Problem 8.44 The arrival times of two signals at a receiver are uniformly distributed over the interval [0,T]. The receiver will be jammed if the time difference in the arrivals is less than τ . Find the probability that the receiver will be jammed.

Solution

Let *X* and *Y* be random variables representing the arrival times of the two signals. The probability density functions of the random variables are

$$f_{X}(x) = \begin{cases} \frac{1}{T} & 0 < x < T \\ 0, & \text{otherwise} \end{cases}$$

and $f_Y(y)$ is similarly defined. Then the probability that the time difference between arrivals is less than τ is given by

$$\mathbf{P}[|X - Y| < \tau] = \mathbf{P}[|X - Y| < \tau \mid X > Y]\mathbf{P}[X > Y] + \mathbf{P}[|X - Y| < \tau \mid Y > X]\mathbf{P}[Y > X]$$
$$= \mathbf{P}[|X - Y| < \tau \mid X > Y]$$

where the second line follows from the symmetry between the random variables *X* and *Y*, namely, $\mathbf{P}[\underline{X} > Y] = \mathbf{P}[Y > X]$. If we only consider the case X > Y, then we have the conditions: 0 < X < T and $0 < Y < X < \tau + Y$. Combining these conditions we have $Y < X < \min(T, \tau + Y)$. Consequently,

$$\mathbf{P}[[X - Y] < \tau] = \int_{0}^{T} \int_{y}^{\min(T, \tau + y)} f_{X}(x) f_{Y}(y) dx dy$$
$$= \int_{0}^{T} \int_{y}^{\min(T, \tau + y)} \left(\frac{1}{T}\right)^{2} dx dy$$
$$= \frac{1}{T^{2}} \int_{0}^{T} \{\min(T, \tau + y) - y\} dy$$

Combining the two terms of the integrand,

$$P[[X - Y] < \tau] = \frac{1}{T^2} \int_0^T \min(T - y, \tau) dy$$
$$= \frac{1}{T^2} \min\left(Ty - \frac{y^2}{2}, \tau y\right)_0^T$$
$$= \min\left(\frac{1}{2}, \frac{\tau}{T}\right)$$

Excerpts from this work may be reproduced by instructors for distribution on a not-for-profit basis for testing or instructional purposes only to students enrolled in courses for which the textbook has been adopted. Any other reproduction or translation of this work beyond that permitted by Sections 107 or 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful. page...8-54