Problem 8.15 For the discrete-time process of Problem 8.14, use the discrete Fourier transform to approximate the corresponding spectrum. That is,

$$S_{Y}(k) = \sum_{n=0}^{N-1} R_{Y}(n) W^{kn}$$

If the sampling in the time domain is at n/T_s where n = 0, 1, 2, ..., N-1. What frequency does k correspond to?

Solution

Let $\beta_0 = (\alpha_0^2 + \alpha_1^2)\sigma^2$ and $\beta_1 = \alpha_0\alpha_1\sigma^2$. Then

$$S_{Y}(k) = \sum_{n=0}^{N-1} [\beta_{0}\delta(n) + \beta_{1}(\delta(n-1) + \delta(n+1))] W^{kn}$$

= $\beta_{0}W^{0} + \beta_{1}(W^{-k} + W^{+k})$
= $\beta_{0} + \beta_{1}\left(e^{\frac{-j2\pi k}{N}} + e^{\frac{+j2\pi k}{N}}\right)$
= $\beta_{0} + 2\beta_{1}\cos\left(\frac{2\pi k}{N}\right)$

The term $S_{Y}(k)$ corresponds to frequency $\frac{kf_{s}}{N}$ where $f_{s} = \frac{1}{T_{s}}$.

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