Problem 9.1 In practice, we often cannot measure the signal by itself but must measure the signal plus noise. Explain how the SNR would be calculated in this case.

Solution

Let $r(t) = s(t) + n(t)$ be the received signal plus noise. Assuming the signal is independent of the noise, we have that the received power is

$$\begin{aligned} R_0 &= \mathbf{E}\left[r^2(t)\right] \\ &= \mathbf{E}\left[\left(s(t)+n(t)\right)^2\right] \\ &= \mathbf{E}\left[s^2(t)\right] + 2\mathbf{E}\left[s(t)n(t)\right] + \mathbf{E}\left[n^2(t)\right] \\ &= \mathbf{E}\left[s^2(t)\right] + 2\mathbf{E}\left[s(t)\right]\mathbf{E}\left[n(t)\right] + \mathbf{E}\left[n^2(t)\right] \\ &= S + 0 + N \end{aligned}$$

where S is the signal power and N is the average noise power. We then measure the noise alone

$$\begin{aligned} R_1 &= \mathbf{E}\left[n^2(t)\right] \\ &= N \end{aligned}$$

and the SNR is given by

$$\text{SNR} = \frac{R_0 - R_1}{R_1}$$