Problem 8.50 Find the spectral density $S_Z(f)$ if

$$Z(t) = X(t)Y(t)$$

where X(t) and Y(t) are independent zero-mean random processes with

$$R_X(\tau) = a_1 e^{-\alpha_1|\tau|}$$
 and $R_Y(\tau) = a_2 e^{-\alpha_2|\tau|}$.

Solution

The autocorrelation of Z(t) is given by

$$R_{Z}(\tau) = \mathbf{E}[Z(t)Z(t+\tau)]$$

= $\mathbf{E}[X(t)X(t+\tau)Y(t)Y(t+\tau)]$
= $\mathbf{E}[X(t)X(t+\tau)]\mathbf{E}[Y(t)Y(t+\tau)]$
= $R_{X}(\tau)R_{Y}(\tau)$

By the Wiener-Khintchine relations, the spectrum of Z(t) is given by

$$S_{Z}(f) = \mathbf{F}^{-1}[R_{X}(\tau)R_{Y}(\tau)]$$

= $\mathbf{F}^{-1}[a_{1}a_{2}\exp(-(\alpha_{1} + \alpha_{2})|\tau|)]$
= $\frac{2a_{1}a_{2}(\alpha_{1} + \alpha_{2})}{(\alpha_{1} + \alpha_{2})^{2} + (2\pi f)^{2}}$

where the last line follows from the Fourier transform of the double-sided exponential (See Example 2.3).

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