies the condition that symbol 0 was sent.

Figure 2(a) shows the bell-shaped curve for the probability density function of the correlator output, given that symbol 0 was transmitted. The probability of the receiver deciding in favor of symbol 1 is given by the area shown shaded in Fig. 2(a). The part of the y-axis covered by this area corresponds to the condition that the correlator output y is in excess of the threshold λ defined by Eq. (1). Let P_{e0} denote the *conditional probability of error, given that symbol 0 was sent.* Hence, we may write

$$\begin{aligned} p_{10} &= \int_{\lambda}^{\infty} f_0(y)\, dy \\ &= \frac{1}{\sqrt{\pi N_0 T_b}\, a} \int_{A^2T_b/2}^{\infty} \exp\left(- \frac{y^2}{N_0 T_b A^2} \right) dy \end{aligned} \tag{10}$$

Define

$$z = \frac{y}{\sqrt{N_0 T_b}\, A} \tag{11}$$

We may then rewrite Eq. (10) in terms of the new variable z as

$$p_{10} = \frac{1}{\sqrt{\pi}} \int_{\sqrt{A^2T_b/2N_0}}^{\infty} \exp(-z^2)\, dz \tag{12}$$

which may be reformulated in terms of

complementary error function

$$\text{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^\infty \exp(-z^2)\, dz \tag{13}$$

Accordingly, we may redefine the conditional probability of error P_{e0} as

$$p_{10} = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{A^2T_b}{4N_0}}\right) \tag{14}$$

Consider next the second kind of error that occurs when symbol 1 is sent and the receiver chooses symbol 0. Under this condition, corresponding to hypothesis H_1, the correlator input consists of a rectangular pulse of amplitude A and duration T_b plus the channel noise w(t). We may thus apply Eq. (2) to write

$$H_1 : y = A \int_0^{T_b} [A + w(t)]\, dt \tag{15}$$

The fixed quantity A in the integrand of Eq. (15) serves to shift the correlator output from a mean value of zero volt under hypothesis H_0 to a mean value of A^2T_b under hypothesis H_1. However, the conditional variance of the correlator output under hypothesis H_1 has the same value as that under hypothesis H_0. Moreover, the correlator output is Gaussian distributed as before. In summary, the correlator output under hypothesis H_1 is a Gaussian random variable with mean A^2T_b and variance $N_0T_b^2/2$, as depicted in Fig. 2(b), which corresponds to those values of the correlator output less than the threshold λ set at $A^2T_b/2$, From the symmetric nature of the Gaussian density function, it is clear that

$$p_{01} = p_{10} \tag{16}$$

Note that this statement is only true when the a priori probabilities of binary symbols 0 and 1 are equal; this assumption was made in calculating the threshold λ.

To determine the average probability of error of the PCM receiver, we note that the two possible kinds of error just considered are mutually exclusive events. Thus, with the a priori probability of transmitting a 0 equal to p_0, and the a priori probability of transmitting a 1 equal to p_1, we find that the *average probability of error*, P_e, is given by

$$P_e = p_0 p_{10} + p_1 p_{01} \tag{17}$$

Since $p_{01} = p_{10}$, and $p_0 + p_1 = 1$, Eq. (17) simplifies as

$$P_e = p_{10} = p_{01}$$

or

$$P_e = \frac{1}{2} \operatorname{erfc}\left(\frac{1}{2} \sqrt{\frac{A^2 T_b}{N_0}} \right) \tag{18}$$

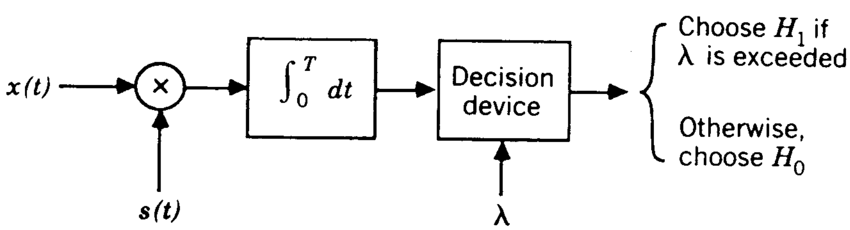


Figure 1

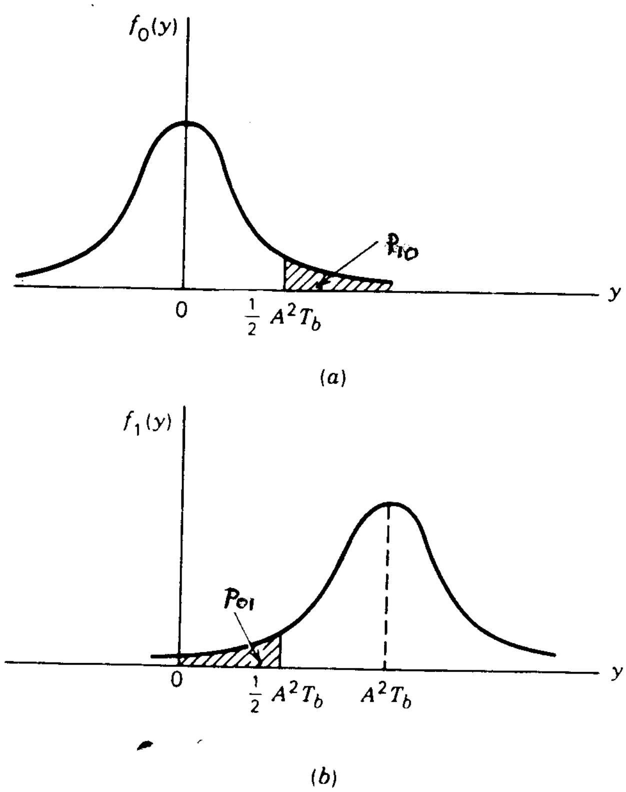


Figure 2

Problem 4.10

For unipolar RZ signaling, we have

Binary symbol 1: $s(t) = +A$ for $0 < t \leq T/2$
and $s(t) = 0$ for $T/2 < t \leq T$

Binary symbol 0: $s(t) = 0$ for $0 < t \leq T$

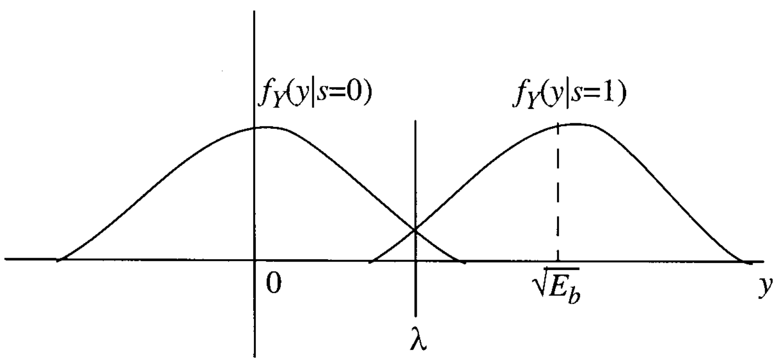
The a priori probabilities of symbols 1 and 0 are assumed to be equal, in which case we have $p_0 = p_1 = 1/2$.

To determine the average probability of error, we consider the two possible kinds of error separately. We begin by considering the first kind of error that occurs when symbol 0 is sent and the receiver chooses symbol 1. In this case, the probability of error is just the probability that the matched filter output will exceed the threshold λ owing to the presence of noise, so the transmitted symbol 0 is mistaken for symbol 1.

$$\text{Energy of symbol 1} = \frac{A^2 T_b}{2} = E_b$$

Energy of symbol 0 = 0

The conditional probability density function of the two signals is given below:



With symbols 1 and 0 assumed to be equiprobable, the optimum threshold is

$$\lambda = \frac{1}{2}\sqrt{E_b} = \frac{1}{2}\sqrt{\frac{A^2 T_b}{2}}$$

Given that symbol 0 was transmitted, the probability of error is simply the probability that $y > \lambda$, as shown by

$$P(\text{error}|0) = \int_{-\infty}^{\infty} f_Y(y|0)dy$$

$$= \frac{1}{\sqrt{\pi N_0}}\int_{-\infty}^{\infty} \exp\left(-\frac{y^2}{N_0}\right)dy$$

Define a new variable z as

$$z = \frac{y}{\sqrt{N_0}}$$

We then have

$$P(\text{error}|0) = \frac{1}{\sqrt{\pi}}\int_{\lambda/\sqrt{N_0}}^{\infty} \exp(-z^2)dz$$

$$= \frac{1}{2}\text{erfc}\left(\frac{\lambda}{\sqrt{N_0}}\right)$$

$$= \frac{1}{2}\text{erfc}\left(\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right)$$

$$= \frac{1}{2}\text{erfc}\left(\frac{1}{2}\sqrt{\frac{A^2T_b}{2N_0}}\right)$$

Similarly, $P(\text{error}|1) = \int_{-\infty}^{\lambda} f_Y(y|(1)dy$

$$= \frac{1}{\sqrt{\pi N_0}}\int_{-\infty}^{\lambda} \exp\left[-\frac{(y-\sqrt{E_b})^2}{N_0}\right]dy$$

Define $z = \dfrac{\sqrt{E_b}-y}{N_0}$, and so write

$$P(\text{error}|1) = \frac{1}{\sqrt{\pi}}\int_{\frac{\sqrt{E_b}-\lambda}{\sqrt{N_0}}}^{\infty} \exp(-z^2)dz$$

$$P(\text{error}|1) = \frac{1}{2}\text{erfc}\left(\frac{\sqrt{E_b} - \lambda}{\sqrt{N_0}}\right)$$

$$= \frac{1}{2}\text{erfc}\left(\frac{\sqrt{E_b}}{2\sqrt{N_0}}\right)$$

$$= \frac{1}{2}\text{erfc}\left(\frac{1}{2}\sqrt{\frac{A^2 T_b}{2N_0}}\right)$$

The average probability of error is therefore

$$P_e = P(1)P(\text{error}|1) + P(\text{error}|0)P(0)$$

$$= \frac{1}{2}\text{erfc}\left(\frac{1}{2}\sqrt{E_b/N_0}\right)$$

$$= \frac{1}{2}\text{erfc}\left(\frac{1}{2}\sqrt{\frac{A^2 T_b}{2N_0}}\right) \qquad (1)$$

The average probability of error for on-off (i.e., unipolar NRZ) type of encoded signals is

$$\frac{1}{2}\text{erfc}\left(\frac{1}{2}\sqrt{\frac{A^2 T_b}{N_0}}\right)$$

Comparing this result with that of Eq. (1) for the unipolar RZ type of encoded signals, we immediately see that, for a prescribed noise spectral density N_0, the symbol energy in unipolar RZ signaling has to be doubled in order to achieve the same average probability of error as in unipolar NRZ signaling.

Problem 4.11

Probability of error for bipolar NRZ signal

Binary symbol 1 : $s(t) = \pm A$
Binary symbol 0: $s(t) = 0$
Energy of symbol 1 = $E_b = A^2 T_b$

The absolute value of the threshold is $\lambda = \frac{1}{2}\sqrt{E_b} = \frac{1}{2}\sqrt{A^2 T_b}$.

