Problem 8.46 Four radio signals are emitted successively. The probability of reception for each of them is independent of the reception of the others and equal, respectively, 0.1, 0.2, 0.3 and 0.4. Find the probability that *k* signals will be received where k = 1, 2, 3, 4.

Solution

For one successful reception, the probability is given by the sum of the probabilities of the four mutually exclusive cases

$$P = p_1(1 - p_2)(1 - p_3)(1 - p_4) + (1 - p_1)p_2(1 - p_3)(1 - p_4) + (1 - p_1)(1 - p_2)p_3(1 - p_4) + (1 - p_1)(1 - p_2)(1 - p_3)p_4$$

= .1 \cdot .8 \cdot .7 \cdot .6 + .9 \cdot .2 \cdot .7 \cdot .6 + .9 \cdot .8 \cdot .3 \cdot .6 + .9 \cdot .8 \cdot .7 \cdot .4
= 0.4404

For k = 2, there six mutually exclusive cases

$$P = p_1 p_2 (1 - p_3)(1 - p_4) +$$

$$p_1 (1 - p_2) p_3 (1 - p_4) +$$

$$p_1 (1 - p_2)(1 - p_3) p_4 +$$

$$(1 - p_1) p_2 p_3 (1 - p_4) +$$

$$(1 - p_1) p_2 (1 - p_3) p_4 +$$

$$(1 - p_1)(1 - p_2) p_3 p_4$$

$$= .1 \cdot .2 \cdot .7 \cdot .6 + .1 \cdot .8 \cdot .3 \cdot .6 + .1 \cdot .8 \cdot .7 \cdot .4 + .9 \cdot .2 \cdot .3 \cdot .6 + .9 \cdot .2 \cdot .7 \cdot .4 + .9 \cdot .8 \cdot .3 \cdot .4$$

$$= 0.2144$$

For k = 3 there are four mutually exclusive cases

$$P = p_1 p_2 p_3 (1 - p_4) + p_1 (1 - p_2) p_3 p_4 + p_1 p_2 (1 - p_3) p_4 + (1 - p_1) p_2 p_3 p_4$$

= .1 \cdot .2 \cdot .3 \cdot .6 + .1 \cdot .8 \cdot .3 \cdot .4 + .1 \cdot .2 \cdot .7 \cdot .4 + .9 \cdot .2 \cdot .3 \cdot .4
= 0.0404

For k = 4 there is only one term

$$P = p_1 p_2 p_3 p_4$$
$$= .1 \cdot .2 \cdot .3 \cdot .4$$
$$= 0.0024$$

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