Problem 8.45 A telegraph system (an early version of digital communications) transmits either a dot or dash signal. Assume the transmission properties are such that 2/5 of the dots and 1/3 of the dashes are received incorrectly. Suppose the ratio of transmitted dots to transmitted dashes is 5 to 3. What is the probability that a received signal as the transmitted if:

a) The received signal is a dot?

b) The received signal is a dash?

Solution

(a) Let X represent the transmitted signal and Y represent the received signal. Then by application of Bayes' rule

$$\begin{aligned}\mathbf{P}(Y = \text{dot}) &= \mathbf{P}(X = \text{dot} \mid \text{No error})\mathbf{P}(\text{No dot error}) + \mathbf{P}(X = \text{dash} \mid \text{error})\mathbf{P}(\text{dash error}) \\ &= \tfrac{5}{8}\left(\tfrac{3}{5}\right) + \left(\tfrac{3}{8}\right)\left(\tfrac{1}{3}\right) \\ &= \tfrac{3}{8} + \tfrac{1}{8} = \tfrac{1}{2}\end{aligned}$$

(b) Similarly,

$$\begin{aligned}\mathbf{P}[Y = \text{dash}] &= \mathbf{P}[X = \text{dash} \mid \text{no error}]\mathbf{P}(\text{no dash error}) + \mathbf{P}(X = \text{dot})\mathbf{P}[\text{dot error}] \\ &= \tfrac{3}{8} \cdot \tfrac{2}{3} + \left[\tfrac{5}{8}\right]\tfrac{2}{5} \\ &= \tfrac{2}{8} + \tfrac{2}{8} = \tfrac{1}{2}\end{aligned}$$