Refering to Fig. 1 on the next page, we may write

$$P(\text{error}|\text{s=-A}) = \frac{1}{\sqrt{\pi N_0}}\int_{-\lambda}^{\lambda} \exp\left[\frac{-(y+\sqrt{E_b})^2}{N_0}\right] dy$$

Let $z = \frac{y+\sqrt{E_b}}{\sqrt{N_0}}$

Then,

$$P(\text{error}|s=-A) = \frac{1}{\sqrt{\pi}}\int_{-\frac{\lambda+\sqrt{E_b}}{\sqrt{N_0}}}^{\frac{\lambda+\sqrt{E_b}}{\sqrt{N_0}}} \exp(-z^2)dz$$

$$= \frac{1}{2}\left[\text{erfc}\left(\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right) - \text{erfc}\left(\frac{3}{4}\sqrt{\frac{E_b}{N_0}}\right)\right]$$

Similarly, $P(\text{error}|s=+A) = P(\text{error}|s=-A)$

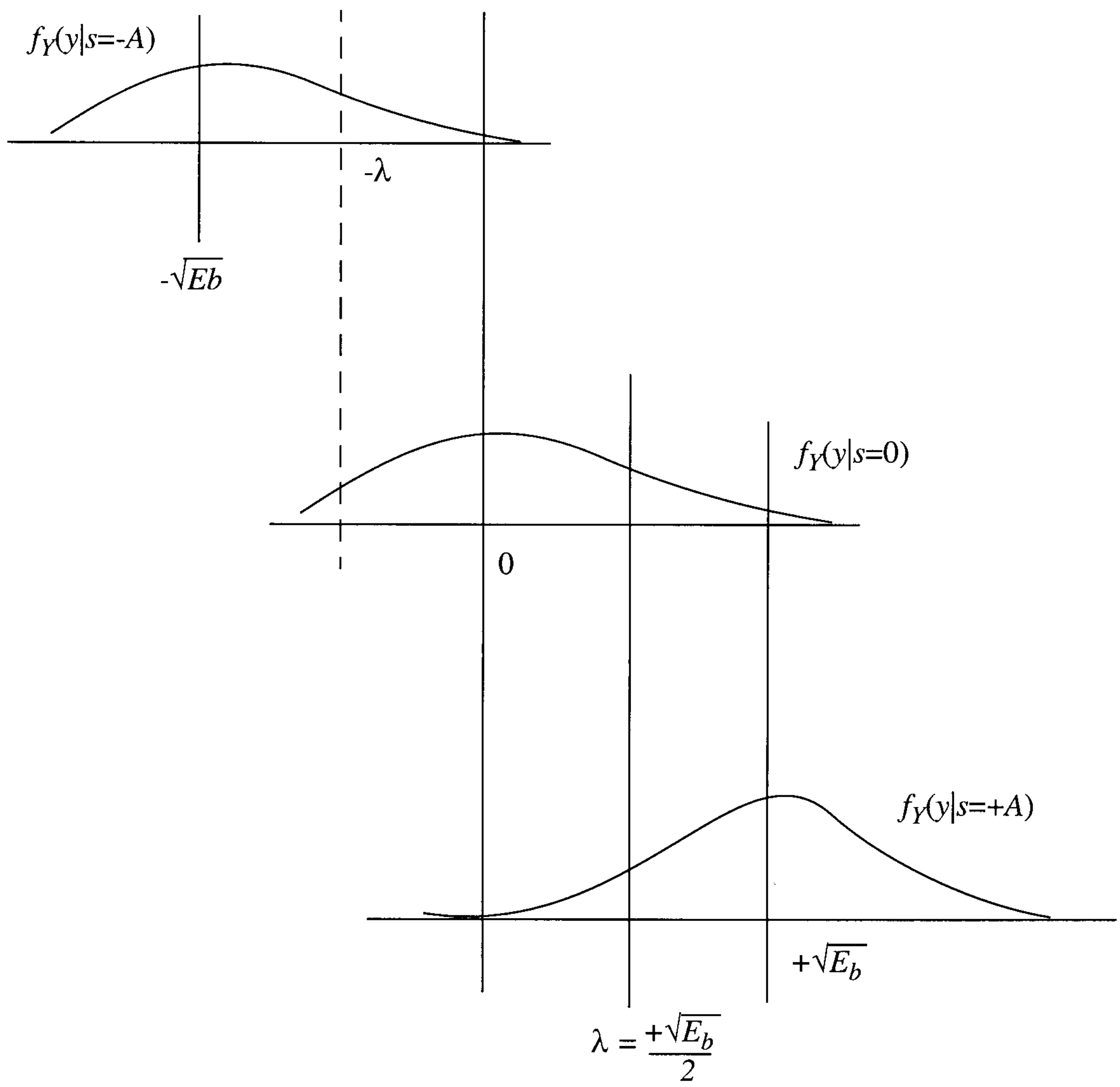
$$P(\text{error}|s=0) = \frac{2\times 1}{\sqrt{\pi N_0}}\int_{\lambda}^{\infty} \exp\left(\frac{-y^2}{N_0}\right) dy$$

$$= \text{erfc}\left(\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right)$$

The average probability of error is therefore

$$P_e = P(s=\pm A)P(\text{error}|s=\pm A) + P(s=0)P(\text{error}|s=0)$$

The conditional probability density functions of symbols 1 and 0 are given in Fig. 1:



$f_Y(y|s\text{=-}A)$
$-\lambda$
$-\sqrt{Eb}$
$f_Y(y|s\text{=}0)$
0
$f_Y(y|s\text{=+}A)$
$+\sqrt{E_b}$
$\lambda = \frac{+\sqrt{E_b}}{2}$

Figure 1

$$P_e = \frac{1}{2} \times \frac{1}{2}\left[\operatorname{erfc}\left(\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right) - \operatorname{erfc}\left(\frac{3}{4}\sqrt{\frac{E_b}{N_0}}\right)\right] + \frac{1}{2}\operatorname{erfc}\left(\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right)$$

$$= \frac{3}{4}\operatorname{erfc}\left(\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right) - \frac{1}{4}\operatorname{erfc}\left(\frac{3}{4}\sqrt{\frac{E_b}{N_0}}\right)$$

Problem 4.12

The rectangular pulse given in Fig. P4.12 is defined by

$g(t) = \text{rec}(t/T)$

The Fourier transform of $g(t)$ is given by

$$G(f) = \int_{-T/2}^{T/2} \exp(-j2\pi ft)dt$$

$$= T\,\text{sinc}\,(fT)$$

We thus have the Fourier-transform pair

$$\text{rec}(t/T) \rightleftharpoons T\,\text{sinc}\,(fT)$$

The magnitude spectrum $|G(f)|/T$ is plotted as the solid line in Fig. 1, shown on the next page.

Consider next a Nyquist pulse (raised cosine pulse with a rolloff factor of zero). The magnitude spectrum of this second pulse is a rectangular function of frequency, as shown by the dashed curve in Fig. 1.

Comparing the two spectral characteristics of Fig. 1, we may say that the rectangular pulse of Fig. P4.12 provides a crude approximation to the Nyquist pulse.

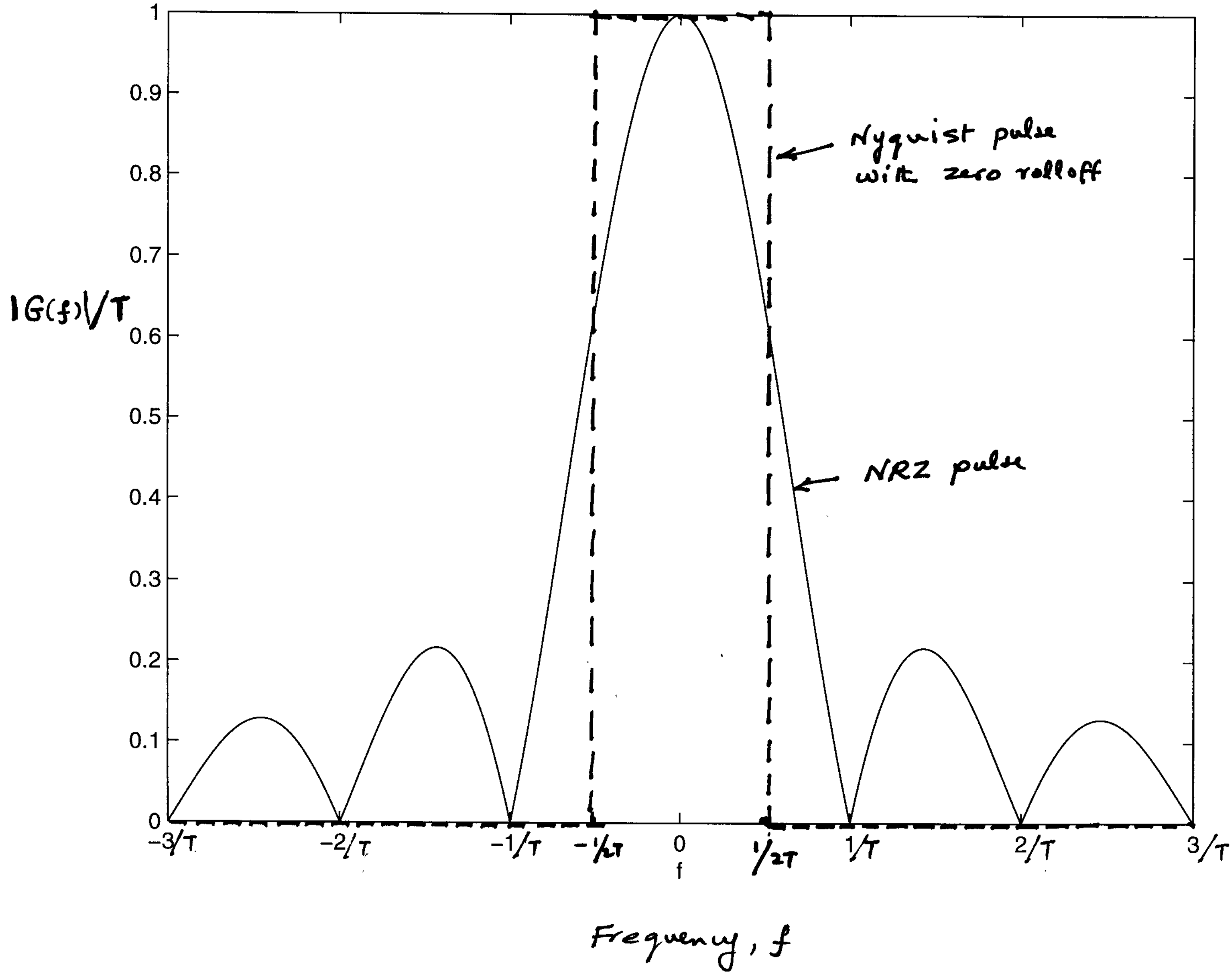


Figure 1 Spectral characteristics

Problem 4.13

Since P(f) is an even real function, its inverse Fourier transform equals

$$p(t) = 2\int_0^{\infty} P(f)\cos(2\pi ft)\,df \tag{1}$$

The P(f) is itself defined by Eq. (7.60) which is reproduced here in the form

$$P(f) = \begin{cases} \dfrac{1}{2W}, & 0 < |f| < f_1 \\ \dfrac{1}{4W}\left\{1 + \cos\left[\dfrac{\pi(|f| - f_1)}{2W - 2f_1}\right]\right\}, & f_1 < f < 2W - f_1 \\ 0, & |f| > 2W - f_1 \end{cases} \tag{2}$$

