Problem 8.39 Assume the narrow-band process X(t) described in Problem 8.38 is Gaussian with zero mean and variance σ_X^2 .

- (a) Calculate σ_X^2 .
- (b) Determine the joint probability density function of the random variables Y and Z obtained by observing the in-phase and quadrature components of X(t) at some fixed time.

Solution

(a) The variance is given by

$$\sigma_X^2 = R(0) = \int_{-\infty}^{\infty} S(f) df$$
$$= 2\left(\frac{1}{2}b_1h_1 + \frac{1}{2}b_2h_2\right)$$
$$= 2\left(\frac{1}{2}\cdot 2\cdot 1 + \frac{1}{2}\cdot 1\cdot 1\right)$$
$$= 3 watts$$

(b) The random variables Y and Z have zero mean, are Gaussian and have variance σ_X^2 . If Y and Z are independent, the joint density is given by

$$f_{Y,Z}(Y,Z) = \frac{1}{\sqrt{2\pi\sigma_X}} \exp\left(\frac{-\frac{y^2}{2\sigma_X^2}}{\frac{1}{\sqrt{2\pi\sigma_X}}} \exp\left(\frac{-\frac{z^2}{2\sigma_X^2}}{\frac{1}{\sqrt{2\pi\sigma_X}}}\right)$$
$$= \frac{1}{2\pi\sigma_X^2} \exp\left(-\frac{y^2 + z^2}{2\sigma_X^2}\right)$$

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