Problem 8.13 Let $X(t)$ be a random process defined by

$$X(t) = A\cos(2\pi ft)$$

where A is uniformly distributed between 0 and 1, and f is constant. Determine the autocorrelation function of X. Is X wide-sense stationary?

Solution

$$\begin{aligned}\mathbf{E}\left[X(t_1)X(t_2)\right] &= \mathbf{E}\left[A^2\right]\cos(2\pi ft_1)\cos(2\pi ft_2) \\ &= \mathbf{E}\left[A^2\right]\left[\cos(2\pi f(t_1 - t_2)) + \cos 2\pi f(t_1 + t_2)\right]\end{aligned}$$

$$\mathbf{E}\left[A^2\right] = \int_0^1 x^2 dx = \left.\frac{x^3}{3}\right|_0^1 = \frac{1}{3}$$

Since the autocorrelation function depends on $t_1 + t_2$ as well as $t_1 - t_2$, the process is not wide-sense stationary.