Hence, using Eq. (2) in (1):

$$p(t) = \frac{1}{W}\int_0^{f_1}\cos(2\pi ft)\,df + \frac{1}{2B}\int_{f_1}^{2W-f_1}\left[1 + \cos\left(\frac{\pi(f-f_1)}{2W\alpha}\right)\right]\cos(2\pi ft)\,df$$

$$= \left[\frac{(\sin(2\pi ft)}{2\pi Wt}\right] + \left[\frac{\sin(2\pi ft)}{4\pi Wt}\right]_{f_1}^{2W-f_1}$$

$$+ \frac{1}{4}W\left[\frac{\sin\left(2\pi ft + \dfrac{\pi(f-f_1)}{2W\alpha}\right)}{2\pi t + \pi/2W\alpha}\right]_{f_1}^{2W-f_1} + \frac{1}{4W}\left[\frac{\sin\left(2\pi ft - \dfrac{\pi(f-f_1)}{2W\alpha}\right)}{2\pi t - \pi/2W\alpha}\right]_{f_1}^{2W-f_1}$$

$$= \frac{\sin(2\pi f_1 t)}{4\pi Wt} + \frac{\sin[2\pi t(2W-f_1)]}{4\pi Wt}$$

$$- \frac{1}{4W}\,\frac{\sin(2\pi f_1 t) + \sin[2\pi t(2W-f_1)]}{2\pi t - \pi/2W\alpha} + \frac{\sin(2\pi f_1 t) + \sin[2\pi t(2W-f_1 t)]}{2\pi t - \pi/2W\alpha}$$

$$= \frac{1}{W}\left[\sin(2\pi f_1 t) + \sin[2\pi t(2W-f_1)]\right]\left[\frac{1}{4\pi t} - \frac{\pi t}{(2\pi t)^2 - (\pi/2W\alpha)^2}\right]$$

$$= \frac{1}{W}[\sin(2\pi Wt)\cos(2\pi\alpha W)]\left[\frac{-(\pi/2W\alpha)^2}{4\pi t\,[(2\pi t)^2 - (\pi/2W\alpha)^2}\right]$$

$$= \text{sinc}(2Wt)\cos(2\pi\alpha Wt)\left[\frac{1}{1 - 16\,\alpha^2W^2\,t^2}\right]$$

Problem 4.14

The minimum bandwidth, B_T, is equal to 1/2T, where T is the pulse duration. For 64 quantization levels, $\log_2 64 = 6$ bits are required.

Problem 4.15

The effect of a linear phase response in the channel is simply to introduce a constant delay τ into the pulse p(t). The delay τ is defined as $-1/(2\pi)$times the slope of the phase response; see Eq. 2.171.

H1

Problem 4.16

The Bandwidth B of a raised cosine pulse spectrum is $2W - f_1$, where $W = 1/2T_b$ and $f_1 = W(1-\alpha)$. Thus $B = W(1+\alpha)$. For a data rate of 56 kilobits per second, W = 28 kHz.

(a) For $\alpha = 0.25$,

B = 28 kHz x 1.25

= 35 kHz

(b) B = 28 kHz x 1.5

= 42 kHz

(c) B = 49 kHz

(d) B = 56 kHz

Problem 4.17

The use of eight amplitude levels ensures that 3 bits can be transmitted per pulse. The symbol period can be increased by a factor of 3. All four bandwidths in problem 7.12 will be reduced to 1/3 of their binary PAM values.

Problem 4.18

(a) For a unity rolloff, raised cosine pulse spectrum, the bandwidth B equals 1/T, where T is the pulse length. Therefore, T in this case is 1/12kHz. Quarternary PAM ensures 2 bits per pulse, so the rate of information is

$$\frac{2 \text{ bits}}{T} = 24 \text{ kilobits per second.}$$

(b) For 128 quantizing levels, 7 bits are required to transmit an amplitude. The additional bit for synchronization makes each code word 8 bits. The signal is transmitted at 24 kilobits/s, so it must be sampled at

$$\frac{24 \text{ kbits/s}}{8 \text{ bits/sample}} = 3 \text{ kHz.}$$

The maximum possible value for the signal's highest frequency component is 1.5 kHz, in order to avoid aliasing.

Problem 4.19

The raised cosine pulse bandwidth $B = 2W - f_1$, where $W = 1/2T_b$. For this channel, $B = 75$ kHz. For the given bit duration, $W = 50$ kHz. Then,

$$f_1 = 2W - B$$

$$= 25 \text{ kHz}$$

$$\alpha = 1 - f_1/B_T$$

$$= 0.5$$

Problem 4.20

The duobinary technique has correlated digits, while the other two methods have independent digits.

Problem 4.21

(a)

binary sequence b_k		0	0	1	1	0	1	0	0	1
polar representation		-1	-1	1	1	-1	1	-1	-1	1
duobinary coder output c_k			-2	0	2	0	0	0	-2	0
receiver output $\hat{b}_k$		-1	-1	1	1	-1	1	-1	-1	1
output binary sequence		0	0	1	1	0	1	0	0	1

(b)

receiver input			0	0	2	0	0	0	-2	0
receiver output $\hat{b}_k$		-1	1	-1	1	-1	1	-1	-1	1
output binary sequence		0	1	0	1	0	1	0	0	1

We see that not only is the second digit in error, but also the error propagates.

Problem 4.22

(a)

binary sequence b_k		0	0	1	1	0	1	0	0	1
coded sequence d_k	1	1	1	0	1	1	0	0	0	1
polar representation	1	1	1	-1	1	1	-1	-1	-1	1
duobinary coder output c_k		2	2	0	0	2	0	-2	-2	0
receiver output		0	0	1	1	0	1	0	0	1

(b)

receiver input	2	0	0	0	2	0	-2	-2	0
receiver output	0	1	1	1	0	1	0	0	1

In this case we see that only the second digit is in error, and there is no error propagation.

Problem 4.23

(a) The correlative coder has output

$$z_n = y_n - y_{n-1}$$

Its impulse response is

$$h_k = \begin{cases} 1 & k = 0 \\ -1 & k = 1 \\ 0 & \text{otherwise.} \end{cases}$$

The frequency response is

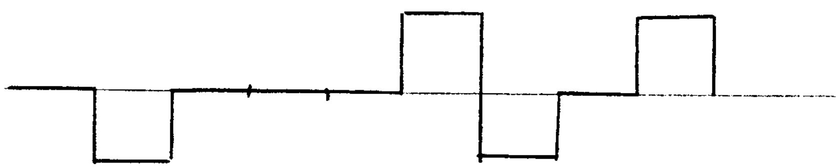
$$H(f) = \sum_{k=-\infty}^{\infty} h_k \exp(-j2\pi fkT_b)$$

$$= 1 - \exp(-j2\pi fT_b)$$

(b) Let the input to the differential encoder be x_n, the input to the correlative coder be y_n, and the output of the correlative coder be z_n. Then, for the sequence 010001101 in its on-off form, we have

x_n		0	1	0	0	0	1	1	0	1
y_n	1	1	0	0	0	0	1	0	0	1
z_n		0	-1	0	0	0	1	-1	0	1

Then z_n has the following waveform



The sequence z_n is a bipolar representation of the input sequence x_n.

Problem 4.24

(a) The output symbols of the modulo-2 adder are independent because:

1. the input sequence to the adder has independent symbols, and therefore
2. knowing the previous value of the adder does not improve prediction of the present value, i.e.

$$f(y_n | y_{n-1}) = f(y_n) ,$$

where y_n is the value of the adder output at time nT_b. The adder output sequence is another on-off binary wave with independent symbols. Such a wave has the power spectral density (from problem 4.10),

$$S_Y(f) = \frac{A^2}{4} \delta(f) + \frac{A^2 T_b}{4} \text{sinc}^2(fT_b) .$$

The correlative coder has the transfer function

$$H(f) = 1 - \exp(-j2\pi f T_b),$$

Hence, the output wave has the power spectral density

$$S_Z(f) = |H(f)|^2 \, S_Y(f)$$

$$= [1 - \exp(-j2\pi f T_b)]\,[1 - \exp(j2\pi f T_b)]\, S_Y(f)$$

$$= [2 - 2\cos(2\pi f T_b)]\, S_Y(f)$$

$$= 4\sin^2(\pi f T_b)\, S_Y(f)$$

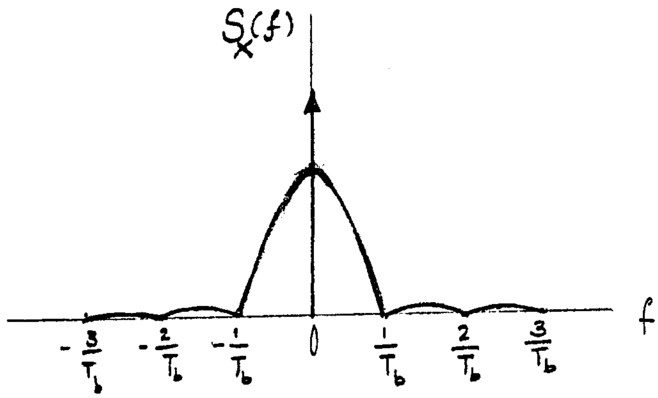
$$= 4\sin^2(\pi f T_b)\left[\frac{A^2}{4}\,\delta(f) + \frac{A^2 T_b}{4}\,\mathrm{sinc}^2(f T_b)\right]$$

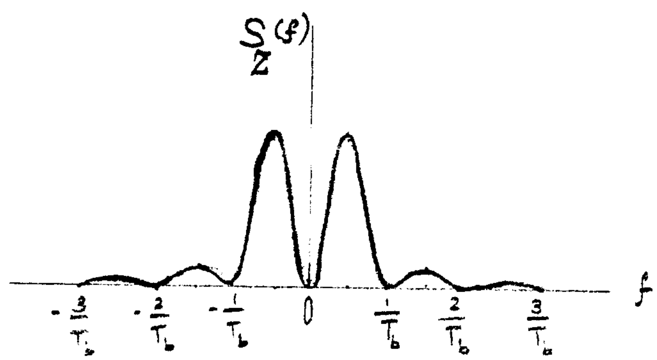
$$= A^2 T_b \sin^2(\pi f T_b)\,\mathrm{sinc}^2(f T_b)$$

In the last line we have used the fact that

$$\sin(\pi f T_b) = 0 \text{ at } f = 0.$$

(b)





Note that the bipolar wave has no dc component.

(Note: The power spectral density of a bipolar signal derived in part (a) assumes the use of a pulse of full duration T_b. On the other hand, the result derived for a bipolar signal in part (d) of Problem 3.11 assumes the use of a pulse of half symbol duration T_b.)

Problem 4.25

The modified duobinary receiver estimate is $\hat{a}_k = c_k + \hat{a}_{k-2}$.

