**Problem 8.5** Determine the mean and variance of a random variable that is uniformly distributed between *a* and *b*.

## <u>Solution</u>

The mean of the uniform distribution is given by

$$\mu = \mathbf{E}[X] = \int_{-\infty}^{\infty} x f_X(x) dx$$
$$= \int_a^b x \frac{1}{b-a} dx$$
$$= \frac{x^2}{2(b-a)} \Big|_a^b$$
$$= \frac{b^2 - a^2}{2(b-a)}$$
$$= \frac{b+a}{2}$$

The variance is given by

$$\mathbf{E}[(X-\mu)^{2}] = \int_{-\infty}^{\infty} (x-\mu)^{2} f_{X}(x) dx$$
$$= \int_{a}^{b} \frac{(x-\mu)^{2}}{b-a} dx$$
$$= \frac{1}{b-a} \frac{(b-\mu)^{3}}{3} - \frac{(a-\mu)^{3}}{3}$$

If we substitute  $\mu = \frac{b+a}{2}$  then

$$\mathbf{E}[(X-\mu)^2] = \frac{1}{b-a} \left[ \frac{(b-a)^3}{24} - \frac{(a-b)^3}{24} \right]$$
$$= \frac{(b-a)^2}{12}$$

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