Problem 8.16 Is the discrete-time process $\{Y_n: n = 1,2,\ldots\}$ defined by: $Y_0 = 0$ and

$$Y_{n+1} = \alpha Y_n + W_n,$$

a Gaussian process, if W_n is Gaussian?

Solution

(Proof by mathematical induction.) The first term $Y_1 = \alpha Y_0 + W_0$ is Gaussian since $Y_0 = 0$ and W_0 are Gaussian. The second term $Y_2 = \alpha Y_1 + W_1$ is Gaussian since Y_1 and W_1 are Gaussian. Assume Y_n is Gaussian. Then $Y_{n+1} = \alpha Y_n + W_n$ is Gaussian since Y_n and W_n are both Gaussian.