Problem 8.3 Continuing with Example 8.6, find the following conditional probabilities: $\mathbf{P}[X=0|Y=1]$ and $\mathbf{P}[X=1|Y=0]$.

Solution

From Bayes' Rule

$$\mathbf{P}[X=0|Y=1] = \frac{\mathbf{P}[Y=1|X=0]\mathbf{P}[X=0]}{\mathbf{P}[Y=1]} = \frac{pp_0}{pp_0 + (1-p)p_1}$$

$$\mathbf{P}[X=1|Y=0] = \frac{\mathbf{P}[Y=0|X=1]\mathbf{P}[X=1]}{\mathbf{P}[Y=0]} = \frac{pp_1}{pp_1 + (1-p)p_0}$$