Problem 8.4 Consider a binary symmetric channel for which the conditional probability of error $p = 10^{-4}$, and symbols 0 and 1 occur with equal probability. Calculate the following probabilities:

- a) The probability of receiving symbol 0.
- b) The probability of receiving symbol 1.
- c) The probability that symbol 0 was sent, given that symbol 0 is received
- d) The probability that symbol 1 was sent, given that symbol 0 is received.

Solution

(a)

$$\mathbf{P}[Y=0] = \mathbf{P}[Y=0 \mid X=0]\mathbf{P}[X=0] + \mathbf{P}[Y=0 \mid X=1]\mathbf{P}[X=1]$$

= (1-p)p₀ + pp₁
= .9999 $\frac{1}{2}$ + .0001 $\frac{1}{2}$
= $\frac{1}{2}$

(b)

$$\mathbf{P}[Y=1] = 1 - \mathbf{P}[Y=0]$$
$$= \frac{1}{2}$$

(c) From Eq.(8.30)

$$\mathbf{P}[X = 0|Y = 0] = \frac{(1-p)p_0}{(1-p)p_0 + pp_1}$$

$$= \frac{(1-10^{-4})\frac{1}{2}}{(1-10^{-4})\frac{1}{2} + 10^{-4}\frac{1}{2}}$$

$$= 1 - 10^{-4}$$

(d) From Prob. 8.3

$$\mathbf{P}[X = 1|Y = 0] = \frac{pp_1}{pp_1 + (1 - p)p_0}$$
$$= \frac{10^{-4} \frac{1}{2}}{10^{-4} \frac{1}{2} + (1 - 10^{-4}) \frac{1}{2}}$$
$$= 10^{-4}$$

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