**Problem 8.10** Determine the mean and variance of the sum of five independent uniformly-distributed random variables on the interval from -1 to +1.

## **Solution**

Let  $X_i$  be the individual uniformly distributed random variables for i = 1,...,5, and let Y be the random variable representing the sum:

$$Y = \sum_{i=1}^{5} X_i$$

Since  $X_i$  has zero mean and  $Var(X_i) = 1/3$  (see Problem 8.5), we have

$$\mathbf{E}[Y] = \sum_{i=1}^{5} \mathbf{E}[X_i] = 0$$

and

$$\operatorname{Var}(Y) = \mathbf{E}\left[\left(Y - \mu_Y\right)^2\right] = \mathbf{E}\left[Y^2\right]$$
$$= \mathbf{E}\left[\left(\sum X_i\right)^2\right]$$
$$= \sum_{i=1}^5 \mathbf{E}\left[X_i^2\right] + \sum_{i \neq j} \mathbf{E}\left[X_i X_j\right]$$

Since the  $X_i$  are independent, we may write this as

$$\operatorname{Var}(Y) = 5\left(\frac{1}{3}\right) + \sum \mathbf{E}[X_i]\mathbf{E}[X_j]$$
$$= \frac{5}{3} + 0$$
$$= \frac{5}{3}$$

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