

Problem 8.10 Determine the mean and variance of the sum of five independent uniformly-distributed random variables on the interval from -1 to +1.

Solution

Let X_i be the individual uniformly distributed random variables for $i = 1, \dots, 5$, and let Y be the random variable representing the sum:

$$Y = \sum_{i=1}^5 X_i$$

Since X_i has zero mean and $\text{Var}(X_i) = 1/3$ (see Problem 8.5), we have

$$\mathbf{E}[Y] = \sum_{i=1}^5 \mathbf{E}[X_i] = 0$$

and

$$\begin{aligned} \text{Var}(Y) &= \mathbf{E}[(Y - \mu_Y)^2] = \mathbf{E}[Y^2] \\ &= \mathbf{E}\left[\left(\sum X_i\right)^2\right] \\ &= \sum_{i=1}^5 \mathbf{E}[X_i^2] + \sum_{i \neq j} \mathbf{E}[X_i X_j] \end{aligned}$$

Since the X_i are independent, we may write this as

$$\begin{aligned} \text{Var}(Y) &= 5\left(\frac{1}{3}\right) + \sum \mathbf{E}[X_i] \mathbf{E}[X_j] \\ &= \frac{5}{3} + 0 \\ &= \frac{5}{3} \end{aligned}$$