(a)	binary sequence a_k	0	1	1	1	0	0	1	0	1
	bipolar representation	−1	1	1	1	−1	−1	1	−1	1
	modified duobinary c_k			2	0	−2	−2	2	0	0
	receiver output $\hat{a}_k$	−1	1	1	1	−1	−1	1	−1	1
	output binary sequence	0	1	1	1	0	0	1	0	1
(b)	receiver input			0	0	−2	−2	2	0	0
	receiver output $\hat{a}_k$	−1	1	−1	1	−3	−1	−1	−1	−1
	output binary sequence	0	1	0	1	0	0	0	0	0

Here we see that not only is the third digit in error, but also the error propagates.

Problem 4.26

(a)	binary sequence b_k			0	1	1	1	0	0	1	0	1
	coded sequence a_k	0	0	0	1	1	0	1	0	0	0	1
	polar representation	−1	−1	−1	1	1	−1	1	−1	−1	−1	1
	modified duobinary c_k			0	2	2	−2	0	0	−2	0	2
	receiver output $\hat{b}_k = \|c_k\|$			0	2	2	2	0	0	2	0	2
	output binary sequence			0	1	1	1	0	0	1	0	1
(b)	receiver input			0	2	0	−2	0	0	−2	0	2
	receiver output			0	2	0	2	0	0	2	0	2
	output binary sequence			0	1	0	1	0	0	1	0	1

This time we find that only the third digit is in error, and there is no error propagation.

Problem 4.27

(a) Polar Signalling (M=2)

In this case, we have

$$m(t) = \sum_n A_n \operatorname{sinc}(\frac{t}{T} - n)$$

where $A_n = \pm A/2$. Digits 0 and 1 are thus represented by $-A/2$ and $+A/2$, respectively. The Fourier transform of m(t) is

$$M(f) = \sum_n A_n F[\operatorname{sinc}(\frac{t}{T} - n)]$$

$$= T \operatorname{rect}(fT) \sum_n A_n \exp(-j2\pi nfT)$$

Therefore, m(t) is passed through the ideal low-pass filter with no distortion.

The noise appearing at the low-pass filter output has a variance given by

$$\sigma^2 = \frac{N_0}{2T}$$

Suppose we transmit digit 1. Then, at the sampling instant, we obtain a random variable at the input of the decision device, defined by

$$X = \frac{A}{2} + N$$

where N denotes the contribution due to noise. The decision level is 0 volts. If $X > 0$, the decision device chooses symbol 1, which is a correct decision. If $X < 0$, it chooses symbol 0, which is in error. The probability of making an error is

$$P(X<0) = \int_{-\infty}^{0} f_X(x)\,dx$$

The expected value of X is A/2, and its variance is σ^2. Hence,

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp[-\frac{(x - \frac{A}{2})^2}{2\sigma^2}]$$

$$P(X<0) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{0} \exp(-\frac{(x - \frac{A}{2})^2}{2\sigma^2})\, dx$$

$$= \frac{1}{2}\,\mathrm{erfc}(\frac{A}{2\sqrt{2}\sigma})$$

Similarly, if we transmit symbol 0, an error is made when X > 0, and the probability of this error is

$$P(X>0) = \frac{1}{2}\,\mathrm{erfc}(\frac{A}{2\sqrt{2}\sigma})$$

Since the symbols 1 and 0 are equally probable, we find that the average probability of error is

$$P_e = \frac{1}{2} P(X<0 \mid \text{transmit } 1) + \frac{1}{2} P(X>0 \mid \text{transmit } 0)$$

$$= \frac{1}{2}\,\mathrm{erfc}(\frac{A}{2\sqrt{2}\sigma})$$

(b) Polar ternary signaling

In this case we have

$$m(t) = \sum_n A_n \,\mathrm{sinc}(\frac{t}{T} - n)$$

where

$$A_n = 0, \pm A.$$

The 3 digits are defined as follows

Digit	Level
0	-A
1	0
2	+A

Suppose we transmit digit 2, which, at the input of the decision device, yields the random variable

$$X = A + N$$

The probability density function of X is

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp(-\frac{(x-A)^2}{2\sigma^2})$$

The decision levels are set at -A/2 and A/2 volts. Hence, the probability of choosing digit 1 is

$$P(-\frac{A}{2} < X < \frac{A}{2}) = \int_{-A/2}^{A/2} \frac{1}{\sqrt{2\pi}\,\sigma} \exp[-\frac{(x-A)^2}{2\sigma^2}]\, dx$$

$$= \frac{1}{2}\,[\mathrm{erfc}(\frac{A}{2\sqrt{2}\,\sigma}) - \mathrm{erfc}(\frac{3A}{2\sqrt{2}\,\sigma})]$$

Next, the probability of choosing digit 0 is

$$P(X < -\frac{A}{2}) = \frac{1}{2}\,\mathrm{erfc}(\frac{3A}{2\sqrt{2}\,\sigma})$$

If we transmit digit 1, the random variable at the input of the decision device is

$$X = N$$

The probability density function of X is therefore

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp(-\frac{x^2}{2\sigma^2})$$

The probability of choosing digit 2 is

$$P(X > \frac{A}{2}) = \frac{1}{2}\,\mathrm{erfc}(\frac{A}{2\sqrt{2}\,\sigma})$$

The probability of choosing digit 0 is

$$P(X < -\frac{A}{2}) = \frac{1}{2}\,\mathrm{erfc}(\frac{A}{2\sqrt{2}\,\sigma})$$

Next, suppose we transmit digit 0. Then, the random variable at the input of the decision device is

$$X = -A + N$$

The probability density function of X is therefore

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp[-\frac{(x+A)^2}{2\sigma^2}]$$

The probability of choosing digit 1 is

$$P(-\frac{A}{2} < X < \frac{A}{2}) = \frac{1}{2} [\text{erfc}(\frac{A}{2\sqrt{2}\,\sigma}) - \text{erfc}(\frac{3A}{2\sqrt{2}\,\sigma})]$$

The probability of choosing digit 2 is

$$P(X > \frac{A}{2}) = \frac{1}{2} \text{erfc}(\frac{3A}{2\sqrt{2}\sigma})$$

Assuming that digits 0, 1, and 2 are equally probable, the average probability of error is

$$P_e = \frac{1}{3} [\frac{1}{2} \text{erfc}(\frac{A}{2\sqrt{2}\,\sigma}) - \frac{1}{2} \text{erfc}(\frac{3A}{2\sqrt{2}\,\sigma})] + \frac{1}{3} \cdot \frac{1}{2} [\text{erfc}(\frac{3A}{2\sqrt{2}\,\sigma})]$$

$$+ \frac{1}{3} \cdot \frac{1}{2} [\text{erfc}(\frac{A}{2\sqrt{2}\,\sigma})] + \frac{1}{3} \cdot \frac{1}{2} [\text{erfc}(\frac{A}{2\sqrt{2}\,\sigma})]$$

$$+ \frac{1}{3} \cdot \frac{1}{2} [\text{erfc}(\frac{A}{2\sqrt{2}\,\sigma}) - \text{erfc}(\frac{3A}{2\sqrt{2}\,\sigma})] + \frac{1}{3} \cdot \frac{1}{2} \text{erfc}(\frac{3A}{2\sqrt{2}\,\sigma})$$

$$= \frac{2}{3} \text{erfc}(\frac{A}{2\sqrt{2}\,\sigma})$$

(c) Polar quaternary signaling

In this case, we have

$$A_n = \pm \frac{A}{2}, \pm \frac{3A}{2}$$

and the 4 digits are represented as follows

Digit	Level
0	$-\frac{3A}{2}$
1	$-\frac{A}{2}$
2	$+\frac{A}{2}$
3	$+\frac{3A}{2}$

Suppose we transmit digit 3, which, at the input of the decision device, yields the random variable:

$$X = \frac{3A}{2} + N.$$

The decision levels are 0, ± A. The probability of choosing digit 2 is

$$P(0 < X < A) = \frac{1}{\sqrt{2\pi}\,\sigma} \int_0^A \exp[- \frac{(x - \frac{3A}{2})^2}{2\sigma^2}]\, dx$$

$$= \frac{1}{2} [\mathrm{erfc}(\frac{A}{2\sqrt{2}\,\sigma}) - \mathrm{erfc}(\frac{3A}{2\sqrt{2}\,\sigma})]$$

The probability of choosing digit 1 is

$$P(-A < X < 0) = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-A}^0 \exp[- \frac{(x - \frac{3A}{2})^2}{2\sigma^2}]\, dx$$

$$= \frac{1}{2} [\mathrm{erfc}(\frac{3A}{2\sqrt{2}\,\sigma}) - \mathrm{erfc}(\frac{5A}{2\sqrt{2}\,\sigma})]$$

The probability of choosing digit 0 is

$$P(X < -A) = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{-A} \exp[- \frac{(x - \frac{3A}{2})^2}{2\sigma^2}]\, dx$$

$$= \frac{1}{2} \mathrm{erfc}(\frac{5A}{2\sqrt{2}\,\sigma}).$$

Suppose next we transmit digit 2, obtaining

$$X = \frac{A}{2} + N.$$

The probability of choosing digit 3 is

$$P(X > A) = \frac{1}{\sqrt{2\pi}\,\sigma} \int_A^{\infty} \exp[- \frac{(x - \frac{A}{2})^2}{2\sigma^2}]\, dx$$

$$= \frac{1}{2} \mathrm{erfc}(\frac{A}{2\sqrt{2}\,\sigma}).$$

The probability of choosing digit 1 is

$$P(-A < X < 0) = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-A}^{0} \exp\left(-\frac{(x - \frac{A}{2})^2}{2\sigma^2}\right) dx$$

$$= \frac{1}{2} \left[\operatorname{erfc}\left(\frac{A}{2\sqrt{2}\,\sigma}\right) - \operatorname{erfc}\left(\frac{3A}{2\sqrt{2}\,\sigma}\right)\right]$$

The probability of choosing digit 0 is

$$P(X < -A) = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{A} \exp\left[-\frac{(x - \frac{A}{2})^2}{2\sigma^2}\right] dx$$

$$= \frac{1}{2} \operatorname{erfc}\left(\frac{3A}{2\sqrt{2}\,\sigma}\right).$$

Suppose next we transmit digit 1, obtaining

$$X = -\frac{A}{2} + N$$

The probability of choosing digit 0 is

$$P(X < -A) = \frac{1}{2} \operatorname{erfc}\left(\frac{A}{2\sqrt{2}\,\sigma}\right)$$

The probability of choosing digit 2 is

$$P(0 < X < A) = \frac{1}{2} \left[\operatorname{erfc}\left(\frac{A}{2\sqrt{2}\,\sigma}\right) - \operatorname{erfc}\left(\frac{3A}{2\sqrt{2}\,\sigma}\right)\right]$$

The probability of choosing digit 3 is

$$P(X > A) = \frac{1}{2} \operatorname{erfc}\left(\frac{3A}{2\sqrt{2}\,\sigma}\right).$$

Finally, suppose we transmit digit 0, obtaining

$$X = -\frac{3A}{2} + N$$

The probability of choosing digit 1 is

$$P(-A < X < 0) = \frac{1}{2} [\text{erfc}(\frac{A}{2\sqrt{2}\ \sigma}) - \text{erfc}(\frac{3A}{2\sqrt{2}\ \sigma})]$$

