Problem 8.6 Let X be a random variable and let $Y = (X-\mu_X)/\sigma_X$. What is the mean and variance of the random variable Y?

Solution

$$\mathbf{E}[Y] = \mathbf{E}\left[\frac{X-\mu_X}{\sigma_X}\right] = \frac{\mathbf{E}[X]-\mu_X}{\sigma_X} = \frac{0}{\sigma_X} = 0$$

$$\mathbf{E}(Y-\mu_Y)^2 = \mathbf{E}[Y^2] = \mathbf{E}\left(\frac{X-\mu_X}{\sigma_X}\right)^2$$

$$= \frac{\mathbf{E}(X-\mu_X)^2}{{\sigma_X}^2} = \frac{{\sigma_X}^2}{{\sigma_X}^2} = 1$$