Problem 8.36 The power spectral density of a random process $X(t)$ is shown in Fig. 8.27.

(a) Determine and sketch the autocorrelation function $R_X(\tau)$ of the $X(t)$.
(b) What is the dc power contained in $X(t)$?
(c) What is the ac power contained in $X(t)$?
(d) What sampling rates will give uncorrelated samples of X(t)? Are the samples statistically independent?

Solution

(a) Using the results of Problem 8.35, and the linear properties of the Fourier transform

$$R(\tau) = 1 + \tfrac{1}{2}\,\mathrm{sinc}^2(f_0\tau)$$

(b) The *dc* power is given by power centered on the origin

$$\begin{aligned} dc\ \text{power} &= \lim_{\delta\to 0}\int_{-\varepsilon}^{\varepsilon} S_X(f)\,df \\ &= \lim_{\varepsilon\to 0}\int_{-\varepsilon}^{\varepsilon} \delta(f)\,df \\ &= 1 \end{aligned}$$

(c) The *ac* power is the total power minus the dc power

$$\begin{aligned} ac\ power &= R_X(0) - dc\ \text{power} \\ &= R_X(0) - 1 \\ &= \tfrac{1}{2} \end{aligned}$$

(d) The correlation function $R_X(\tau)$ is zero if samples are spaced at multiples of $1/f_0$.