The probability of choosing digit 2 is

$$P(0 < X < A) = \frac{1}{2} [\text{erfc}(\frac{3A}{2\sqrt{2}\ \sigma}) - \text{erfc}(\frac{5A}{2\sqrt{2}\ \sigma})]$$

The probability of choosing digit 3 is

$$P(X > A) = \frac{1}{2} \text{erfc}(\frac{5A}{2\sqrt{2}\ \sigma})$$

Since all 4 digits are equally probable, with a probability of occurence equal to 1/4, we find that the average probability of error is

$$P_e = \frac{1}{4} \cdot 2 \cdot \frac{1}{2} \{\text{erfc}(\frac{A}{2\sqrt{2}\ \sigma}) - \text{erfc}(\frac{3A}{2\sqrt{2}\ \sigma})$$

$$+ \text{erfc}(\frac{3A}{2\sqrt{2}\ \sigma}) - \text{erfc}(\frac{5A}{2\sqrt{2}\ \sigma})$$

$$+ \text{erfc}(\frac{5A}{2\sqrt{2}\ \sigma})$$

$$+ \text{erfc}(\frac{A}{2\sqrt{2}\ \sigma})$$

$$+ \text{erfc}(\frac{A}{2\sqrt{2}\ \sigma}) - \text{erfc}(\frac{3A}{2\sqrt{2}\ \sigma})$$

$$+ \text{erfc}(\frac{3A}{2\sqrt{2}\ \sigma}) \}$$

$$= \frac{3}{4} \text{erfc}(\frac{A}{2\sqrt{2}\ \sigma}) .$$

Problem 4.28

The average probability of error is (from the solution to Problem 7.23)

$$P_e = (1 - \frac{1}{M}) \text{erfc}(\frac{A}{2\sqrt{2}\ \sigma}) \quad (1)$$

The received signal-to-noise ratio is

$$(SNR)_R = \frac{A^2(M^2 - 1)}{12\ \sigma^2}$$

That is

$$\frac{A}{\sigma} = \sqrt{\frac{12(SNR)_R}{M^2 - 1}} \qquad (2)$$

Substituting Eq. (2) in (1), we get

$$P_e = (1 - \frac{1}{M})\ \text{erfc}(\sqrt{\frac{3(SNR)_R}{2(M^2-1)}})$$

With $P_e = 10^{-6}$, we may thus write

$$10^{-6} = (1 - \frac{1}{M})\ \text{erfc}(u) \qquad (3)$$

where

$$u^2 = \frac{3(SNR)_R}{2(M^2-1)}$$

For a specified value of M, we may solve Eq. (3) for the corresponding value of u. We may thus construct the following table:

M	u
2	3.37
4	3.42
8	3.45
16	3.46

We thus find that to a first degree of approximation, the minimum value of received signal-to-noise ratio required for $P_e < 10^{-6}$ is given by

$$\frac{3(SNR)_{R,min}}{2(M^2-1)} \simeq (3.42)^2$$

That is, $(SNR)_{R,\min} \approx 7.8\ \ (M^2 - 1)$

Problem 4.29

Typically, a cable contains many twisted pairs. Therefore, the received signal can be written as

$$r(n) = \sum_{i=1}^{N} v_i(n) + d(n), \qquad \text{large } N$$

where $d(n)$ is the desired signal and $\sum_{i=1}^{N} v_i(n)$ is due to cross-talk. Typically, the v_i are statistically independent and identically distributed. Hence, by using the central limit theorem, as N becomes infinitely large, the term $\sum_{i=1}^{N} v_i(n)$ is closely approximated by a Gaussian random variable for each time instant n.

Problem 4.30

(a) The power spectral density of the signal generated by the NRZ transmitter is given by

$$S(f) = \frac{\sigma^2}{T}|G(f)|^2 \tag{1}$$

where σ^2 is the symbol variance, T is the symbol duration, and

$$G(f) = \int_{-T/2}^{T/2} 1 \cdot e^{-j2\pi ft} dt = T \operatorname{sinc}(fT) = \frac{1}{R}\operatorname{sinc}\left(\frac{f}{R}\right) \tag{2}$$

is the Fourier transform of the generating function for NRZ symbols. Here, we have used the fact that the symbol rate $R = 1/T$. A 2B1Q code is a multi-level block code where each block has 2 bits and the bit rate $R = 2/T$ (i.e., m/T, where m is the number of bits in a block). Since the 2B1Q pulse has the shape of an NRZ pulse, the power spectral density of 2B1Q signals is given by

$$S_{\text{2B1Q}} = \frac{\sigma^2}{T}|G_{\text{2B1Q}}(f)|^2$$

where

$$G_{\text{2B1Q}}(f) = \frac{\sin(2\pi(f/R))}{\sqrt{2}\pi f}$$

The factor $\sqrt{2}$ in the denominator is introduced to make the average power of the 2B1Q signal equal to the average power of the corresponding NRZ signal. Hence,

$$S_{2B1Q}(f) = \frac{\sigma^2}{T}\left(\frac{\sin(2\pi(f/R))}{\sqrt{2}\pi f}\right)^2$$

$$= \frac{2\sigma^2}{R}\operatorname{sinc}^2(2(f/R)) \tag{3}$$

(b) The transfer functions of pulse-shaping filters for the Manchester code, modified duobinary code, and bipolar return-to-zero code are as follows:

(i) Manchester code:

$$G(f) = \frac{j}{\pi f}\left[1 - \cos\left(\pi\frac{f}{R}\right)\right] \tag{4}$$

(ii)Modified duobinary code:

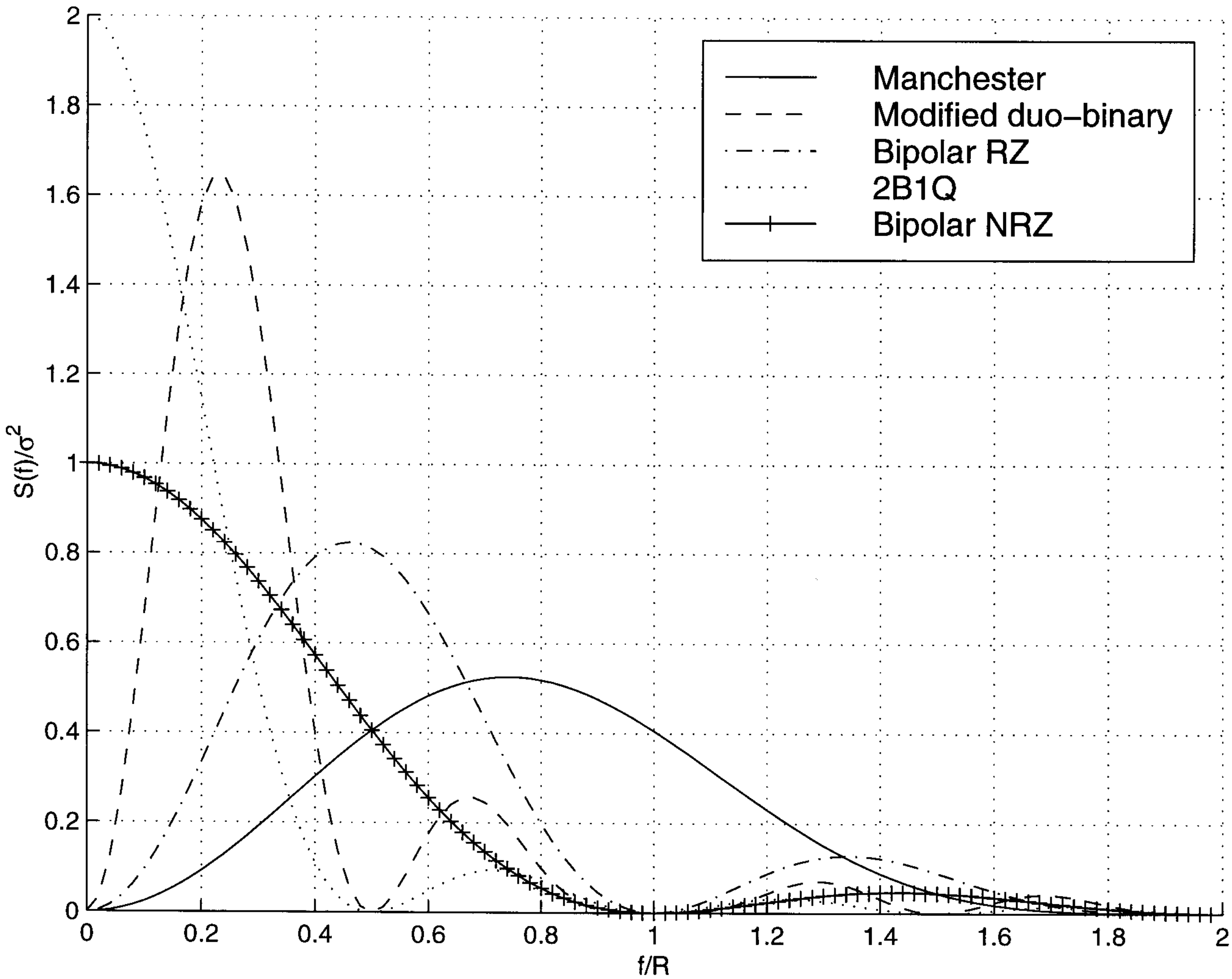
$$G(f) = \frac{1}{j\sqrt{2}\pi f}\left[\cos\left(3\pi\frac{f}{R}\right) - \cos\left(\pi\frac{f}{R}\right)\right] \tag{5}$$

(iii)Bipolar return-to-zero code:

$$G(f) = \frac{2}{\pi f}\left[\sin\left(\pi\frac{f}{2R}\right) \times \sin\left(\pi\frac{f}{R}\right)\right] \tag{6}$$

Hence, using Eqs. (4), (5), and (6) in the formula of Eq. (1) for the power spectral density of PAM line codes, we get the normalized spectral plots shown in Fig. 1. In this figure, the spectral density is normalized with respect to the symbol variance σ^2 and the frequency is normalized with respect to the data rate R.

From Fig. 1, we may make the following observations: Among the four line codes displayed here, the 2B1Q code has much of its power concentrated inside the frequency band $-R/2 \le f \le R/2$, which is much more compact than all the other three codes: Manchester code, modified duobinary code, and bipolar return-to-zero code.

