Problem 8.51 Consider a random process $X(t)$ defined by

$$X(t) = \sin(2\pi f_c t)$$

where the frequency f_c is a random variable uniformly distributed over the interval $[0,W]$. Show that $X(t)$ is nonstationary. *Hint*: Examine specific sample functions of the random process $X(t)$ for, say, the frequencies $W/4$, $W/2$, and W.

Solution

To be stationary to first order implies that the mean value of the process $X(t)$ must be constant and independent of t. In this case,

$$\begin{aligned} \mathbf{E}[X(t)] &= \mathbf{E}[\sin(2\pi f_c t)] \\ &= \frac{1}{W}\int_0^W \sin(2\pi w t)\,dw \\ &= \left.\frac{-\cos(2\pi w t)}{2\pi W t}\right|_0^W \\ &= \frac{1-\cos(2\pi W t)}{2\pi W t} \end{aligned}$$

This mean value clearly depends on t, and thus the process $X(t)$ is nonstationary.