Problem 8.5 Determine the mean and variance of a random variable that is uniformly distributed between *a* and *b*.

Solution

The mean of the uniform distribution is given by

$$\begin{aligned}\mu = \mathbf{E}[X] &= \int_{-\infty}^{\infty} x f_X(x)dx \\ &= \int_a^b x \frac{1}{b-a} dx \\ &= \left.\frac{x^2}{2(b-a)}\right|_a^b \\ &= \frac{b^2-a^2}{2(b-a)} \\ &= \frac{b+a}{2}\end{aligned}$$

The variance is given by

$$\begin{aligned}\mathbf{E}\left[(X-\mu)^2\right] &= \int_{-\infty}^{\infty} (x-\mu)^2 f_X(x)dx \\ &= \int_a^b \frac{(x-\mu)^2}{b-a} dx \\ &= \frac{1}{b-a}\,\frac{(b-\mu)^3}{3} - \frac{(a-\mu)^3}{3}\end{aligned}$$

If we substitute $\mu = \frac{b+a}{2}$ then

$$\begin{aligned}\mathbf{E}\left[(X-\mu)^2\right] &= \frac{1}{b-a}\left[\frac{(b-a)^3}{24} - \frac{(a-b)^3}{24}\right] \\ &= \frac{(b-a)^2}{12}\end{aligned}$$