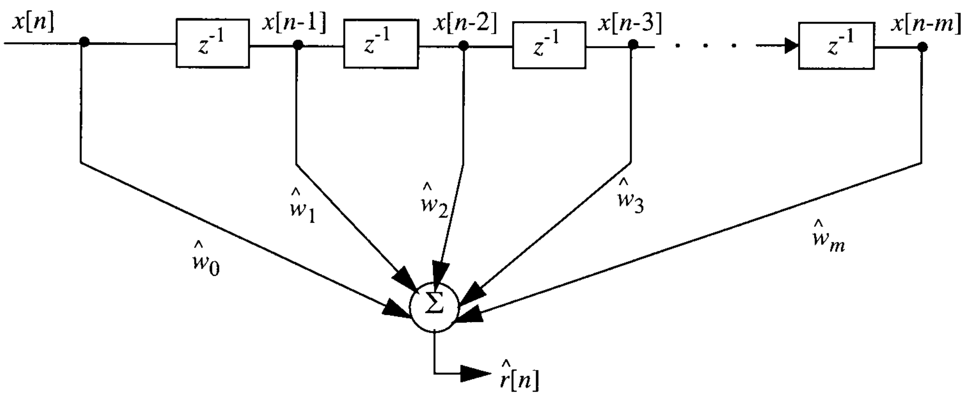


Manchester
Modified duo-binary
Bipolar RZ
2B1Q
Bipolar NRZ
S(f)/σ²
f/R
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

Figure 1

Problem 4.31

The tapped-delay-line section of the adaptive filter is shown below:



$\hat{r}[n] = \mathbf{x}^T[n]\hat{\mathbf{w}}[n]$

$d[n] = x[n] + r[n]$

Error signal $e[n] = d[n] - \hat{r}[n]$

$\hat{\mathbf{w}}[n+1] = \hat{\mathbf{w}}[n] + \mu\mathbf{x}[n](d[n] - \mathbf{x}^T[n]\hat{\mathbf{w}}[n])$

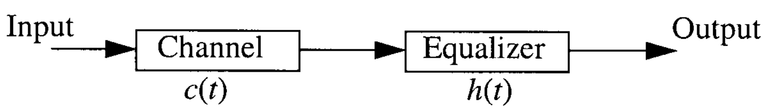
where $\hat{\mathbf{w}}[n] = [\hat{w}_0[n], \cdots, \hat{w}_m[n]^T]$

$\mathbf{x}[n] = [x[n], x[n-1], \cdots, x[n-m]^T]$

μ = learning parameter

Problem 4.32

(a)



The $h(t)$ is defined by

$$h(t) = \sum_{k=-N}^{N} w_k \delta(t - kT)$$

The impulse response of the cascaded system is given by the convolution sum

$$p_n = \sum_{j=-N}^{N} w_j c_{n-j}$$

where $p_n = p(nT)$. The kth sample of the output of the cascaded system due to the input sequence $\{I_n\}$ is defined by

$$\hat{I}_k = p_0 I_k + \sum_{n \neq k} I_n p_{k-n}$$

where $p_0 I_k$ is a scaled version of the desired symbol I_k. The summation term $\sum_{n \neq k} I_n p_{k-n}$ is the intersymbol interference.

The peak value of the interference is given by

$$D(N) = \sum_{\substack{n=-N \\ n \neq 0}}^{N} |p_n| = \sum_{\substack{n=-N \\ n \neq 0}}^{N} \left| \sum_{k=-N}^{N} w_k c_{n-k} \right|$$

To make the ISI equal to zero, we require

$$p_n = \sum_{k=-N}^{N} w_k c_{n-k} = \left\{ \begin{matrix} 1, & n = 0 \\ 0, & n \neq 0 \end{matrix} \right\}$$

(b) By taking the z-transform of the convolution sum

$$\sum_{k=-N}^{N} w_k c_{n-k}$$

and recalling that convolution in the discrete-time domain is transformed into multiplication in the z-domain, we get

$$P(z) = H(z)C(z)$$

For the zero-forcing condition, we require that $P(z) = 1$. Under this condition, we have

$$H(z) = 1/C(z)$$

which represents the transfer function of an inverse filter.

If the channel contains a spectral null at $f = 1/2T$ in its frequency response, the linear zero-forcing equalizer attempts to compensate for this null by introducing an infinite gain at frequency $f = 1/2T$. However, the channel distortion is compensated at the expense of enhancing additive noise: With $H(z) = 1/C(z)$, we find that when $C(z) = 0$,

$$H(z) = \infty$$

which results in noise enhancement.

Similarly, when the channel spectral response takes a smaller value, the equalizer will introduce a high gain at that frequency. Again, this tends to enhance the additive noise.

Problem 4.33

(a) Consider Eq. (4.108) of the textbook, which is rewritten as

$$\int_{-\infty}^{\infty}\left(R_q(t-\tau)+\frac{N_0}{2}\delta(t-\tau)\right)c(\tau)d\tau = q(-t)$$

Expanding the left-hand side:

$$\int_{-\infty}^{\infty}R_q(t-\tau)c(\tau)d\tau+\int_{-\infty}^{\infty}\frac{N_0}{2}\delta(t-\tau)c(\tau)d\tau = q(-t)$$

Applying the Fourier transform:

$$F\left\{\int_{-\infty}^{\infty}R_q(t-\tau)c(\tau)d\tau\right\} = F\{R_q(t-\tau)\} \times F\{(c(\tau))\}$$

$$= S_q(f)C(f)$$

$$F\left\{\int_{-\infty}^{\infty}\frac{N_0}{2}\delta(t-\tau)c(\tau)d\tau\right\} = \frac{N_0}{2}C(f)$$

$$F\{q(-t)\} = Q(-f) = Q^*(f)$$

In these three relations we have used the fact that convolution in the time domain corresponds to multiplication in the frequency domain.

Putting these results together, we get

$$S_q(f)C(f) + \frac{N_0}{2}C(f) = Q^*(f)$$

or

$$\left(S_q(f) + \frac{N_0}{2}\right)C(f) = Q^*(f)$$

which is the desired result.

(b) The autocorrelation function of the sequence is given by

$$R_q(\tau_1, \tau_2) = \sum_k q(kT_b - \tau_1)q(kT_b - \tau_2)$$

Using the fact that the autocorrelation function and power spectral density (PSD) form a Fourier transform pair, we may write

$$\begin{aligned}
\text{PSD} &= F\{R_q(\tau_1, \tau_2)\} \\
&= F\left\{\sum_k q(kT_b - \tau_1)q(kT_b - \tau_2)\right\} \\
&= \frac{1}{T_b}\sum_k \left|Q\left(f + \frac{k}{T_b}\right)\right|^2
\end{aligned}$$

where $F\{q(t)\} = Q(f)$

Problem 4.34

(a) The channel output is

$$x(t) = \alpha_1 s(t-t_{01}) + \alpha_2 s(t-t_{02})$$

Taking the Fourier transform of both sides:

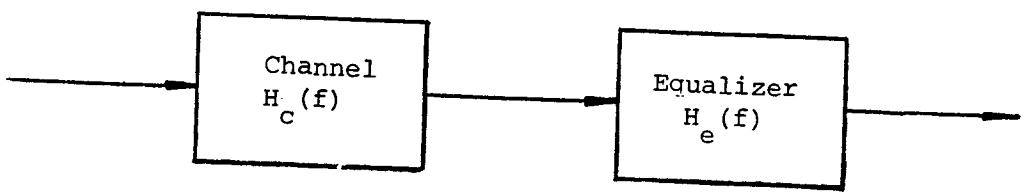
$$X(f) = [\alpha_1 \exp(-j2\pi f t_{01}) + \alpha_2 \exp(-j2\pi f t_{02})] S(f)$$

The transfer function of the channel is

$$H_c(f) = \frac{X(f)}{S(f)}$$

$$= \alpha_1 \exp(-j2\pi f t_{01}) + \alpha_2 \exp(-j2\pi f t_{02})$$

(b)



Ideally, the equalizer should be designed so that

$$H_c(f)\, H_e(f) = K_0 \exp(-j2\pi f t_0)$$

where K_0 is a constant gain and t_0 is the transmission delay. The transfer function of the equalizer is

$$H_e(f) = w_0 + w_1 \exp(-j2\pi fT) + w_2 \exp(-j4\pi fT)$$

$$= w_0 \left[1 + \frac{w_1}{w_0} \exp(-j2\pi fT) + \frac{w_2}{w_0} \exp(-j4\pi fT)\right] \quad (1)$$

Therefore

$$H_e(f) = \frac{K_0 \exp(-j2\pi f t_0)}{H_c(f)}$$

$$= \frac{K_0 \exp(-j2\pi f t_0)}{\alpha_1 \exp(-j2\pi f t_{01}) + \alpha_2 \exp(-j2\pi f t_{02})}$$

$$= \frac{(K_0/a_1)\ \exp[-j2\pi f(t_0 - t_{01})]}{1 + \frac{a_2}{a_1}\exp[-j2\pi f(t_{02} - t_{01})]}$$

Since $a_2 \ll a_1$, we may approximate $H_e(f)$ as follows

$$H_e(f) = \frac{K_0}{a_1}\exp[-j2\pi f(t_0 - t_{01})]\ \{1 - \frac{a_2}{a_1}\exp[-j2\pi f(t_{02} - t_{01})]$$

$$+ (\frac{a_2}{a_1})^2 \exp[-j4\pi f(t_{02} - t_{01})]\} \qquad (2)$$

Comparing Eqs. (1) and (2), we deduce that

$$\frac{K_0}{a_1} \simeq w_0$$

$$t_0 - t_{01} \simeq 0$$

$$-\frac{a_2}{a_1} \simeq \frac{w_1}{w_0}$$

$$(\frac{a_2}{a_1})^2 \simeq \frac{w_2}{w_0}$$

$$T \simeq t_{02} - t_{01}$$

Choosing $K_0 = a_1$, we find that the tap weights of the equalizer are as follows

$$w_0 = 1$$

$$w_1 = -\frac{a_2}{a_1}$$

$$w_2 = (\frac{a_2}{a_1})^2$$

Problem 4.35

The Fourier transform of the tapped-delay-line equalizer output is defined by

$$Y_{out}(f) = H(f)\, X_{in}(f) \qquad (1)$$

where H(f) is the equalizer's transfer function and $X_{in}(f)$ is the Fourier transform of the input signal. The input signal consists of a uniform sequence of samples, denoted by {x(nT)}. We may therefore write (see Eq. 6.2):

$$X_{in}(f) = \frac{1}{T}\sum_{k} X(f-\frac{k}{T}) \qquad (2)$$

where T is the sampling period and s(t) is the signal from which the sequence of samples is derived. For perfect equalization, we require that

$$Y_{out}(f) = 1 \qquad \text{for all } f.$$

From Eqs. (1) and (2) we therefore find that

$$H(f) = \frac{T}{\sum_{k} X(f-k/T)} \qquad (3)$$

Let the impulse response (sequence) of the equalizer be denoted by $\{w_n\}$. Assuming an infinite number of taps, we have

$$H(f) = \sum_{n=-\infty}^{\infty} w_n \exp(j2\pi fT)$$

We now immediately see that H(f) is in the form of a complex Fourier series with real coefficients defined by the tap weights of the equalizer. The tap-weights are themselves defined by

$$w_n = \frac{1}{T}\int_{-1/2T}^{1/2T} H(f)\exp(-j2\pi t/T), \qquad n=0, +1, +2,...$$

The transfer function H(f) is itself defined in terms of the input signal by Eq, (3). Accordingly, a tapped-delay-line equalizer of infinite length can approximate any function in the frequency interval (-1/2T, 1/2T).

