

Problem 8.41 Consider the random experiment of selecting a number uniformly distributed over the range $\{1, 2, 3, \dots, 120\}$. Let A , B , and C be the events that the selected number is a multiple of 3, 4, and 6, respectively.

- a) What is the probability of event A , i.e. $\mathbf{P}[A]$?
- b) What is $\mathbf{P}[B]$?
- c) What is $\mathbf{P}[A \cap B]$?
- d) What is $\mathbf{P}[A \cup B]$?
- e) What is $\mathbf{P}[A \cap C]$?

Solution

(a) From a counting argument, $\mathbf{P}(A) = \frac{40}{120} = \frac{1}{3}$

(b) $\mathbf{P}(B) = \frac{30}{120} = \frac{1}{4}$

(c) $\mathbf{P}(A \cap B) = \frac{12}{120} = \frac{1}{10}$

(d)
$$\begin{aligned}\mathbf{P}(A \cup B) &= \mathbf{P}(A) + \mathbf{P}(B) - \mathbf{P}(A \cap B) \\ &= \frac{1}{3} + \frac{1}{4} - \frac{1}{10} = \frac{20 + 15 - 6}{60} = \frac{29}{60}\end{aligned}$$

(e) $\mathbf{P}(A \cap C) = \mathbf{P}(C) = \frac{20}{120} = \frac{1}{6}$