Problem 8.22 Consider a random variable *X* defined by the double-exponential density where *a* and *b* are constants.

$$f_x(x) = a \exp(-b|x|) \qquad -\infty < x < \infty$$

(a) Determine the relationship between a and b so that $f_X(x)$ is a probability density function.

(**b**) Determine the corresponding distribution function $F_X(x)$.

(c) Find the probability that the random variable *X* lies between 1 and 2.

Solution

(a)

$$\int_{-\infty}^{\infty} f_x(x)dx = 1 \implies 2\int_{0}^{\infty} a \exp(-bx)dx = 1$$
$$-\frac{2a}{b} \exp(-bx)\Big|_{0}^{\infty} = 1$$
$$\implies \frac{2a}{b} = 1 \quad or \quad b = 2a$$

(b)

$$F_{x}(x) = \int_{-\infty}^{x} a \exp(-b|s|) ds$$

$$= \begin{cases} -\frac{a}{b} \exp(-b(-s)) \Big|_{-\infty}^{x} - \infty < x < 0 \\ \frac{1}{2} + -\frac{a}{b} \exp(-bs) \Big|_{0}^{x} \quad 0 < x < \infty \end{cases}$$

$$= \begin{cases} \frac{a}{b} \exp(bs) & -\infty < x < 0 \\ \frac{1}{2} + \frac{a}{b} - \frac{a}{b} \exp(-bs) \quad 0 \le x < \infty \end{cases}$$

$$= \begin{cases} \frac{1}{2} \exp(bx) & -\infty < x < 0 \\ 1 - \frac{1}{2} \exp(-bx) & 0 \le x < \infty \end{cases}$$

(c) The probability that $1 \le X \le 2$ is

$$F_{X}(2) - F_{X}(1) = \frac{1}{2} [\exp(-b) - \exp(-2b)]$$

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