Problem 4.36

(a) As an example, consider the following single-parameter model of a noisy system:

$$d[n] = w_0[n]x[n] + v[n]$$

where $x[n]$ is the input signal and $v[n]$ is additive noise. To track variations in the parameter $w_0[n]$, we may use the LMS algorithm, which is described by

$$\hat{w}[n+1] = \hat{w}[n] + \mu x[n]\left(\overbrace{(d[n] - \hat{w}[n]x[n])}^{\text{Error signal}}\right)$$

$$= (1 - \mu x^2[n])\hat{w}[n] + \mu x[n]d[n] \qquad (1)$$

To simplify matters, we assume that $\hat{w}[n]$ is independent of $x[n]$. Hence, taking the expectation of both sides of Eq. (1):

$$E[\hat{w}[n+1]] = (1 - \mu\sigma_x^2)E[\hat{w}[n]] + \mu r_{dx} \qquad (2)$$

where E is the statistical expectation operator, and

$$\sigma_x^2 = E[x^2[n]]$$

$$r_{dx} = E[d[n]x[n]]$$

Equation (2) represents a first-order difference equation in the mean value $E[\hat{w}[n]]$. For this difference equation to be convergent (i.e., for the system to be stable), we require that

$$|1 - \mu\sigma_x^2| < 1$$

or equivalently

(i) $1 - \mu\sigma_x^2 < 1$, i.e., $\mu > 0$

(ii) $-1 + \mu\sigma_x^2 < 1$, i.e., $\mu < \dfrac{2}{\sigma_x^2}$

Stated in yet another way, the LMS algorithm for the example considered herein is stable provided that the step-size parameter μ satisfies the following conditions:

$$0 < \mu < \frac{2}{\sigma_x^2}$$

where σ_x^2 is the variance of the input signal.

(b) When a small value is assigned to μ, the adaptation is slow, which is equivalent to the LMS algorithm having a long "memory". The excess mean-squared error after adaptation is small, on the average, because of the large amount of data used by the algorithm to estimate the gradient vector. On the other hand, when μ is large, the adaptation is relatively fast, but at the expense of an increase in the excess mean-squared error after adaptation. In this case, less data enter the estimation, hence a degraded estimation error performance. Thus, the reciprocal of the parameter μ may be viewed as the memory of the LMS algorithm.

Problem 4.37

A *decision-feedback equalizer* consists of a feedforward section, a feedback section, and a decision device connected together as shown in Fig. 1. The feed-forward section consists of a tapped-delay-line filter whose taps are spaced at the reciprocal of the signaling rate. The data sequence to be equalized is applied to this section. The feedback section consists of another tapped-delay-line filter whose taps are also spaced at the reciprocal of the signaling rate. The input applied to the feedback section consists of the decisions made on previously detected symbols of the input sequence. The function of the feedback section is to subtract out that portion of the intersymbol interference produced by previously detected symbols from the estimates of future samples.

Note that the inclusion of the decision device in the feedback loop makes the equalizer intrinsically *nonlinear* and therefore more difficult to analyze than an ordinary tapped-delay-line equalizer. Nevertheless, the mean-square error criterion can be used to obtain a mathematically tractable optimization of a decision-feedback equalizer. Indeed, the LMS algorithm can be used to jointly adapt both the feedforward tap-weights and the feedback tap-weights based on a *common* error signal. To be specific, let the augmented vector $\mathbf{c}_n$ denote the combination of the feedforward and feedback tap-weights, as shown by

$$\mathbf{c}_n = \begin{bmatrix} \hat{\mathbf{w}}_n^{(1)} \\ \hat{\mathbf{w}}_n^{(2)} \end{bmatrix} \qquad (1)$$

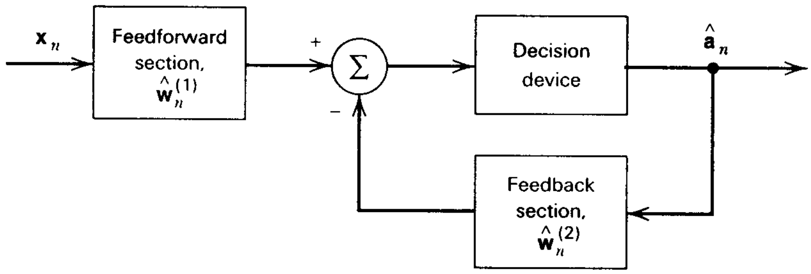


Figure 1

where the vector $\hat{\mathbf{w}}_n^{(1)}$ denotes the tap-weights of the feedforward section, and $\hat{\mathbf{w}}_n^{(2)}$ denotes the tap-weights of the feedback section. Let the augmented vector $\mathbf{v}_n$ denote the combination of input samples for both sections:

$$\mathbf{v}_n = \begin{bmatrix} \mathbf{x}_n \\ \hat{\mathbf{a}}_n \end{bmatrix} \tag{2}$$

where $\mathbf{x}_n$ is the vector of tap-inputs in the feedforward section, and $\hat{\mathbf{a}}_n$ is the vector of tap-inputs (i.e., present and past decisions) in the feedback section. The common error signal is defined by

$$e_n = a_n - \mathbf{c}_n^T \mathbf{v}_n \tag{3}$$

where the superscript T denotes matrix transposition and a_n is the polar representation of the nth transmitted binary symbol. The LMS algorithm for the decision-feedback equalizer is described by the update equations:

$$\hat{\mathbf{w}}_{n+1}^{(1)} = \hat{\mathbf{w}}_n^{(1)} + \mu_1 e_n \mathbf{x}_n$$

$$\hat{\mathbf{w}}_{n+1}^{(2)} = \hat{\mathbf{w}}_n^{(2)} + \mu_2 e_n \hat{\mathbf{a}}_n$$

where μ_1 and μ_2 are the step-size parameters for the feedforward and feedback sections, respectively.

Problem 4.38

Matlab codes

```
% Problem 4.38, CS: haykin
% Eyediagram
% baseband PAM transmission, M=4
%  Mathini Sellathurai
clear all

% Define the M-ary number, calculation sample frequency
M=4; Fs=20;

% Define the number of points in the calculation
Pd=500;

% Generate an integer message in range [0, M-1].
msg_d = exp_randint(Pd,1,M);

% Use square constellation PAM method for modulation
msg_a = exp_modmap(msg_d,Fs,M);

% nonlinear channel
alpha=0.0
```

```
msg_a=msg_a +alpha*msg_a.^2;

%raised cosine filtering
rcv_a=raisecos_n(msg_a,Fs);

% eye pattern
eyescat(rcv_a,0.5,Fs)
axis([-0.5 2.5 -1.5 1.5])
```

```
function y = exp_modmap(x, Fs,M);
% PAM modulation
% used in Problem 4.38
% Mathini Sellathurai

x=x-(M-1)/2;
x=2*x/(M-1)
y=zeros(length(x)*Fs,1);

p=0;
for k=1:Fs:length(y)
p=p+1;
y(k:(k+Fs-1))=x(p)*ones(Fs,1);
end
```

```
function out = exp_randint(p, q, r);
% random interger generator
%used for Problem 4.38
% Mathini Sellathurai

r = [0, r-1];
r = sort(r);
r(1) = ceil(r(1));
r(2) = floor(r(2));
if r(1) == r(2)
    out = ones(p, q) * r(1);
    return;
end;

d = r(2) - r(1);

r1 = rand(p, q);

out = ones(p,q)*r(1);

for i = 1:d
    index = find(r1 >= i/(d+1));
  out(index) = (r(1) + i) * index./index;
end;
```

