

Problem 8.4 Consider a binary symmetric channel for which the conditional probability of error $p = 10^{-4}$, and symbols 0 and 1 occur with equal probability. Calculate the following probabilities:

- The probability of receiving symbol 0.
- The probability of receiving symbol 1.
- The probability that symbol 0 was sent, given that symbol 0 is received
- The probability that symbol 1 was sent, given that symbol 0 is received.

Solution

(a)

$$\begin{aligned} \mathbf{P}[Y = 0] &= \mathbf{P}[Y = 0 | X = 0]\mathbf{P}[X = 0] + \mathbf{P}[Y = 0 | X = 1]\mathbf{P}[X = 1] \\ &= (1 - p)p_0 + pp_1 \\ &= .9999 \frac{1}{2} + .0001 \frac{1}{2} \\ &= \frac{1}{2} \end{aligned}$$

(b)

$$\begin{aligned} \mathbf{P}[Y = 1] &= 1 - \mathbf{P}[Y = 0] \\ &= \frac{1}{2} \end{aligned}$$

(c) From Eq.(8.30)

$$\begin{aligned} \mathbf{P}[X = 0|Y = 0] &= \frac{(1 - p)p_0}{(1 - p)p_0 + pp_1} \\ &= \frac{(1 - 10^{-4})\frac{1}{2}}{(1 - 10^{-4})\frac{1}{2} + 10^{-4}\frac{1}{2}} \\ &= 1 - 10^{-4} \end{aligned}$$

(d) From Prob. 8.3

$$\begin{aligned} \mathbf{P}[X = 1|Y = 0] &= \frac{pp_1}{pp_1 + (1 - p)p_0} \\ &= \frac{10^{-4}\frac{1}{2}}{10^{-4}\frac{1}{2} + (1 - 10^{-4})\frac{1}{2}} \\ &= 10^{-4} \end{aligned}$$