Problem 8.9 Show that for a Gaussian random variable *X* with mean μ_X and variance σ_X^2 the transformation $Y = (X - \mu_X)/\sigma_X$, converts *X* to a normalized Gaussian random variable.

Solution

Let $y = \frac{x - \mu_X}{\sigma_X}$. Then

$$\mathbf{E}[Y] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} y \exp\left(-\frac{y^2}{2}\right) dy$$
$$= 0$$

by the odd symmetry of the integrand. If $\mathbf{E}[Y] = 0$, then from the definition of *Y*, $\mathbf{E}[X] = \mu_X$. In a similar fashion

$$\mathbf{E}[Y^{2}] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} y^{2} \exp\left(-\frac{y^{2}}{2}\right) dy$$
$$= \frac{(-)y}{\sqrt{2\pi}} \cdot \exp\left\{-\frac{y^{2}}{2}\right\} \Big|_{-\infty}^{\infty} + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-\frac{y^{2}}{2}\right) dy$$
$$= 1$$

where we use integration by parts as in Problem 8.8. This result implies

$$E\left(\frac{x-\mu_X}{\sigma_X}\right)^2 = 1$$

and hence $\mathbf{E}(x-\mu_x)^2 = \sigma_x^2$

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