**Problem 8.26** A Gaussian random variable has zero mean and a standard deviation of 10 V. A constant voltage of 5 V is added to this random variable.

**(a)** Determine the probability that a measurement of this composite signal yields a positive value. **(b)** Determine the probability that the arithmetic mean of two independent measurements of this signal is positive.

## **Solution**

(a) Let *Z* represent the initial Gaussian random variable and *Y* the composite random variable. Then

$$
Y=5+Z
$$

and the density function of *Y* is given by

$$
f_{y}(y) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\left(y - \mu\right)^{2} \left/2\sigma^{2}\right\}\right\}
$$

where  $\mu$  corresponds to a mean of 5V and  $\sigma$  corresponds to a standard deviation of 10V. The probability that *Y* is positive is

$$
P[Y > 0] = \frac{1}{\sqrt{2\pi}\sigma} \int_0^\infty \exp\left\{-\left(y - \mu\right)/2\sigma^2\right\} dy
$$

$$
= \frac{1}{\sqrt{2\pi}} \int_{-\frac{\mu}{\sigma}}^\infty \exp\left(-\frac{s^2}{2}\right) ds
$$

$$
= Q\left(\frac{-\mu}{\sigma}\right)
$$

where, in the second line, we have made the substitution

$$
s=\frac{y-\mu}{\sigma}
$$

Making the substitutions for  $\mu$  and  $\sigma$ , we have  $P[Y>0] = Q(-1/2)$ . We note that in Fig. 8.11, the values of  $Q(x)$  are not shown for negative x; to obtain a numerical result, we use the fact that  $O(-x) = 1 - O(x)$ . Consequently,  $O(-\frac{1}{2}) = 1 - 0.3 = 0.7$ .

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## **Problem 8.26 continued**

(b) Let *W* represent the arithmetic mean of two measurements  $Y_1$  and  $Y_2$ , that is

$$
W = \frac{Y_1 + Y_2}{2}
$$

It follows that *W* is a Gaussian random variable with  $\mathbf{E}[W] = \mathbf{E}[Y] = 5$ . The variance of *W* is given by

$$
\begin{aligned} \mathbf{Var}(W) &= \mathbf{E} \Big[ (W - \mathbf{E}(W))^2 \Big] \\ &= \mathbf{E} \Big[ \Big( \frac{Y_1 + Y_2}{2} - \frac{\mathbf{E}(Y_1) + \mathbf{E}(Y_2)}{2} \Big)^2 \Big] \\ &= \frac{1}{4} \Big( \mathbf{E} \Big[ (Y_1 - \mathbf{E}(Y_1))^2 + (Y_2 - \mathbf{E}(Y_2))^2 + 2(Y_1 - \mathbf{E}(Y_1)) (Y_2 - \mathbf{E}(Y_2)) \Big] \Big) \end{aligned}
$$

The first two terms correspond to the variance of *Y*. The third term is zero because the measurements are independent. Making these substitutions, the variance of *W* reduces to

$$
Var[W] = \frac{\sigma^2}{2}
$$

Using the result of part (a), we then have

$$
\mathbf{P}[W > 0] = Q \left( \frac{-\mu}{\left( \frac{\sigma}{\sqrt{2}} \right)} \right) = Q \left( -\frac{1}{\sqrt{2}} \right)
$$

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