

**Problem 8.17** A discrete-time white noise process  $\{W_n\}$  has an autocorrelation function given by  $R_W(n) = N_0\delta(n)$ .

- (a) Using the discrete Fourier transform, determine the power spectral density of  $\{W_n\}$ .
- (b) The white noise process is passed through a discrete-time filter having a discrete-frequency response

$$H(k) = \frac{1 - (\alpha W^k)^N}{1 - \alpha W^k}$$

where, for a  $N$ -point discrete Fourier transform,  $W = \exp\{j2\pi/N\}$ . What is the spectrum of the filter output?

### Solution

The spectrum of the discrete white noise process is

$$\begin{aligned} S(k) &= \sum_{n=0}^{N-1} R(n) W^{nk} \\ &= \sum_{n=0}^{N-1} N_0 \delta(n) W^{nk} \\ &= N_0 \end{aligned}$$

The spectrum of the process after filtering is

$$\begin{aligned} S_Y(k) &= |H(k)|^2 S(k) \\ &= N_0 \left| \frac{1 - (\alpha W^k)^N}{1 - \alpha W^k} \right|^2 \end{aligned}$$