**Problem 8.40** Find the probability that the last two digits of the cube of a natural number (1, 2, 3, ...) will be 01.

## **Solution**

Let a natural number be represented by concatenation xy where y represents last two digits and x represents the other digits. For example, the number 1345 has y = 45 and x = 13. Then

$$(xy)^3 = (x00 + y)^3 = (x^3000000) + 3(x^20000)y + 3(x00)y^2 + y^3$$

where we have used the binomial expansion of  $(a+b)^3$ . The last digits of the first three terms on the right are clearly 00. Consequently, it is the last two digits of  $y^3$  which determines the last two digits of  $(xy)^3$ . Checking the cube of all two digit numbers for 00 to 99, we find that: (a)  $y^3$  ends in 1, only if y ends in 1; and (b) only the number  $(01)^3$  gives 01 as the last two digits. From this counting argument, the probability is 0.01.

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