Problem 8.12 Show that a random process that is stationary to the second order is also stationary to the first order.

Solution

Let the distribution F be stationary to second order

$$F_{X(t_1)X(t_2)}(x_1, x_2) = F_{X(t_1+\tau)X(t_2+\tau)}(x_1, x_2)$$

Then,

$$\begin{aligned} F_{X(t_1)X(t_2)}(x_1, \infty) &= F_{X(t_1)}(x_1) \\ &= F_{X(t_1+\tau)X(t_2+\tau)}(x_1, \infty) \\ &= F_{X(t_1+\tau)}(x_1) \end{aligned}$$

Thus the first order distributions are stationary as well.