Problem 8.46 Four radio signals are emitted successively. The probability of reception for each of them is independent of the reception of the others and equal, respectively, 0.1, 0.2, 0.3 and 0.4. Find the probability that k signals will be received where k = 1, 2, 3, 4.

Solution

For one successful reception, the probability is given by the sum of the probabilities of the four mutually exclusive cases

$$\begin{aligned} P = {} & p_1(1-p_2)(1-p_3)(1-p_4)+ \\ & (1-p_1)p_2(1-p_3)(1-p_4)+ \\ & (1-p_1)(1-p_2)p_3(1-p_4)+ \\ & (1-p_1)(1-p_2)(1-p_3)p_4 \\ = {} & .1\cdot.8\cdot.7\cdot.6+.9\cdot.2\cdot.7\cdot.6+.9\cdot.8\cdot.3\cdot.6+.9\cdot.8\cdot.7\cdot.4 \\ = {} & 0.4404 \end{aligned}$$

For k = 2, there six mutually exclusive cases

$$\begin{aligned} P = {} & p_1p_2(1-p_3)(1-p_4)+ \\ & p_1(1-p_2)p_3(1-p_4)+ \\ & p_1(1-p_2)(1-p_3)p_4+ \\ & (1-p_1)p_2p_3(1-p_4)+ \\ & (1-p_1)p_2(1-p_3)p_4+ \\ & (1-p_1)(1-p_2)p_3p_4 \\ = {} & .1\cdot.2\cdot.7\cdot.6+.1\cdot.8\cdot.3\cdot.6+.1\cdot.8\cdot.7\cdot.4+.9\cdot.2\cdot.3\cdot.6+.9\cdot.2\cdot.7\cdot.4+.9\cdot.8\cdot.3\cdot.4 \\ = {} & 0.2144 \end{aligned}$$

For k =3 there are four mutually exclusive cases

$$\begin{aligned} P = {} & p_1p_2p_3(1-p_4)+ \\ & p_1(1-p_2)p_3p_4+ \\ & p_1p_2(1-p_3)p_4+ \\ & (1-p_1)p_2p_3p_4 \\ = {} & .1\cdot.2\cdot.3\cdot.6+.1\cdot.8\cdot.3\cdot.4+.1\cdot.2\cdot.7\cdot.4+.9\cdot.2\cdot.3\cdot.4 \\ = {} & 0.0404 \end{aligned}$$

For k = 4 there is only one term

$$\begin{aligned} P &= p_1p_2p_3p_4 \\ &= .1\cdot.2\cdot.3\cdot.4 \\ &= 0.0024 \end{aligned}$$