Problem 8.35 Consider a wide-sense stationary process X(t) having the power spectral density $S_X(f)$ shown in Fig. 8.26. Find the autocorrelation function $R_X(\tau)$ of the process X(t).

Solution

The Wiener-Khintchine relations imply the autocorrelation is given by the inverse Fourier transform of the power spectral density, thus

$$R(\tau) = \int_{-\infty}^{\infty} S_X(f) \exp(j2\pi f t) df$$
$$= \int_{0}^{1} (1 - f) \cos(2\pi f t) df$$

where we have used the symmetry properties of the spectrum to obtain the second line. Integrating by parts, we obtain

$$R_X(\tau) = (1 - f) \frac{\sin(2\pi f\tau)}{2\pi\tau} \Big|_0^1 + \int_0^1 \frac{\sin(2\pi f\tau)}{2\pi\tau} df$$
$$= 0 + \frac{-\cos(2\pi f\tau)}{(2\pi\tau)^2} \Big|_0^1$$
$$= \frac{1 - \cos(2\pi \tau)}{(2\pi\tau)^2}$$

Using the half-angle formula $\sin^2(\theta) = \frac{1}{2}(1-\cos(2\theta))$, this result simplifies to

$$R_X(\tau) = \frac{2\sin^2(\pi\tau)}{(2\pi\tau)^2}$$
$$= \frac{1}{2}\operatorname{sinc}^2(\tau)$$

where $\operatorname{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$.

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