Problem 8.2 Suppose the packet of the Problem 8.1 includes an error-correcting code that can correct up to three errors located anywhere in the packet. What is the probability that a particular packet is received in error in this case?

Solution

The probability of a packet error is equal to the probability of more than three bit errors. This is equivalent to 1 minus the probability of 0, 1, 2, or 3 errors:

$$
\begin{aligned}
1-\mathbf{P}[x\le 3] &= 1-\left(\mathbf{P}[x=0]+\mathbf{P}[x=1]+\mathbf{P}[x=2]+\mathbf{P}[x=3]\right)\\
&= 1-(1-p)^n-\binom{n}{1}p(1-p)^{n-1}-\binom{n}{2}p^2(1-p)^{n-2}-\binom{n}{3}p^3(1-p)^{n-3}\\
&= 1-(1-p)^{n-3}\left[(1-p)^3+np(1-p)^2+\frac{n(n-1)}{2}p^2(1-p)+\frac{n(n-1)(n-2)}{6}p^3\right]\\
&= 5.5\times 10^{-5}
\end{aligned}
$$

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