Problem 8.37 Consider the two linear filters shown in cascade as in Fig. 8.28. Let $X(t)$ be a stationary process with autocorrelation function $R_X(\tau)$. The random process appearing at the first filter output is $V(t)$ and that at the second filter output is $Y(t)$.

(a) Find the autocorrelation function of $V(t)$.
(b) Find the autocorrelation function of $Y(t)$.

Solution

Expressing the first filtering operation in the frequency domain, we have

$$V(f) = H_1(f)X(f)$$

where $H_1(f)$ is the Fourier transform of $h_1(t)$. From Eq. (8.87) it follows that the spectrum of $V(t)$ is

$$S_V(f) = |H_1(f)|^2 S_X(f)$$

By analogy, we have

$$S_Y(f) = |H_2(f)|^2 S_V(f)$$

$$= |H_2(f)|^2 |H_1(f)|^2 S_X(f)$$

Consequently, apply the convolution properties of the Fourier transform, we have

$$R_Y(\tau) = g_2(t) * g_1(t) * R_X(f)$$

where * denotes convolution; $g_2(t)$ and $g_1(t)$ are the inverse Fourier transforms of $|H_2(f)|^2$ and $|H_1(f)|^2$, respectively.