

**Problem 8.11** A random process is defined by the function

$$X(t, \theta) = A \cos(2\pi ft + \theta)$$

where  $A$  and  $f$  are constants, and  $\theta$  is uniformly distributed over the interval  $0$  to  $2\pi$ . Is  $X$  stationary to the first order?

**Solution**

Denote

$$Y = X(t_1, \theta) = A \cos(2\pi ft_1 + \theta)$$

for any  $t_1$ . From Problem 8.7, the distribution of  $Y$  and therefore of  $X$  for any  $t_1$  is

$$F_{X(t_1)}(y) = \begin{cases} 0 & y < -A \\ \frac{2\pi - 2\cos^{-1}(y/A)}{2\pi} & |y| < A \\ 1 & y > A \end{cases}$$

Since the distribution is independent of  $t$  it is stationary to first order.