Problem 8.14 A discrete-time random process $\{Y_n: n = \dots, -1, 0, 1, 2, \dots\}$ is defined by

$$Y_n = \alpha_0 Z_n + \alpha_1 Z_{n-1}$$

where $\{Z_n\}$ is a random process with autocorrelation function $R_Z(n) = \sigma^2 \delta(n)$. What is the autocorrelation function $R_Y(n,m) = \mathbf{E}[Y_n Y_m]$? Is the process $\{Y_n\}$ wide-sense stationary?

Solution

We implicitly assume that Z_n is stationary and has a constant mean μ_Z . Then the mean of Y_n is given by

$$\mathbf{E}[Y_n] = \alpha_0 \mathbf{E}[Z_n] + \alpha_1 \mathbf{E}[Z_{n-1}] \\ = (\alpha_0 + \alpha_1) \mu_Z$$

The autocorrelation of *Y* is given by

$$\mathbf{E}[Y_{n}Y_{m}] = \mathbf{E}[(\alpha_{0}Z_{n} + \alpha_{0}Z_{n-1})(\alpha_{0}Z_{m} + \alpha_{1}Z_{m-1})] \\ = \alpha_{0}^{2}\mathbf{E}[Z_{n}Z_{m}] + \alpha_{1}\alpha_{0}\mathbf{E}[Z_{n}Z_{m-1}] + \alpha_{0}\alpha_{1}\mathbf{E}[Z_{n-1}Z_{m}] + \alpha_{1}^{2}\mathbf{E}[Z_{m-1}Z_{n-1}] \\ = \alpha_{0}^{2}\sigma^{2}\delta(n-m) + \alpha_{1}\alpha_{0}\sigma^{2}\delta(m-1-n) + \alpha_{0}\alpha_{1}\sigma^{2}\delta(n-1-m) + \alpha_{1}^{2}\delta(m-1-(n-1)) \\ = (\alpha_{0}^{2} + \alpha_{1}^{2})\sigma^{2}\delta(n-m) + \alpha_{0}\alpha_{1}\sigma^{2}[\delta(n-m-1) + \delta(m-n-1)]$$

Since the autocorrelation only depends on the time difference n-m, the process is widesense stationary with

$$R_Y(n) = \left(\alpha_0^2 + \alpha_1^2\right)\sigma^2\delta(n) + \alpha_0\alpha_1\sigma^2\left(\delta(n-1) + \delta(n+1)\right)$$

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