Problem 8.7 What is the probability density function of the random variable *Y* of Example 8.8? Sketch this density function.

Solution

From Example 8.8, the distribution of *Y* is

$$F_Y(y)=\begin{cases}0 & y<-1\\ \dfrac{2\pi-2\cos^{-1}(y)}{2\pi} & |y|<1\\ 1 & y>1\end{cases}$$

Thus, the density of *Y* is given by

$$\frac{dF_Y(y)}{dy}=\begin{cases}0 & y<-1\\ \dfrac{1}{\pi\sqrt{1-y^2}} & |y|<1\\ 0 & y>1\end{cases}$$

This density is sketched in the following figure.