Problem 8.48 If X has a density $f_X(x)$, find the density of Y where

a) $Y = aX + b$ for constants a and b.

b) $Y = X^2$.

c) $Y = \sqrt{X}$, assuming X is a non-negative random variable.

Solution

(a) If $Y = aX + b$, using the results of Section 8.3 for $Y = g(X)$

$$f_Y(y) = f_X\left(g^{-1}(y)\right)\left|\frac{dg^{-1}(y)}{dy}\right|$$

$$= f_X\left(\frac{y-b}{a}\right)\frac{1}{a}$$

(b) If $Y = X^2$, then

$$f_Y(y) = \left(f_X\left(-\sqrt{y}\right) + f_X\left(+\sqrt{y}\right)\right)\left(\frac{1}{2\sqrt{y}}\right)$$

(c) If $Y = \sqrt{X}$, then we must assume X is positive valued so, this is a one-to-one mapping and

$$f_Y(y) = f_X\left(y^2\right)\cdot 2y$$