Problem 8.42 A message consists of ten "0"s and "1"s.

- a) How many such messages are there?
- b) How many such messages are there that contain exactly four "1"s?
- c) Suppose the 10th bit is not independent of the others but is chosen such that the modulo-2 sum of all the bits is zero. This is referred to as an even parity sequence. How many such even parity sequences are there?
- d) If this ten-bit even-parity sequence is transmitted over a channel that has a probability of error p for each bit. What is the probability that the received sequence contains an undetected error?

<u>Solution</u>

(a) A message corresponds to a binary number of length 10, there are thus 2^{10} possibilities.

(b) The number of messages with four "1"s is

$$\binom{10}{4} = \frac{10!}{4!6!} = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2} = 10 \times 3 \times 7 = 210$$

(c) Since there are only 9 independent bits in this case, the number of such message is 2^9 .

(d) The probability of an undetected error corresponds to the probability of 2, 4, 6, 8, or 10 errors. The received message corresponds to a Bernoulli sequence, so the corresponding error probabilities are given by the binomial distribution and is

$$\binom{10}{2}p^{2}(1-p)^{8} + \binom{10}{4}p^{4}(1-p)^{6} + \binom{10}{6}p^{6}(1-p)^{4} + \binom{10}{8}p^{8}(1-p)^{2} + \binom{10}{10}p^{10}$$

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