**Problem 8.26** A Gaussian random variable has zero mean and a standard deviation of 10 V. A constant voltage of 5 V is added to this random variable.

(a) Determine the probability that a measurement of this composite signal yields a positive value.(b) Determine the probability that the arithmetic mean of two independent measurements of this signal is positive.

## **Solution**

(a) Let *Z* represent the initial Gaussian random variable and *Y* the composite random variable. Then

$$Y = 5 + Z$$

and the density function of Y is given by

$$f_{y}(y) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left\{-\left(y-\mu\right)^{2}/2\sigma^{2}\right\}$$

where  $\mu$  corresponds to a mean of 5V and  $\sigma$  corresponds to a standard deviation of 10V. The probability that Y is positive is

$$P[Y > 0] = \frac{1}{\sqrt{2\pi\sigma}} \int_0^\infty \exp\{-(y-\mu)/2\sigma^2\} dy$$
$$= \frac{1}{\sqrt{2\pi}} \int_{\frac{-\mu}{\sigma}}^\infty \exp\left(-\frac{s^2}{2}\right) ds$$
$$= Q\left(\frac{-\mu}{\sigma}\right)$$

where, in the second line, we have made the substitution

$$s = \frac{y - \mu}{\sigma}$$

Making the substitutions for  $\mu$  and  $\sigma$ , we have  $\mathbf{P}[Y>0] = Q(-\frac{1}{2})$ . We note that in Fig. 8.11, the values of Q(x) are not shown for negative x; to obtain a numerical result, we use the fact that Q(-x) = 1 - Q(x). Consequently,  $Q(-\frac{1}{2}) = 1 - 0.3 = 0.7$ .

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## **Problem 8.26 continued**

(b) Let W represent the arithmetic mean of two measurements  $Y_1$  and  $Y_2$ , that is

$$W = \frac{Y_1 + Y_2}{2}$$

It follows that *W* is a Gaussian random variable with  $\mathbf{E}[W] = \mathbf{E}[Y] = 5$ . The variance of *W* is given by

$$Var(W) = \mathbf{E} \Big[ (W - \mathbf{E}(W))^2 \Big]$$
  
=  $\mathbf{E} \Big[ \Big( \frac{Y_1 + Y_2}{2} - \frac{\mathbf{E}(Y_1) + \mathbf{E}(Y_2)}{2} \Big)^2 \Big]$   
=  $\frac{1}{4} \Big( \mathbf{E} \Big[ (Y_1 - \mathbf{E}(Y_1))^2 + (Y_2 - \mathbf{E}(Y_2))^2 + 2(Y_1 - \mathbf{E}(Y_1))(Y_2 - \mathbf{E}(Y_2)) \Big] \Big)$ 

The first two terms correspond to the variance of Y. The third term is zero because the measurements are independent. Making these substitutions, the variance of W reduces to

$$\operatorname{Var}[W] = \frac{\sigma^2}{2}$$

Using the result of part (a), we then have

$$\mathbf{P}[W > 0] = Q\left(\frac{-\mu}{\left(\frac{\sigma}{\sqrt{2}}\right)}\right) = Q\left(-\frac{1}{\sqrt{2}}\right)$$

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