Problem 8.29 A random process is defined by

$$X(t) = A\cos(2\pi f_c t)$$

where A is a Gaussian random variable of zero mean and variance σ^2. This random process is applied to an ideal integrator, producing an output $Y(t)$ defined by

$$Y(t) = \int_0^t X(\tau)d\tau$$

(a) Determine the probability density function of the output at a particular time.
(b) Determine whether or not is stationary.

Solution

(a) The output process is given by

$$Y(t) = \int_0^t X(\tau)d\tau$$
$$= \int_0^t A\cos(2\pi f_c \tau)d\tau$$
$$= \frac{A}{2\pi f_c}\sin(2\pi f_c t)$$

At time t_0, it follows that $Y(t_0)$ is Gaussian with zero mean, and variance

$$\frac{\sigma^2}{(2\pi f_c)^2}\sin^2(2\pi f_c t_0)$$

(b) No, the process $Y(t)$ is not stationary as $F_{Y(t_0)} \neq F_{Y(t_1)}$ for all t_1 and t_0.