

Problem 8.27 Consider a random process defined by

$$X(t) = \sin(2\pi Wt)$$

in which the frequency W is a random variable with the probability density function

$$f_w(w) = \begin{cases} \frac{1}{B} & 0 < w < B \\ 0 & \text{otherwise} \end{cases}$$

Show that $X(t)$ is nonstationary.

Solution

At time $t = 0$, $X(0) = 0$ and the distribution of $X(0)$ is

$$F_{X(0)}(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases}$$

At time $t = 1$, $X(1) = \sin(2\pi w)$, and the distribution of $X(1)$ is clearly not a step function so

$$F_{X(1)}(x) \neq F_{X(0)}(x)$$

And the process $X(t)$ is not first-order stationary, and hence nonstationary.