Problem 9.13 Assume a message signal $m(t)$ has power spectral density

$$S_M(f) = \begin{cases} a\dfrac{|f|}{W} & |f| \le W \\ 0 & \text{otherwise} \end{cases}$$

where a and W are constants. Find the expression for post-detection SNR of the receiver when

(a) The signal is transmitted by DSB-SC.
(b) The signal is transmitted by envelope modulation with $k_a = 0.3$.
(c) The signal is transmitted with frequency modulation with k_f = 500 hertz per volt.

Assume that white Gaussian noise of zero mean and power spectral density $N_0/2$ is added to the signal at the receiver input.

Solution

(a) with DSB-SC modulation and detection, the post-detection SNR is given by

$$SNR^{DSB} = \frac{A_c^2 P}{2N_0 W}$$

For the given message spectrum, the power is

$$\begin{aligned} P &= \int_{-\infty}^{\infty} S_M(f)df \\ &= 2\int_0^W a\frac{f}{W}df \\ &= aW \end{aligned}$$

where we have used the even-symmetry of the message spectrum on the second line. Consequently, the post-detection SNR is

$$SNR^{DSB} = \frac{A_c^2 a}{2N_0}$$

(b) for envelope detection with $k_a = 0.3$, the post-detection SNR is

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Problem 9.13 continued

$$\text{SNR}^{\text{AM}} = \frac{A_c^2 k_a^2 P}{2N_0 W}$$
$$= \frac{A_c^2 a}{2N_0} k_a^2$$
$$= 0.09 \frac{A_c^2 a}{2N_0}$$

(c) for frequency modulation and detection with k_f = 500 Hz/V, the post-detection SNR is

$$\text{SNR}^{\text{FM}} = \frac{3A_c^2 k_f^2 P}{2N_0 W^3}$$
$$= \frac{A_c^2 a}{2N_0} 3\left(\frac{k_f}{W}\right)^2$$