Problem 8.23 Show that the expression for the variance of a random variable can be expressed in terms of the first and second moments as

$$\mathrm{Var}(X) = \mathbf{E}\left[X^2\right] - \left(\mathbf{E}[X]\right)^2$$

Solution

$$\begin{aligned}
\mathrm{Var}(X) &= \mathbf{E}\left[\left(X - \mathbf{E}(X)\right)^2\right] \\
&= \mathbf{E}\left(X^2 - 2X\mathbf{E}(X) + \left(\mathbf{E}[X]\right)^2\right) \\
&= \mathbf{E}\left[X^2\right] - 2\mathbf{E}[X]\mathbf{E}[X] + \left(\mathbf{E}[X]\right)^2 \\
&= \mathbf{E}\left[X^2\right] - \left(\mathbf{E}[X]\right)^2
\end{aligned}$$