Problem 8.42 A message consists of ten "0"s and "1"s.

a) How many such messages are there?
b) How many such messages are there that contain exactly four "1"s?
c) Suppose the 10th bit is not independent of the others but is chosen such that the modulo-2 sum of all the bits is zero. This is referred to as an even parity sequence. How many such even parity sequences are there?
d) If this ten-bit even-parity sequence is transmitted over a channel that has a probability of error p for each bit. What is the probability that the received sequence contains an undetected error?

Solution

(a) A message corresponds to a binary number of length 10, there are thus 2^{10} possibilities.

(b) The number of messages with four "1"s is

$$\binom{10}{4} = \frac{10!}{4!6!} = \frac{10\times 9\times 8\times 7}{4\times 3\times 2} = 10\times 3\times 7 = 210$$

(c) Since there are only 9 independent bits in this case, the number of such message is 2^9.

(d) The probability of an undetected error corresponds to the probability of 2, 4, 6, 8, or 10 errors. The received message corresponds to a Bernoulli sequence, so the corresponding error probabilities are given by the binomial distribution and is

$$\binom{10}{2}p^2(1-p)^8 + \binom{10}{4}p^4(1-p)^6 + \binom{10}{6}p^6(1-p)^4 + \binom{10}{8}p^8(1-p)^2 + \binom{10}{10}p^{10}$$