Problem 8.37 Consider the two linear filters shown in cascade as in Fig. 8.28. Let X(t) be a stationary process with autocorrelation function $R_X(\tau)$. The random process appearing at the first filter output is V(t) and that at the second filter output is Y(t).

(a) Find the autocorrelation function of V(t).

(b) Find the autocorrelation function of Y(t).

Solution

Expressing the first filtering operation in the frequency domain, we have

$$V(f) = H_1(f)X(f)$$

where $H_1(f)$ is the Fourier transform of $h_1(t)$. From Eq. (8.87) it follows that the spectrum of V(t) is

$$S_V(f) = |H_1(f)|^2 S_X(f)$$

By analogy, we have

$$S_{Y}(f) = |H_{2}(f)|^{2} S_{V}(f)$$
$$= |H_{2}(f)|^{2} |H_{1}(f)|^{2} S_{X}(f)$$

Consequently, apply the convolution properties of the Fourier transform, we have $R_Y(\tau) = g_2(t) * g_1(t) * R_X(f)$

where * denotes convolution; $g_2(t)$ and $g_1(t)$ are the inverse Fourier transforms of $|H_2(f)|^2$ and $|H_1(f)|^2$, respectively.

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