Answer to Problem 4.38

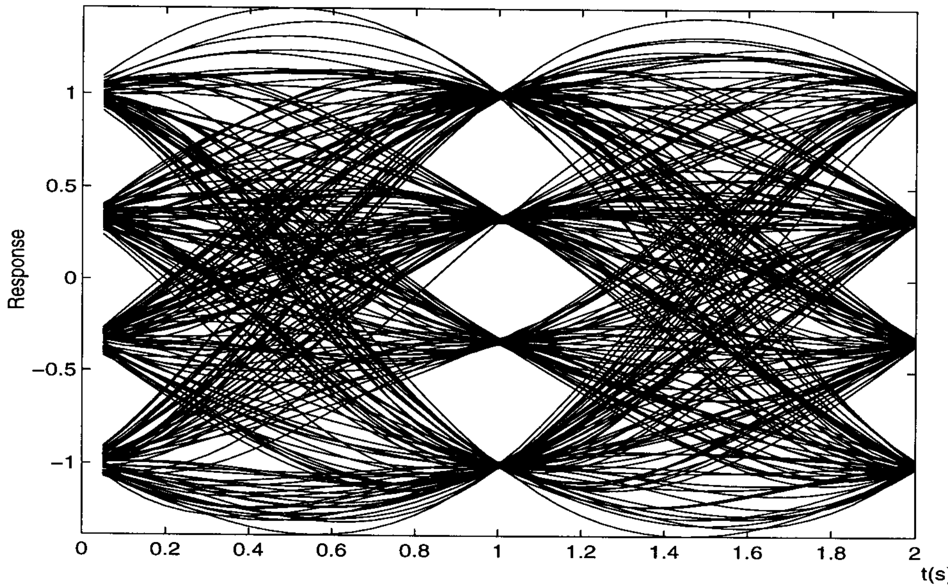


Figure 4 : Eye pattern for α=0

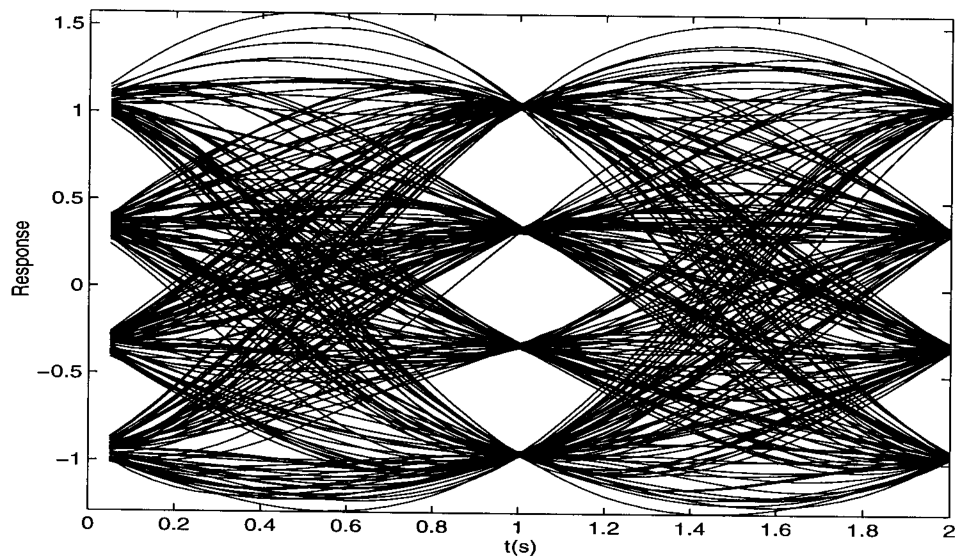


Figure 2: Eye pattern for α=0.05

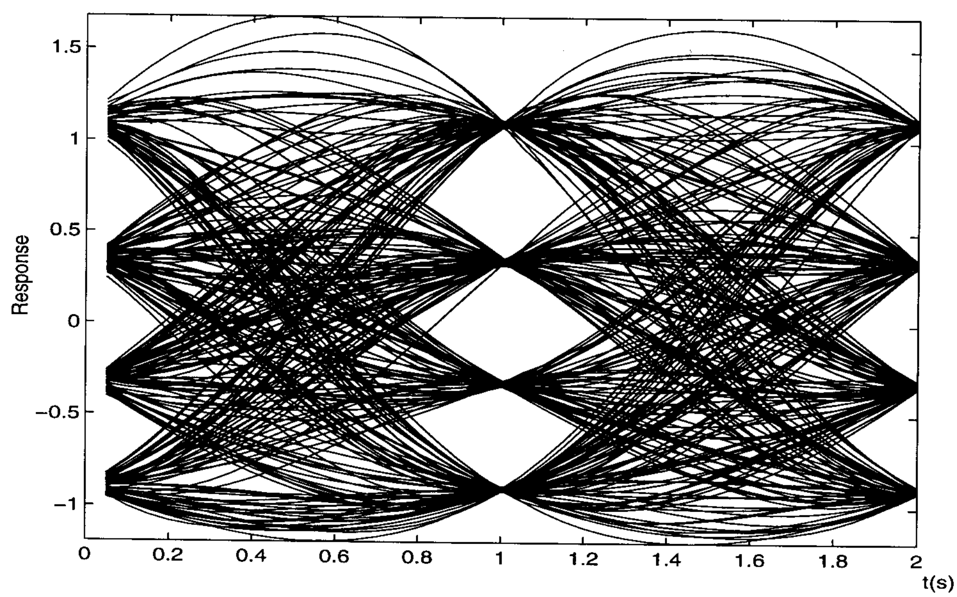


Figure 3: Eye pattern for α=0.1

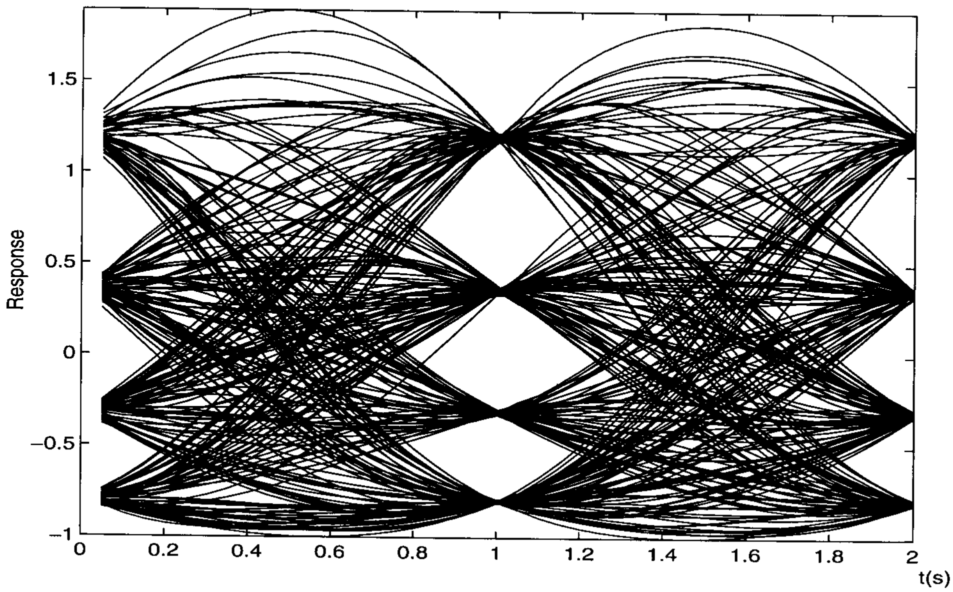


Figure 4 : Eye pattern for α=0.2

Problem 4.39

Matlab codes

```
% problem 4.39, CS: Haykin
%  root raised-cosine and raised cosine sequences
% M. Sellathurai

Data=[1 0 1 1 0 0]';
% sample frequency 20
sample_freq=20;

%generate antipodal signal
syms=PAM_mod(Data, sample_freq, 2);

% root raised cosine pulse
r_c_r = raisecos_sqrt(syms, sample_freq );

% normal raised cosine pulse
r_c_n= raisecos_n(syms, sample_freq );

% plots
t=length(r_c_r)-1;
figure; hold on
```

```
plot(0:1/20:t/20, r_c_r);
plot(0:1/20:t/20, r_c_n,'--');
xlabel('time')
legend('root raised-cosine','raised-cosine')
hold off
```

```
function osyms = raisecos_n(syms, sample_freq )
% function to generate raised-cosine sequence
% used in Problem 4.39, CS: Haykin
%M. Sellathurai

% size of data
[l_syms, w_syms] = size(syms);

% data
R=0.3;
W_T=[3, 3*3];

% Calculation of  Raised cosine pulse
W_T(1) = -abs(W_T(1));
time_T = [0 : 1/sample_freq : max(W_T(2), abs(W_T(1)))];
time_T_R = R * time_T;


    den = 1 - (2 * time_T_R).^2;
   index1 = find(den~= 0);
   index2 = find(den == 0);

    % when denominator not equal to zero
  b(index1) = sinc(time_T(index1)) .* cos(pi * time_T_R(index1)) ./ den(index1);

    % when denominator  equal to zero, (using  L'Hopital rule)
    if ~isempty(index2)
         b(index2) = 0;
  end;

b = [b(sample_freq * abs(W_T(1))+1 : -1 : 1), b(2 : sample_freq * W_T(2)+1)];
b=b(:)';
% filter parameters
order= floor(length(b)/2);
bb=[];
for i = 1: order
    bb = [bb; b(1+i:order+i)];
end;

[u, d, v] = svd(bb);
d = diag(d);

index = find(d/d(1) < 0.01);
    if isempty(index)
        o = length(bb);
    else
```

```
        o = index(1)-1;
    end;

a4 = bb(1);
u1 = u(1 : length(bb)-1, 1 : o);
v1 = v(1 :  length(bb)-1, 1 : o);
u2 = u(2 :  length(bb),   1 : o);

dd = sqrt(d(1:o));
vdd = 1 ./ dd;

uu = u1' * u2;
a1 = uu .* (vdd * dd');
a2 = dd .* v1(1, :)';
a3 = u1(1, :) .* dd';

[num, den] = ss2tf(a1, a2, a3, a4, 1);

 fsyms = zeros(l_syms+3*sample_freq, w_syms);
    for i = 1 : sample_freq : l_syms
        fsyms(i, :) = syms(i, :);
    end;

% filtering
for i = 1:w_syms
    fsyms(:, i) = filter(num, den, fsyms(:, i));
end;

 osyms = fsyms(( (3 - 1) * sample_freq  + 2):(size(fsyms, 1) - (sample_freq - 1)), :);
```

```
function osyms = raisecos_sqrt(syms, sample_freq )
% function to generate root raised-cosine sequence
% used in Problem 4.39, CS: Haykin
%M. Sellathurai

% size of data
[l_syms, w_syms] = size(syms);

% rolloff factor
R=0.3;
% window
W_T=[3, 3*3];

% Calculation of  Raised cosine pulse
W_T(1) = -abs(W_T(1));
time_T = [0 : 1/sample_freq : max(W_T(2), abs(W_T(1)))];

    den = 1 - (4 * time_T*R).^2;
   index1 = find(den ~= 0);
   index2 = find(den == 0);

    % when denominator not equal to zero
b(index1)=( cos((1 + R) * pi * time_T(index1))+...
(sinc((1-R)*time_T(index1))*(1-R)*pi/4/R))./den(index1)*4*R/ pi ;


    % when denominator  equal to zero t=\pm T/4/alpha
    if ~isempty(index2)
b(index2)=((1+2/pi)*sin(pi/4/R)+(1-2/pi)*cos(pi/4/R))*R/sqrt(2)
    end;

b(1)=1-R+4*R/pi; %t=0;

b = [b(sample_freq * abs(W_T(1))+1 : -1 : 1), b(2 : sample_freq * W_T(2)+1)];
b=b(:)';

% filter parameters
order= floor(length(b)/2);
bb=[];
for i = 1: order
    bb = [bb; b(1+i:order+i)];
end;

[u, d, v] = svd(bb);
d = diag(d);
```

```
index = find(d/d(1) < 0.01);
    if isempty(index)
        o = length(bb);
    else
        o = index(1)-1;
    end;

a4 = bb(1);
u1 = u(1 : length(bb)-1, 1 : o);
v1 = v(1 :  length(bb)-1, 1 : o);
u2 = u(2 :  length(bb),   1 : o);

dd = sqrt(d(1:o));
vdd = 1 ./ dd;

uu = u1' * u2;
a1 = uu .* (vdd * dd');
a2 = dd .* v1(1, :)';
a3 = u1(1, :) .* dd';

[num, den] = ss2tf(a1, a2, a3, a4, 1);

 fsyms = zeros(l_syms+3*sample_freq, w_syms);
    for i = 1 : sample_freq : l_syms
        fsyms(i, :) = syms(i, :);
    end;

% filtering
for i = 1:w_syms
    fsyms(:, i) = filter(num, den, fsyms(:, i));
end;

 osyms = fsyms(( (3 - 1) * sample_freq  + 2):(size(fsyms, 1) - (sample_freq - 1)), :);
```

Answer to Problem 4.39

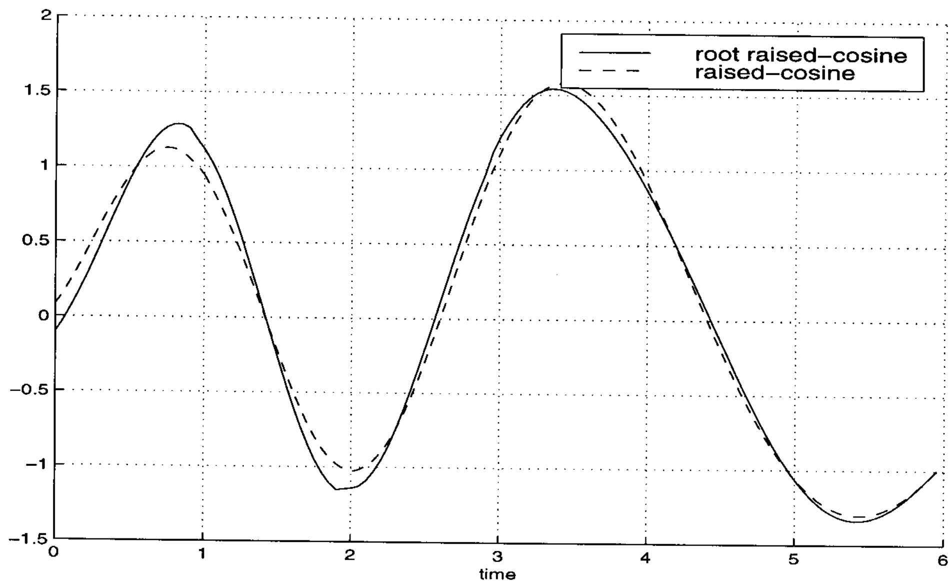


Figure 1: Raised-cosine and root raised-cosine pulse for sequence [101100]