Problem 8.29 A random process is defined by

$$X(t) = A\cos(2\pi f_c t)$$

where A is a Gaussian random variable of zero mean and variance σ^2 . This random process is applied to an ideal integrator, producing an output Y(t) defined by

$$Y(t) = \int_{0}^{t} X(\tau) d\tau$$

(a) Determine the probability density function of the output at a particular time.

(**b**) Determine whether or not is stationary.

Solution

(a) The output process is given by $Y(t) = \int_{0}^{t} Y(\tau) d\tau$

$$Y(t) = \int_0^t X(\tau) d\tau$$
$$= \int_0^t A \cos(2\pi f_c \tau) d\tau$$
$$= \frac{A}{2\pi f_c} \sin(2\pi f_c t)$$

At time t_0 , it follows that $Y(t_0)$ is Gaussian with zero mean, and variance

$$\frac{\sigma^2}{\left(2\pi f_c\right)^2}\sin^2\left(2\pi f_c t_0\right)$$

(b) No, the process Y(t) is not stationary as $F_{Y(t_0)} \neq F_{Y(t_1)}$ for all t_1 and t_0 .

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